Gold Level

Model Answers 6

Level	IGCSE
Subject	Maths
Exam Board	Edexcel
Difficulty Level	Gold
Booklet	Model Answers 6

Time Allowed: 57 minutes

Score: / 47

Percentage: /100

1 Show that $(6 - \sqrt{8})^2 = 44 - 24\sqrt{2}$

Show each stage of your working clearly.

Foil

$$(6 - \sqrt{8})(6 - \sqrt{8})$$

$$36 - 6\sqrt{8} - 6\sqrt{8} + 8$$

$$44 - 12\sqrt{4 \times 2} \rightarrow 44 - 2(12)\sqrt{2} \rightarrow 44 - 24\sqrt{2}$$
$$QED$$

(Total for Question is 3 marks)

2 Solve $\dfrac{5}{(x+2)} + \dfrac{9}{(x-2)} = 2$

Show clear algebraic working.

$$5 + \frac{9(x+2)}{(x-2)} = 2 \; (x+2)(x-2)$$

$$5(x-2) + 9(x+2) = 2(x^2 - 4)$$

$$5x - 10 + 9x + 18 = 2x^2 - 8$$

$$14x + 8 = 2x^2 - 8$$

$$2x^2 - 14x - 16 = 0$$

$$x^2 - 7x - 8 = 0$$

$$(x + 1)(x - 8) = 0$$

$$x = -1, 8$$

$$x = -1, x = 8$$
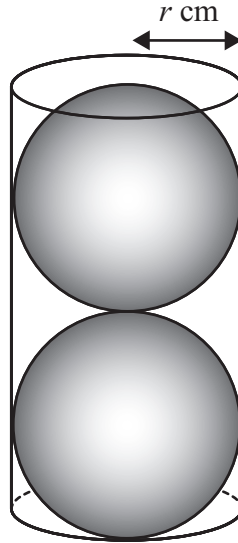
(Total for Question is 5 marks)

3

r cm

Diagram **NOT**
accurately drawn

Two solid spheres, each of radius *r* cm, fit exactly inside a hollow cylinder.
The radius of the cylinder is *r* cm.
The height of the cylinder is equal to 4*r* cm.

The volume of the space inside the cylinder, not occupied by the spheres, is $\frac{125}{6}\pi$ cm³

Calculate the value of *r*.

Show your working clearly.

$$\text{Vol of spheres} = \text{Vol of C} - \frac{125}{6}\pi$$

$$V_S = \pi r^2 (4r) - \frac{125}{6}\pi$$

$$2\left(\frac{4}{3}\pi r^3\right) = 4\pi r^3 - \frac{125}{6}\pi \rightarrow \frac{125}{6}\pi = 4\pi r^3 - \frac{8}{3}\pi r^3$$

