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MATHEMATICS **9709/62**

Paper 6 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

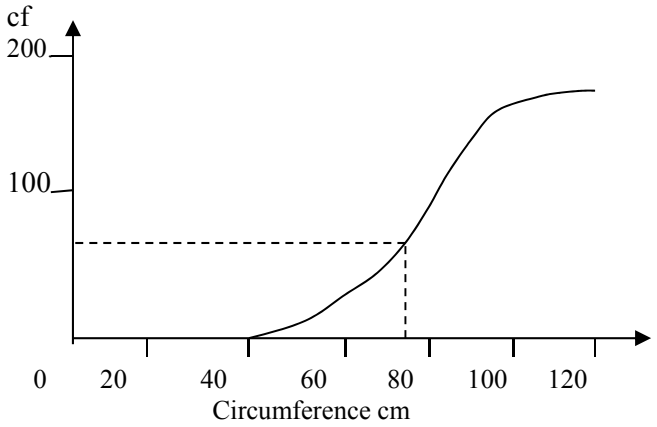
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER:* $(\Sigma x =)\ 11.5n = 27 + 10n$	**(M1**	Expanding brackets and forming a three term equation involving 27 and at least one term in n, without x
		M1	$10n$ or $11.5n$ seen in expression without x ($1.5n = 27$ implies **M2**)
	$n = 18$	**A1)**	
	OR: $11.5 = \frac{27}{n} + 10$	**(M1**	Dividing coded sum by n and forming a three term equation involving 11.5 and at least one term in n, without x
		M1	$27/n$ seen in expression without x ($1.5 = \frac{27}{n}$ implies **M2**)
	$n = 18$	**A1)**	
		3	

Question	Answer	Marks	Guidance
2(i)	points (50, 14), (80, 62), (100, 132), (120, 140)	**B1**	Correct cfs values seen listed, in or by table or on graph, 0 not required
		**B1**	Axes labelled 'cumulative frequency' (or cf) and 'circumference [or cir or c etc.] (in) cm'. Linear scales – c.f. 0–140 circumference 40–120 (ignore <40 on circ.) At least 3 values stated on each axis, but (0,0) can be implied without stating.
		B1	All points plotted accurately
		3	
2(ii)	140 – 54 = 86	**M1**	Finding correct value from graph (checked ±1 mm) or linear interpolation. Subtraction from 140 can be implied
	Percentage = 61.4%	**A1**	60.5% ⩽ Ans ⩽ 64.5%
		2	

Question	Answer	Marks	Guidance
3(i)	*EITHER:* $P(X = 3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4}$	**(M1**	probabilities in order $\frac{2}{p} \times \frac{1}{q} \times \frac{4}{r}$, $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$, accept $\times 1$ as $\frac{4}{r}$.
	$= \frac{1}{15}$ AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
	OR1: $P(X = 3) = P(RRB) = \frac{{}^2C_2}{{}^6C_2} \times \frac{{}^4C_1}{{}^4C_1}$	**(M1**	probabilities stated clearly, $\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ included
	$= \frac{1}{15}$ AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
	OR2: $P(X = 3) = P(RRB) = \frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^5C_1} \times \frac{{}^4C_1}{{}^4C_1}$	**(M1**	probabilities in order $\frac{{}^2C_1}{{}^pC_1} \times \frac{{}^1C_1}{{}^qC_1} \times \frac{{}^4C_1}{{}^rC_1}$ $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$ ($\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ acceptable)
	= 1/15 AG	**A1)**	Needs either P(RRB) OE stated or identified on tree diagram.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">3(ii)</td><td rowspan="2">$P(1) = P(B) = \frac{4}{6}\ (\frac{2}{3} = 0.667)$
$P(2) = P(RB) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}\ (= 0.267)$
$P(3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} = \frac{1}{15}\ (= 0.0667)$
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P</td><td>$\frac{10}{15}$</td><td>$\frac{4}{15}$</td><td>$\frac{1}{15}$</td></tr></table></td><td>B1</td><td>Probability distribution table drawn with at least 2 correct x values and at least 1 probability. All probabilities $0 \leqslant p < 1$.</td></tr>
<tr><td>B1</td><td>P(1) or P(2) correct unsimplified, or better, and identified.</td></tr>
<tr><td>B1</td><td>All probabilities in table, evaluated correctly OE. Additional x values must have a stated probability of 0</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(i)	$P(4, 2H) = \frac{1}{4} \times {}^4C_2 \times (\frac{1}{3})^2(\frac{2}{3})^2$	**M1**	Multiplying their 2H expression by ¼ [P(4)]
		M1	Remaining factor is $(\frac{1}{3})^2(\frac{2}{3})^2$ [or $\frac{4}{81}$] multiplied by integer value $k \geqslant 1$ OE
	$= \frac{2}{27}$ (0.0741)	**A1**	
		3	
4(ii)	$P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.00926)	**B1**	
		1	
4(iii)	$P(1, 1H) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ (0.08333) $P(2, 2H) = \frac{1}{4} \times (\frac{1}{3})^2 = \frac{1}{36}$ (0.02778) $P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.009259) $P(4, 4H) = \frac{1}{4} \times (\frac{1}{3})^4 = \frac{1}{324}$ (0.003086)	**M1**	Correct expression for 1 of P(1, 1H), P(2, 2H), P(4, 4H) Unsimplified (or better)
		M1	Summing their values for 3 or 4 appropriate outcomes for the 'game' with no additional outcomes.
	Prob = $\frac{10}{81}$ (0.123)	**A1**	
		3	

