## Pure Mathematics P4 Mark scheme

| Question | Scheme | Marks |
|---|---|---|
| 1 | $\left\{\dfrac{1}{(2+5x)^3} =\right\}\ (2+5x)^{-3}$ | M1 |
| | $=\underline{(2)^{-3}}\left(1+\dfrac{5x}{2}\right)^{-3} = \underline{\dfrac{1}{8}}\left(1+\dfrac{5x}{2}\right)^{-3}$ | B1 |
| | $=\left\{\dfrac{1}{8}\right\}\left[1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$<br>$=\left\{\dfrac{1}{8}\right\}\left[1+(-3)\left(\dfrac{5x}{2}\right)+\dfrac{(-3)(-4)}{2!}\left(\dfrac{5x}{2}\right)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{5x}{2}\right)^3+\ldots\right]$<br>$=\dfrac{1}{8}\left[1-\dfrac{15}{2}x+\dfrac{75}{2}x^2-\dfrac{625}{4}x^3+\ldots\right]$<br>$=\dfrac{1}{8}[\,1-7.5x+37.5x^2\ -156.25\,x^3+\ldots]$ | M1 A1 |
| | $=\dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$<br>**or** $\dfrac{1}{8}-\dfrac{15}{16}x;+4\dfrac{11}{16}x^2-19\dfrac{17}{32}x^3+\ldots$ | A1 A1 |
| | | **(6)** |

**Notes:**

**M1:** Mark can be implied by a constant term of $(2)^{-3}$ or $\dfrac{1}{8}$.

**B1:** $\underline{2^{-3}}$ or $\underline{\dfrac{1}{8}}$ outside brackets or $\dfrac{1}{8}$ as candidate's constant term in their binomial expansion.

**M1:** Expands $(\ldots+kx)^{-3}$, $k$ = a value $\neq 1$ to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+(-3)(kx)$ or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1+\ldots+\dfrac{(-3)(-4)}{2!}(kx)^2$ or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.

**A1:** A correct simplified or un-simplified $1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent $(kx)$. Note that $(kx)$ must be consistent and $k$ = a value $\neq 1$. (on the RHS, not necessarily the LHS) in a candidate's expansion.

**A1:** For $\dfrac{1}{8}-\dfrac{15}{16}x$ **(simplified)** or also allow $0.125-0.9375x$.

**A1:** Accept only $\dfrac{75}{16}x^2-\dfrac{625}{32}x^3$ or $4\dfrac{11}{16}x^2-19\dfrac{17}{32}x^3$ or $4.6875x^2-19.53125x^3$

| Question | Scheme | Marks |
|---|---|---|
| **2(a)** | $x^3 + 2xy - x - y^3 - 20 = 0$ | |
| | $\left\{\dfrac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}}\cancel{\times}\right\}\quad \underline{3x^2} + \left(\underline{\underline{2y + 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}}}\right)\underline{-1 - 3y^2\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0}$ | M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$ |
| | $3x^2 + 2y - 1 + \left(2x - 3y^2\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ | dM1 |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3x^2 + 2y - 1}{3y^2 - 2x}$ **or** $\dfrac{1 - 3x^2 - 2y}{2x - 3y^2}$ **cso** | A1 |
| | | **(5)** |
| **(b)** | At P(3, −2), $\mathrm{m}(\mathbf{T}) = \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \dfrac{22}{6}$ or $\dfrac{11}{3}$ <br> **and either T**: $y - -2 = \dfrac{"11"}{3}\left(x - 3\right)$ **or** $(-2) = \left(\dfrac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$ | M1 |
| | **T**: $11x - 3y - 39 = 0$ **or** $K(11x - 3y - 39) = 0$ **cso** | A1 |
| | | **(2)** |
| | | **(7 marks)** |

**Notes:**

**(a)**

**M1:** Differentiates implicitly to include either $2y\dfrac{\mathrm{d}x}{\mathrm{d}y}$ or $x^3 \to \pm kx^2\dfrac{\mathrm{d}x}{\mathrm{d}y}$ or $-x \to -\dfrac{\mathrm{d}x}{\mathrm{d}y}$

(Ignore $\left(\dfrac{\mathrm{d}x}{\mathrm{d}y} = \right)$).