$$\rightarrow 125 = 24 r^3 - 16 r^3 = 8r^3$$

$$r^3 = \frac{125}{8} \qquad \therefore r = \sqrt[3]{\frac{125}{8}} = 2.5$$

r = 2.5
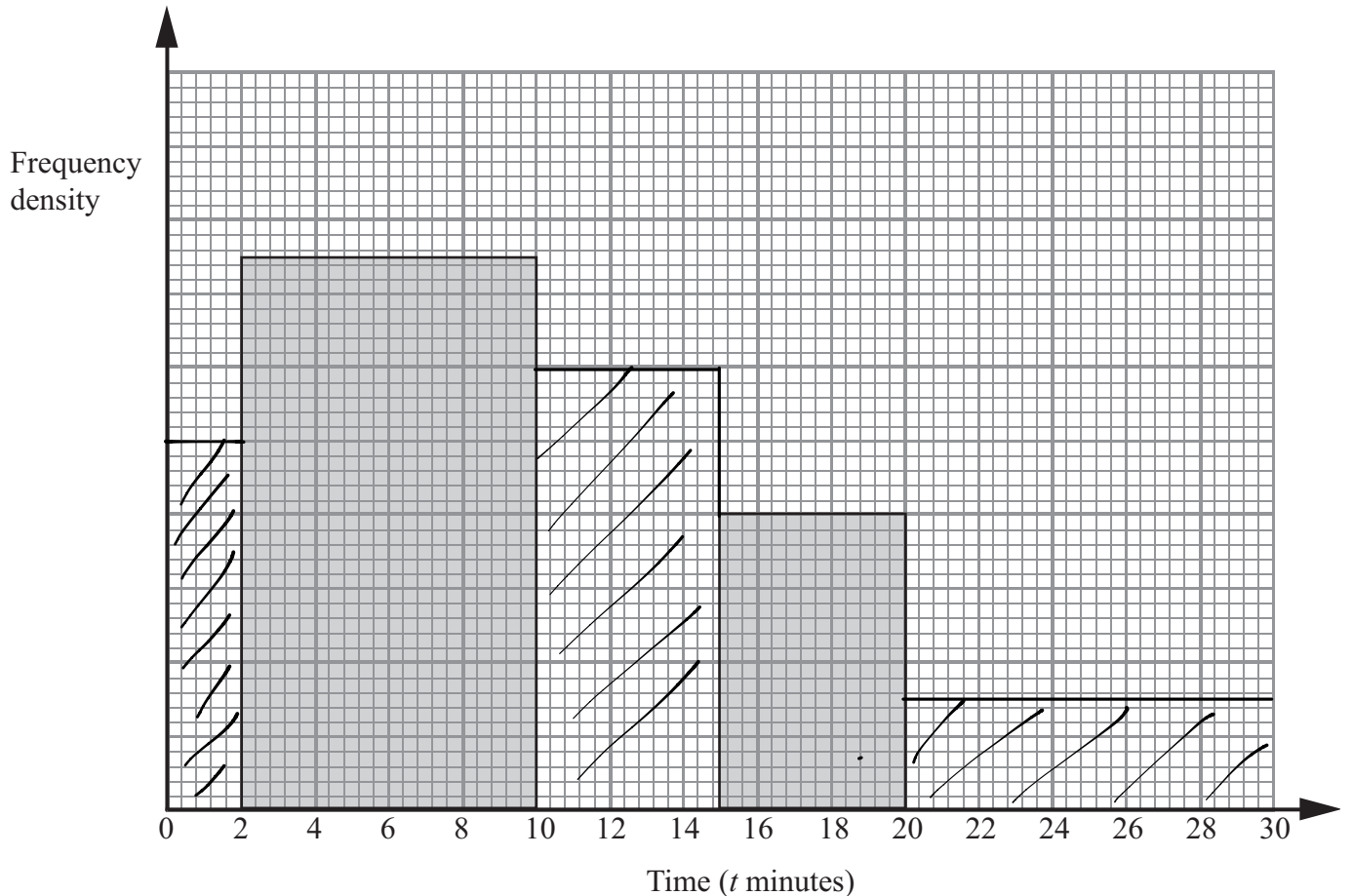
(Total for Question is 5 marks)

4 The incomplete table and histogram show information about the lengths of time, *t* minutes, students spent waiting for their school bus one morning.

Time (*t* minutes)	Number of students
$0 < t \leqslant 2$	20
$2 < t \leqslant 10$	120
$10 < t \leqslant 15$	60
$15 < t \leqslant 20$	40
$20 < t \leqslant 30$	30

(i) Use the histogram to complete the table.