Question	Answer	Marks	Guidance
5(i)	*EITHER:* P(> 2) = 1 – P(0, 1, 2)	**(M1**	Binomial term of form ${}^{30}C_x p^x (1-p)^{30-x}$, $0 < p < 1$ any p
	$= 1 - (0.96)^{30} - {}^{30}C_1(0.04)(0.96)^{29} - {}^{30}C_2(0.04)^2(0.96)^{28}$ (= 1 – 0.2938… – 0.3673… – 0.2219…)	**A1**	Correct unsimplified answer
	= 1-0.883103 = 0.117 (0.116896)	**A1)**	
	OR: P(> 2) = P(3,4,5,6,….30)	**(M1**	Binomial term of form ${}^{30}C_x p^x (1-p)^{30-x}$, $0 < p < 1$ any p
	$= {}^{30}C_3(0.04)^3(0.96)^{27} + {}^{30}C_4(0.04)^4(0.96)^{26} + \ldots + (0.04)^{30}$	**A1**	Correct unsimplified answer
	= 0.117	**A1)**	
		3	

Question	Answer	Marks	Guidance
5(ii)	$np = 280 \times 0.1169 = 32.73$, $npq = 280 \times 0.1169 \times 0.8831 = 28.9$	**M1 FT**	Correct unsimplified np and npq, FT their p *from* **(i)**,
	$P(\geqslant 30) = P\left(z > \frac{29.5 - 32.73}{\sqrt{28.9}}\right) = P(z > -0.6008)$	**M1**	Substituting *their* μ and σ ($\sqrt{npq}$ only) into the Standardisation Formula
		M1	Using continuity correction of 29.5 or 30.5
		M1	Appropriate area Φ from standardisation formula P(z >….) in final solution
	= 0.726	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)(i)	*EITHER:* 3**, 4**, 6**, 8**	**(M1**	${}^5P_2$ or ${}^5C_2 \times 2!$ or 5×4 OE (considering final 2 digits)
	options $4 \times 5 \times 4 = 80$	**M1**	Mult by 4 or summing 4 options (considering first digit)
		A1)	Correct final answer
	OR: Total number of values: $6 \times 5 \times 4 = 120$	**(M1**	Calculating total number of values (with subtraction seen)
	Number of values less than 300: $2 \times 5 \times 4 = 40$	**M1**	Calculating number of unwanted values
	Number of evens = $120 - 40 = 80$	**A1)**	Correct final answer
		3	

Question	Answer	Marks	Guidance
6(a)(ii)	3**, 4**, 6**, 8** *EITHER:* options 4 × 6 × 4 (last)	**(M1**	6 linked to considering middle digit e.g. multiplied or in list
		M1	Multiply an integer by 4 × 4 (condone × 16) (No additional figures present for both M's to be awarded)
	= 96	**A1)**	
	OR: Total number of values 4 × 6 × 6 = 144	**(M1**	Calculating total number of values (with subtraction seen)
	Number of odd values 4 × 6 × 2 = 48	**M1**	Calculating number of unwanted values
	Number of evens = 144 – 48 = 96	**A1)**	
		3	
6(b)(i)	252	**B1**	
		1	

Question	Answer	Marks	Guidance
6(b)(ii)	B (6)G(4) 5 0 in ${}^6C_5$ (×${}^4C_0$) = 6 × 1 = 6 4 1 in ${}^6C_4$ × ${}^4C_1$ = 15 × 4 = 60 3 2 in ${}^6C_3$ × ${}^4C_2$ = 20 × 6 = 120		
		M1	Multiplying 2 combinations ${}^6C_q$ × ${}^4C_r$, $q + r = 5$, or ${}^6C_5$ seen alone
		M1	Summing 2 or 3 appropriate outcomes, involving perm/comb, no extra outcomes.
	Total = 186 ways	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	$P(> 65) = P\left(z > \frac{65 - 61.4}{12.3}\right) = P(z > 0.2927)$	**M1**	Standardising no continuity correction, no square or square root, condone ± standardisation formula
		M1	Correct area (< 0.5)
	= 1 – 0.6153 = 0.385	**A1**	
		3	

Question	Answer	Marks	Guidance
7(ii)	P (< 65) = 0.6153 so P(< k) = 0.25 + 0.6153 = 0.8653	**B1**	
	$z = 1.105$	**B1**	$z = \pm 1.105$ seen or rounding to 1.1
	$1.105 = \frac{k - 61.4}{12.3}$	**M1**	standardising allow ±, cc, sq rt, sq. Need to see use of tables backwards so must be a z-value, not $1 - z$ value.
	$k = 75.0$	**A1**	Answers which round to 75.0. Condone 75 if supported.
		4	
7(iii)	$2.326 = \frac{97.2 - \mu}{\sigma}$	**B1**	± 2.326 seen (Use of critical value)
	$-0.44 = \frac{55.2 - \mu}{\sigma}$	**B1**	± 0.44 seen
		M1	An equation with a z-value, μ, σ and 97.2 or 55.2, allow $\sqrt{\sigma}$ or σ^2
		M1	Algebraic elimination μ or σ from *their* two simultaneous equations
	$\mu = 61.9$ $\sigma = 15.2$	**A1**	both correct answers
		5	