**A1:** $x^3 \to 3x^2\dfrac{\mathrm{d}x}{\mathrm{d}y}$ **and** $-x - y^3 - 20 = 0 \to -\dfrac{\mathrm{d}x}{\mathrm{d}y} - 3y^2 = 0$

**B1:** $2xy \to 2y\dfrac{\mathrm{d}x}{\mathrm{d}y} + 2x$

**dM1:** **Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are ***at least two terms*** in $\dfrac{\mathrm{d}x}{\mathrm{d}y}$.

**A1:** For $\dfrac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\dfrac{3x^2 + 2y - 1}{3y^2 - 2x}$

**(b)**

**M1:** **Some** attempt to substitute **both** $x = 3$ **and** $y = -2$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ which contains both $x$ and $y$ to find $m_T$ **and**

- **either** applies $y - -2 = \left(\text{their } m_T\right)\left(x - 3\right)$, where $m_T$ is a numerical value.
- **or** finds $c$ by solving $(-2) = \left(\text{their } m_T\right)(3) + c$, where $m_T$ is a numerical value.

**A1:** Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.

| Question | Scheme | Marks |
|---|---|---|
| 3(a) | $1 = A(3x-1)^2 + Bx(3x-1) + Cx$ | B1 |
| | $x \to 0 \quad (1 = A)$ | M1 |
| | $x \to \frac{1}{3} \quad 1 = \frac{1}{3}C \Rightarrow \; C = 3$ any two constants correct coefficients of $x^2$ | A1 |
| | $0 = 9A + 3B \Rightarrow B = -3$ all three constants correct | A1 |
| | | **(4)** |
| (b)(i) | $\int \left( \frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$<br>$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$<br>$\left( = \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$ | M1<br>A1ft<br>A1ft |
| | | **(3)** |
| (b)(ii) | $\int_1^2 f(x)\,dx = \left[ \ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$ | |
| | $= \left( \ln 2 - \ln 5 - \frac{1}{5} \right) - \left( \ln 1 - \ln 2 - \frac{1}{2} \right)$ | M1 |
| | $= \ln \frac{2 \times 2}{5} + \ldots$ | M1 |
| | $= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$ | A1 |
| | | **(3)** |
| | | **(10 marks)** |

**Notes:**

**(a)**

**B1:** Obtaining $1 = A(3x-1)^2 + Bx(3x-1) + Cx$ at any stage. This will usually be at the beginning of the solution but, if the cover-up rule is used, it could appear later.

**M1:** A complete method of finding any one of the three constants. If either $A = 1$ or $C = 3$ is given without working or, at least, without incorrect working, allow this M1 – use of the cover-up rule is acceptable. In principle, an alternative method is equating coefficients (or substituting three values other than 0 and $\frac{1}{3}$), obtaining a sufficient set of equations and solving for any one of the three constants.

**A1:** Any two of $A$, $B$ and $C$ correct. These will usually, but not always, be $A$ and $C$.

**A1:** All three of $A$, $B$ and $C$ correct. If all three constants are correct and the answers do not clearly conflict with any working, allow all 4 marks (including the B1) bod. There are a number of possible ways of finding $B$ but, as long as the M has been gained, you need not consider the method used.

| **Question 3 notes** *continued* |
|---|
| **(b)(ii)**<br>**M1:** Dependent upon the M mark in (b). Substituting in the correct limits and subtracting, not necessarily the right way round. There must be evidence that both 1 and 2 have been used but errors in substitution do not lose the mark.<br>**M1:** Dependent upon both previous Ms. Applies the addition and/or subtraction rules of logs to obtain a single logarithm. Either the addition or the subtraction rule of logs must be used correctly at least once to gain this mark and this must be seen in the attempt at (b)(ii).<br>**A1:** The correct answer in the form specified. Accept equivalent fractions including exact decimals for *a* and or *b*.<br>Accept $\ln\frac{4}{5}+\frac{3}{10}$.<br>$\frac{3}{10}-\ln\frac{5}{4}$ is not acceptable. |