5 × 8

(ii) Use the table to complete the histogram.
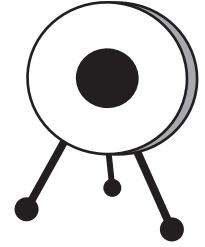

Time (*t* minutes)

(Total for Question is 4 marks)

5 A target has a black circle and a white region.
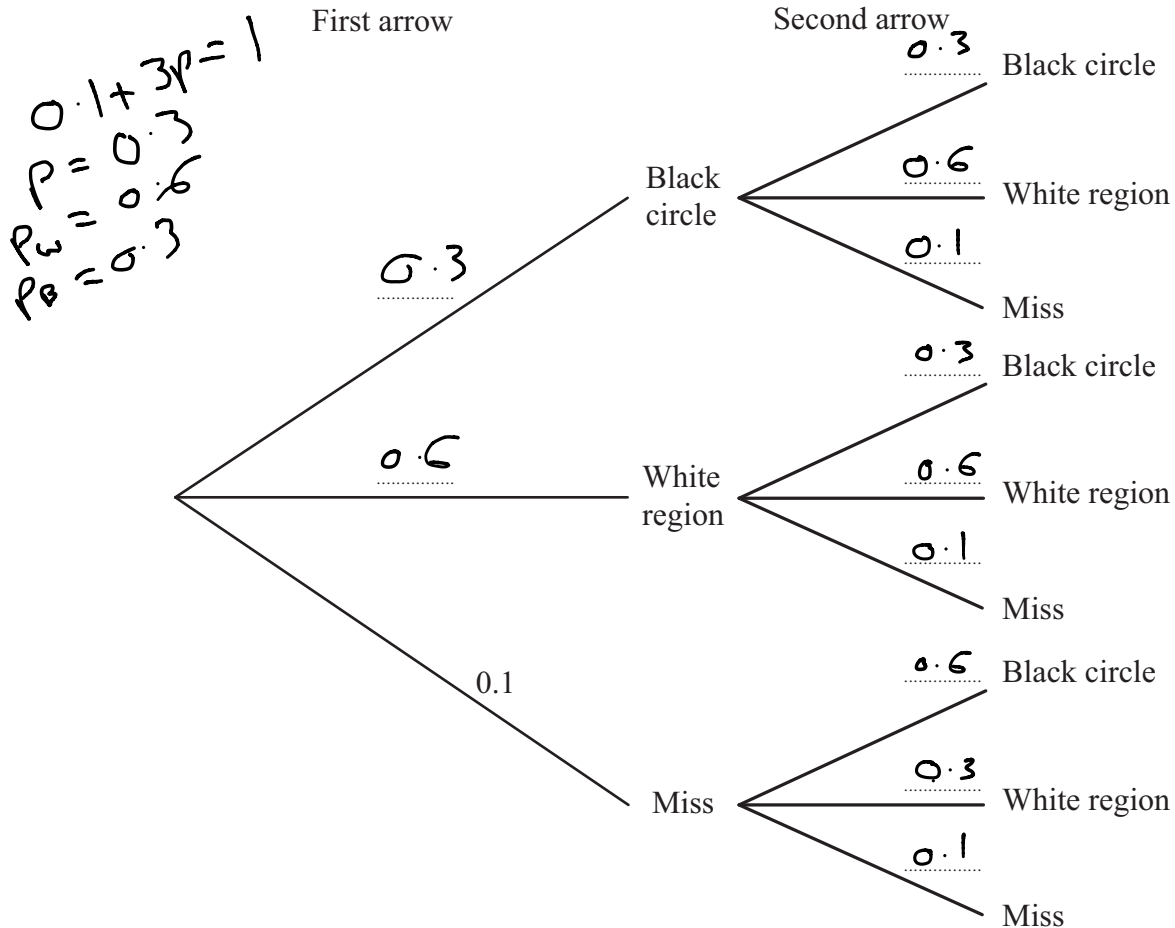Arrows can hit the black circle, the white region or miss the target.

Peter shoots two arrows at the target.
On each shot, the probability that Peter's arrow misses the target is 0.1
On each shot, the probability that Peter's arrow hits the white region is twice the probability that it hits the black circle.

(a) Complete the probability tree diagram for Peter's two arrows.