| Question | Scheme | Marks |
|---|---|---|
| 4(a) | $\dfrac{dx}{dt} = 2\sqrt{3}\cos 2t$ | B1 |
| | $\dfrac{dy}{dt} = -8\cos t\sin t$ | M1 A1 |
| | $\dfrac{dy}{dx} = \dfrac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$ <br> $= -\dfrac{4\sin 2t}{2\sqrt{3}\cos 2t}$ | M1 |
| | $\dfrac{dy}{dx} = -\dfrac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\dfrac{2}{3}\right)$ | A1 |
| | | **(5)** |
| (b) | When $t = \dfrac{\pi}{3}$ $\qquad x = \dfrac{3}{2},\ y = 1$ can be implied | B1 |
| | $m = -\dfrac{2}{3}\sqrt{3}\tan\left(\dfrac{2\pi}{3}\right) \quad (=2)$ | M1 |
| | $y - 1 = 2\left(x - \dfrac{3}{2}\right)$ | dM1 |
| | $y = 2x - 2$ | A1 |
| | | **(4)** |
| | | **(9 marks)** |

**Notes:**

**(a)**

**B1:** The correct $\dfrac{dx}{dt}$

**M1:** $\dfrac{dy}{dt} = \pm k\cos t\sin t$ or $\pm k\sin 2t$, where $k$ is a non-zero constant. Allow $k = 1$

**A1:** $\dfrac{dy}{dt} = -8\cos t\sin t$ or $-4\sin 2t$ or equivalent. In this question, it is possible to get a correct answer after incorrect working, e.g. $2\cos 2t - 2 \to -4\sin 2t$. This should lose this mark and the next A but ignore in part (b).

**M1:** Their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$, or their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$. The answer must be a function of $t$ only.

| **Question 4 notes** *continued* | |
|---|---|
| **A1:** | The correct answer in the form specified. They don't have to explicitly state $k=-\frac{2}{3}$ but there must be evidence that the constant is $-\frac{2}{3}$. Accept equivalent fractions. |
| **(b)** | |
| **B1:** | That when $t=\frac{\pi}{3}$, $x=\frac{3}{2}$ and $y=1$. Exact numerical values are required but the values can be implied, for example by a correct final answer, and can occur anywhere in the question. |
| **M1:** | Substituting $t=\frac{\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. Trigonometric terms, e.g. $\tan\frac{2\pi}{3}$ need not be evaluated. |
| **dM1:** | Dependent on the previous M. Finding an equation of a tangent with their point and their numerical value of the gradient of the tangent, not the normal. Expressions like $\tan\frac{2\pi}{3}$ must be evaluated. The equation must be linear. Using $y-y'=m(x-x')$. They should get $x'$ and $y'$ the right way round. Alternatively writing $y=(\text{their } m)x+c$ and using their point, the right way round, to find $c$. |
| **A1:** | cao. The correct answer in the form specified. |

| Question | Scheme | | Marks |
|---|---|---|---|
| **5(a)** | $y = 4x - xe^{\frac{1}{2}x}, x \geqslant 0$ | | |
| | $\left\{ y = 0 \Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow \right\}$ | | |
| | $e^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$ | Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$ | M1 |
| | | $4\ln 2$ **cao** (Ignore $x = 0$ ) | A1 |
| | | | **(2)** |
| **(b)** | $\left\{ \int xe^{\frac{1}{2}x}\,dx \right\} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$ | $\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, $\alpha > 0$, $\beta > 0$ | M1 |
| | | $2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$, with or without $dx$ | A1 |
| | $= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\ \{+c\}$ | | A1 |
| | | | **(3)** |
| **(c)** | $\left\{ \int 4x\,dx \right\} = 2x^2$ | | B1 |
| | $\left\{ \int_0^{4\ln 2} (4x - xe^{\frac{1}{2}x})\,dx \right\} = \left[ 2x^2 - \left( 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \right) \right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$ | | |
| | $= \left( 2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)} \right) - \left( 2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)} \right)$ | | M1 |
| | $= (32(\ln 2)^2 - 32(\ln 2) + 16) - (4)$<br>$= 32(\ln 2)^2 - 32(\ln 2) + 12$ | | A1 |
| | | | **(3)** |
| | | | **(8 marks)** |

**Notes:**

**(a)**

**M1:** Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$

**A1:** $4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)

**(b)**

**M1:** Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, where $\alpha > 0, \beta > 0$.

(must be in this form) with or without $dx$

**Question 5 notes** *continued*

**A1:** $2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$ or equivalent, with or without $\mathrm{d}x$. **Can be un-simplified.**

**A1:** $2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ or equivalent with or without $+\,c$. **Can be un-simplified.**

**(c)**

**B1:** $4x \to 2x^2$ or $\dfrac{4x^2}{2}$ oe

**M1:** **Complete** method of applying limits of their $x_A$ and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bx\mathrm{e}^{\frac{1}{2}x} \pm C\mathrm{e}^{\frac{1}{2}x}$. (Where $A$ □ $0$, $B$ □ $0$ and $C$ □ $0$) and subtracting the correct way round.

**A1:** A correct three term exact quadratic expression in $\ln 2$. For example allow for A1

- $32(\ln 2)^2 - 32(\ln 2) + 12$
- $8(2\ln 2)^2 - 8(4\ln 2) + 12$
- $2(4\ln 2)^2 - 32(\ln 2) + 12$
- $2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 12$

Note that the constant term of 12 needs to be combined from $4\mathrm{e}^{\frac{1}{2}(4\ln 2)} - 4\mathrm{e}^{\frac{1}{2}(0)}$ o.e.

Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \dfrac{12}{32\ln 2}\right)$ for A1.

Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.

<table>
<tr><th>Question</th><th colspan="3">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="6">6</td><td colspan="3">Assumption: there exists positive real numbers $a$, $b$ such that<br>$a+b<2\sqrt{ab}$</td><td>B1</td></tr>
<tr><td>Method 1<br>$a+b-2\sqrt{ab}<0$<br>$(\sqrt{a}-\sqrt{b})^2<0$</td><td>Method 2<br>$(a+b)^2=(2\sqrt{ab})^2$<br>$a^2+2ab+b^2<4ab$<br>$a^2-2ab+b^2<0$<br>$(a-b)^2<0$</td><td>A complete method for creating<br>$(f(a,b))^2<0$</td><td>M1A1</td></tr>
<tr><td colspan="3">This is a contradiction, therefore</td><td rowspan="2">A1</td></tr>
<tr><td colspan="3">If $a$, $b$ are positive real numbers, then $a+b \geqslant 2\sqrt{ab}$</td></tr>
<tr><td colspan="3"></td><td>(4)</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="5">(4 marks)</td></tr>
<tr><td colspan="5">Notes:</td></tr>
<tr><td>B1:</td><td colspan="4">As this is proof by contradiction, the candidate is required to start their proof by assuming that the contrary. That is "if $a$, $b$ are positive real numbers, then $a+b \geqslant 2\sqrt{ab}$ " is true.<br>Accept, as a minimum, there exists $a$ and $b$ such that $a+b<2\sqrt{ab}$</td></tr>
<tr><td>M1:</td><td colspan="4">For starting with $a+b<2\sqrt{ab}$ and proceeding to either $(\sqrt{a}-\sqrt{b})^2<0$ or $(a-b)^2<0$</td></tr>
<tr><td>A1:</td><td colspan="4">All algebra is required to be correct. Do not accept, for instance, $(a+b)^2=2\sqrt{ab}^{\,2}$ even when followed by correct lines.</td></tr>
<tr><td>A1:</td><td colspan="4">A fully correct proof by contradiction. It must include a statement that $(a-b)^2<0$ is a contradiction so if $a$, $b$ are positive real numbers, then $a+b \geqslant 2\sqrt{ab}$</td></tr>
</table>