First arrow   Second arrow

$0.1 + 3p = 1$
$P = 0.3$
$= 0.6$
$Pw = 0.3$
$PB = 0.3$

Black circle — 0.3 — Black circle
— 0.6 — White region
— 0.1 — Miss

0.3 (first) → Black circle

White region — 0.6 →
  0.3 Black circle
  0.6 White region
  0.1 Miss

0.6 (first) → White region

0.1 → Miss

Miss — 0.6 Black circle
  0.3 White region
  0.1 Miss

(3)

(b) An arrow which hits the black circle scores 10 points.
An arrow which hits the white region scores 5 points.
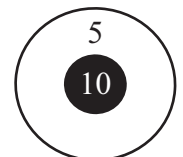An arrow which misses the target scores 0 points.

5
10

Calculate the probability that Peter scores exactly 10 points with his 2 arrows.

Black → miss
white → white
miss → black

0.3×0.1
$+ 0.6 \times 0.6$
$+ 0.1 \times 0.3$
$= 0.42$

0.42

(3)

(Total for Question is 6 marks)

6 Simplify fully $\dfrac{4x^2-25}{6x^2+13x-5}$

Difference of two squares numerator:

$(2x-5)(2x+5)$

Denominator:

$(2x+5)(3x-1)$

$\dfrac{(2x+5)(2x-5)}{(2x+5)(3x-5)} = \dfrac{(2x-5)}{(3x-1)}$

$\dfrac{2x-5}{3x-1}$

(Total for Question is 3 marks)

7 (a) Differentiate with respect to x

    (i) $8x^2$

$$\frac{dy}{dx} = 2(8) x^1$$

                                             $16x$

    (ii) $\frac{2}{x}$ $= 2x^{-1}$

$$\frac{dy}{dx} = (-1)(2) x^{-2}$$

$$= \frac{-2}{x^2}$$

                                             $-2/x^2$

                                              (3)

    (b) The curve with equation $y = 8x^2 + \dfrac{2}{x}$ has one turning point.

    Find the coordinates of this turning point.
    Show your working clearly.

$$\frac{dy}{dx} = 16x - \frac{2}{x^2}$$

$$0 = 16x - \frac{2}{x^2} \quad \rightarrow \quad 8x - \frac{1}{x^2} \neq 0$$

$$8x^3 - 1 = 0 \quad \rightarrow \quad 8x^3 = 1 \quad \therefore x = \frac{1}{2}$$

Sub $x = \frac{1}{2}$ back in $\rightarrow \quad x = \frac{1}{2} \quad y = 6$

                                                     (0.5 , 6)

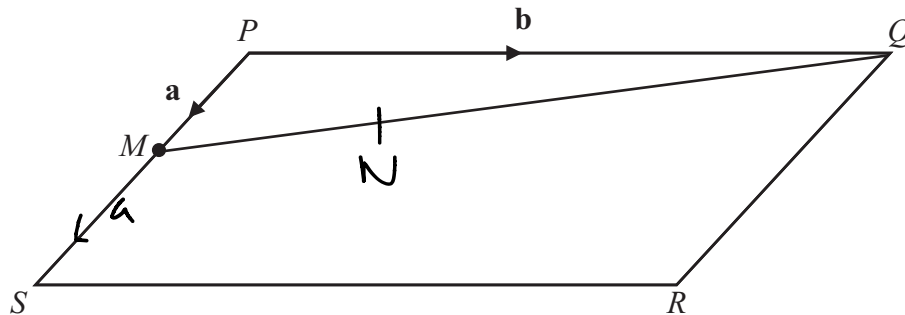   **(4)**

                                        (Total for Question is 7 marks)

8 The diagram shows a parallelogram, *PQRS*.

M is the midpoint of *PS*.