| Question | Scheme | | Marks |
|---|---|---|---|
| 7(a) | $x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$ | | |
| | $x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$ | | M1 |
| | So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$ | Adds their expanded $x$ (which is in terms of $t$) to $2\sin t$ | dM1 |
| | $= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$ | | |
| | $= 2\sqrt{3}\cos t$ * **cso** | | A1* |
| | | | **(3)** |
| (b) | $\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ | Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** $x$'s and $y$'s. | M1 |
| | $\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$ | | |
| | $\Rightarrow (x+y)^2 + 3y^2 = 12$ | $\Rightarrow (x+y)^2 + 3y^2 = 12$ | A1 |
| | | $\{a = 3, b = 12\}$ | **(2)** |
| | **Alternative** | | |
| | $(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$ | | |
| | $(x+y)^2 = 12 - 3y^2$ | Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** $x$'s and $y$'s. | M1 |
| | $\Rightarrow (x+y)^2 + 3y^2 = 12$ | $(x+y)^2 + 3y^2 = 12$ | A1 |
| | | | **(2)** |
| | | | **(5 marks)** |

**Notes:**

**(a)**

**M1:** $\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t + \frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$

**dM1:** Adds their expanded $x$ (which is in terms of $t$) to $2\sin t$.

**A1*:** Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.

**(b)**

**M1:** Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** $x$'s and $y$'s.

**A1:** leading $(x+y)^2 + 3y^2 = 12$

| Question | Scheme | | Marks |
|---|---|---|---|
| 8(a) | $\dfrac{d\theta}{dt} = \lambda(120-\theta),\ \theta \leqslant 100$ | | |
| | $\displaystyle\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$ | | B1 |
| | $-\ln(120-\theta); = \lambda t + c$ | For integrating lhs M1 A1<br>For integrating rhs M1 A1 | M1A1;<br>M1A1 |
| | $\{t=0,\ \theta=20\ \Rightarrow\}\ -\ln(100) = \lambda(0) + c$<br>$\Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$<br>$\Rightarrow\ -\lambda t = \ln(120-\theta) - \ln 100$<br>$\Rightarrow -\lambda t = \ln\left(\dfrac{120-\theta}{100}\right)$ | | M1 |
| | $e^{-\lambda t} = \dfrac{120-\theta}{100}$ | | dddM1 |
| | $100\,e^{-\lambda t} = 120-\theta$<br>leading to $\theta = 120 - 100e^{-\lambda t}$ | | A1* |
| | | | **(8)** |
| (b) | $\{\lambda = 0.01,\ \theta = 100\ \Rightarrow\}$ $\quad 100 = 120 - 100\,e^{-0.01t}$ | | M1 |
| | $\Rightarrow 100\,e^{-0.01t} = 120 - 100\ \Rightarrow$<br>$-0.01t = \ln\left(\dfrac{120-100}{100}\right)$<br>$t = \dfrac{1}{-0.01}\ln\left(\dfrac{120-100}{100}\right)$<br>$\left\{t = \dfrac{1}{-0.01}\ln\left(\dfrac{1}{5}\right) = 100\ln 5\right\}$ | Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = \ldots$ and $t = A\ln B$, where $B > 0$ | dM1 |
| | $t = 160.94379\ldots$ 161 (s) (nearest second) awrt 161 | | A1 |
| | | | **(3)** |
| | | | **(11 marks)** |