Diagram **NOT** accurately drawn

$\overrightarrow{PM} = \mathbf{a}$ $\overrightarrow{PQ} = \mathbf{b}$

(a) Find, in terms of **a** and/or **b**,

(i) $\overrightarrow{PS}$

$2\underline{a}$

(ii) $\overrightarrow{PR}$

$2\underline{a} + \underline{b}$

$2\underline{a} + \underline{b}$

(iii) $\overrightarrow{MQ}$

$-\underline{a} + \underline{b}$

$-\underline{a} + \underline{b}$

(3)

N is the point on *MQ* such that $MN = \dfrac{1}{3}MQ$

(b) Use a vector method to prove that *PNR* is a straight line.

$\overrightarrow{NR} = \dfrac{2}{3}(-\underline{a} + \underline{b}) + 2\underline{a}$

$\overrightarrow{PN} = a + \dfrac{1}{3}(-a + b)$

$\overrightarrow{NR} = \dfrac{2}{3}(2\underline{a} + \underline{b})$ $\rightarrow$ $\overrightarrow{NR} = \dfrac{2}{3}\overrightarrow{PR}$

$\overrightarrow{PN} = \dfrac{2a}{3} + \dfrac{b}{3} = \dfrac{1}{3}(2a + b)$

① $\overrightarrow{PN} = \dfrac{1}{3}PR$

① $\overrightarrow{NR} = \dfrac{2}{3}\overrightarrow{PR}$

∴ $\overrightarrow{PN} + \overrightarrow{NR} = \overrightarrow{PR}$

(2)

(Total for Question is 5 marks)

9 The diagram shows a pyramid with a horizontal rectangular base *PQRS*.
  PQ = 16 cm.
  QR = 10 cm.
  M is the midpoint of the line *PR*.
  The vertex, *T*, is vertically above *M*.
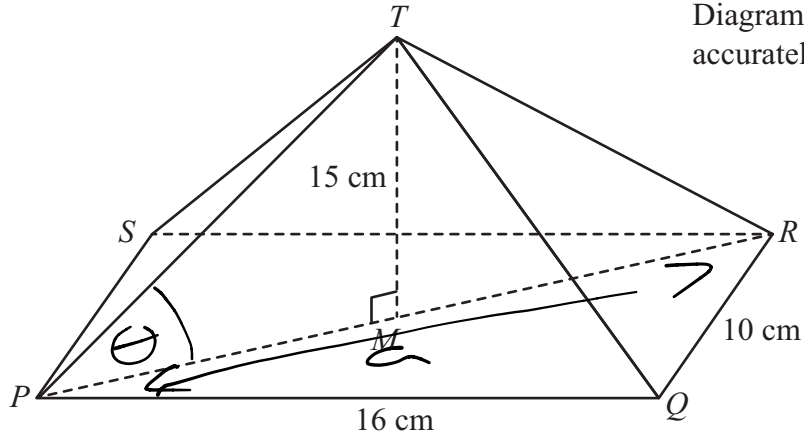 *MT* = 15cm.

Diagram **NOT**
accurately drawn

T

15 cm

S ---------------- R

10 cm

P

16 cm

Q

Calculate the size of the angle between *TP* and the base *PQRS*.
Give your answer correct to 1 decimal place.

$$a^2 = 16^2 + 10^2$$

$$a^2 = \sqrt{16^2 + 10^2}$$

S⁰H C⁴H T⁰a

$$\tan\theta = \frac{15}{\left(\frac{\sqrt{16^2+10^2}}{2}\right)} = \frac{15}{9.43...} \quad \therefore \theta \approx 57.8°$$
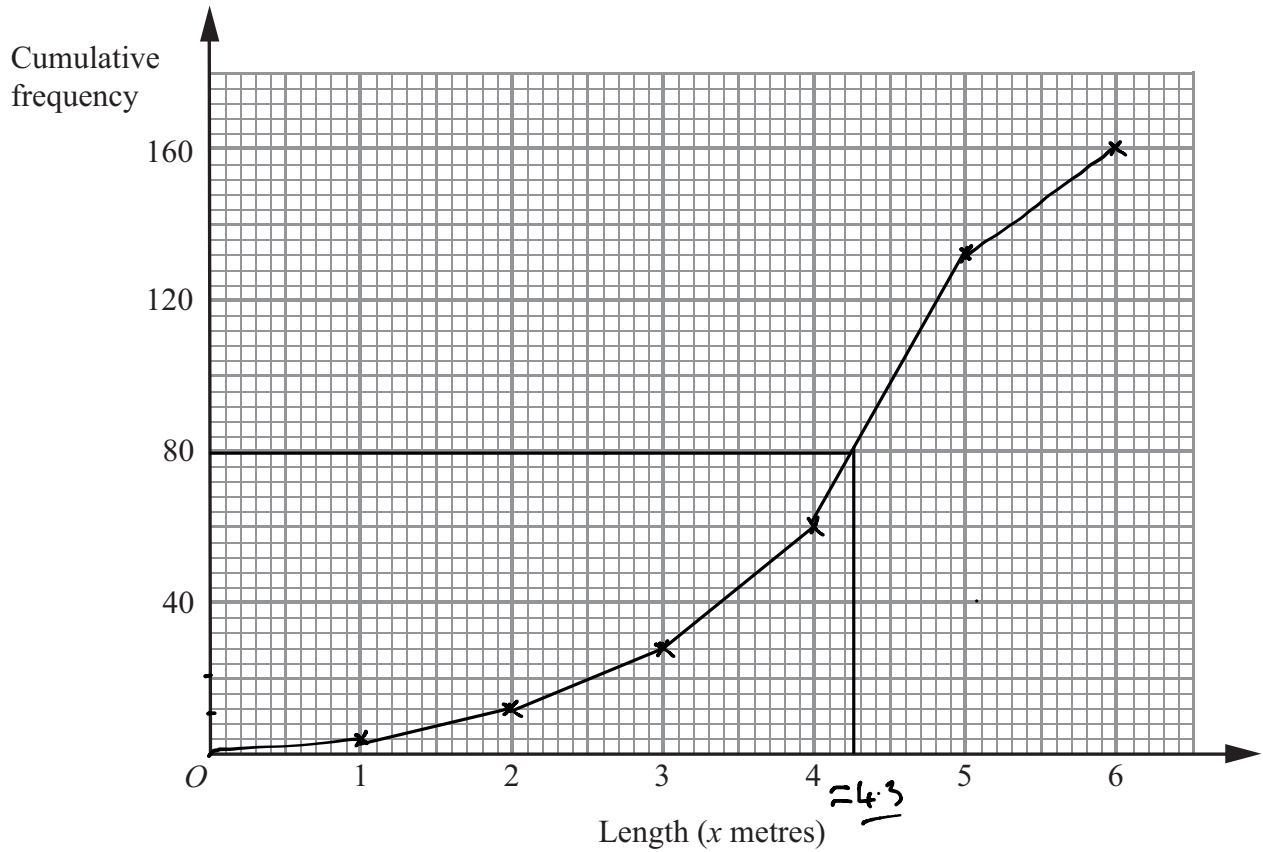
57.8 °

(Total for Question is 4 marks)

10 The grouped frequency table gives information about the lengths of 160 pythons.

Length (*x* metres)	Frequency
$0 < x \leqslant 1$	4
$1 < x \leqslant 2$	8
$2 < x \leqslant 3$	16
$3 < x \leqslant 4$	32
$4 < x \leqslant 5$	72
$5 < x \leqslant 6$	28

(a) Complete the cumulative frequency table.

Length (*x* metres)	Cumulative frequency
$0 < x \leqslant 1$	4
$0 < x \leqslant 2$	12
$0 < x \leqslant 3$	28
$0 < x \leqslant 4$	60
$0 < x \leqslant 5$	132
$0 < x \leqslant 6$	160

(1)

(b) On the grid, draw a cumulative frequency graph for your table.

(2)

(c) Use your graph to find an estimate for the median length of the pythons.

4.3 .... metres

(2)

(Total for Question is 5 marks)