**Notes:**

**(a)**

**B1M1A1M1A1:** Mark as in the scheme.

**M1:** Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation leading to $\pm\lambda t = \ln(f(\theta))$

**dddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.

**A1*:** Correct answer with no errors. This is a given answer

**(b)**

**M1:** Substitutes $\lambda = 0.01,\ \theta = 100$ into given equation

**M1:** See scheme

**A1:** Awrt 161 seconds.

| Question | Scheme | | Marks |
|---|---|---|---|
| 9 (a) | $A(3, 5, 0)$ | | B1 |
| | | | (1) |
| (b) | $\{l_2 :\}\ \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | $\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$ $\mathbf{a} \neq 0, \mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ | M1 |
| | | Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ | A1 |
| | | | (2) |
| (c) | $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix}1\\5\\2\end{pmatrix} - \begin{pmatrix}3\\5\\0\end{pmatrix} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$ | | |
| | $AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$ | Full method for finding $AP$ | M1 |
| | $2\sqrt{2}$ | | A1 |
| | | | (2) |
| (d) | So $\overrightarrow{AP} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow \begin{pmatrix}-2\\0\\2\end{pmatrix}.\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | Realisation that the dot product is required between $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | M1 |
| | $\{\cos\theta =\}\ \dfrac{\overrightarrow{AP}\,.\,\mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert\,\lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix}-2\\0\\2\end{pmatrix}.\begin{pmatrix}-5\\4\\3\end{pmatrix}\right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2}\,.\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ | | dM1 |
| | $\{\cos\theta\} = \dfrac{\pm(10 + 0 + 6)}{\sqrt{8}.\sqrt{50}} = \underline{\dfrac{4}{5}}$ | | A1 cso |
| | | | (3) |
| (e) | $\{\text{Area } APE =\}\ \dfrac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ | | M1 |
| | $= 2.4$ | | A1 |
| | | | (2) |

| Question | Scheme | | Marks |
|---|---|---|---|
| 9(f) | $\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c) | | |
| | $\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$ | This mark can be implied. | M1 |
| | $\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \frac{4}{25} \Rightarrow\right\}\ \lambda = \pm\frac{2}{5}$ | Either $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$ | A1 |
| | $l_2 : \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} \pm \frac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | **dependent on the previous M mark** Substitutes at least one of their values of $\lambda$ into $l_2$. | dM1 |
| | $\left\{\overrightarrow{OE}\right\} = \begin{pmatrix}3\\ \frac{17}{5}\\ \frac{4}{5}\end{pmatrix}$ or $\begin{pmatrix}3\\3.4\\0.8\end{pmatrix}$, $\left\{\overrightarrow{OE}\right\} = \begin{pmatrix}-1\\ \frac{33}{5}\\ \frac{16}{5}\end{pmatrix}$ or $\begin{pmatrix}-1\\6.6\\3.2\end{pmatrix}$ | At least one set of coordinates are correct. | A1 |
| | | Both sets of coordinates are correct. | A1 |
| | | | **(5)** |
| | | | **(15 marks)** |

**Notes:**

**(a)**

**B1:** Allow $A(3, 5, 0)$ or $3\mathbf{i} + 5\mathbf{j}$ or $3\mathbf{i} + 5\mathbf{j} + 0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$

**(b)**

**A1:** Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 =

i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$.

**Note:** Allow the use of parameters $\mu$ or $t$ instead of $\lambda$.

**(c)**

**M1:** Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find $AP$

Note: Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$.

| **Question 9 notes** *continued* |
|---|
| **(d)**<br>**M1:** Realisation that the dot product is required between $\left(\overrightarrow{AP} \text{ or } \overrightarrow{PA}\right)$<br>**dM1:** Full method to find $\cos\theta$ (dependent upon the previous M),<br>**A1:** $\cos\theta = \dfrac{4}{5}$ or exact equivalent |
| **(e)**<br>**M1 A1:** For $\dfrac{1}{2}(2\sqrt{2})^2 \sin(36.869...^\circ)$ or $\dfrac{1}{2}(2\sqrt{2})^2 \sin(180^\circ - 36.869...^\circ)$; = awrt 2.40<br>Candidates must use their $\theta$ from part (d) or apply a correct method of finding their $\sin\theta = \dfrac{3}{5}$ from their $\cos\theta = \dfrac{4}{5}$ |
| **(f)**<br>**M1:** Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working.<br>for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$<br>1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent.<br>1st M1 for $\lambda = \dfrac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \dfrac{2\sqrt{2}}{5\sqrt{2}}$<br>So $\left\{\mathbf{d}_1 = \dfrac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\} \text{"vector"} = \dfrac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1<br>**dM1:** In part (f) can be implied for at least 2 (out of 6) correct *x*, *y*, *z* ordinates from their values of $\lambda$. |