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Mark Scheme (Results)

October 2016

Pearson Edexcel IAL
in Core Mathematics 12 (WMA01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- $\sqsubseteq$ or d... The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
1.	$\mathrm{f}(x) = 3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}$	
	$\int\left(3x^2 + x - 4x^{-\frac{1}{2}} + 6x^{-3}\right)\mathrm{d}x = \frac{3x^3}{3} + \frac{x^2}{2} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-2}}{-2}(+c)$	M1 A1A1A1
	$= x^3 + \frac{x^2}{2} - 8x^{\frac{1}{2}} - 3x^{-2} + c$	A1
		[5]
		5 marks

Notes

M1: Attempt to integrate original f(x)– one power increased $x^n \to x^{n+1}$

A1: Two of the four terms in x correct un simplified or simplified– (ignore no constant here). They may be listed.

$3x^2 \to 3\frac{x^3}{3}$ is acceptable for an un simplified term BUT $3x^2 \to 3\frac{x^{2+1}}{2+1}$ isn't

A1: **Three terms** correct **(may be) unsimplified.** They may be listed separately

A1: All four terms correct (may be) unsimplified on a single line.

A1 **cao: All four** terms **correct simplified** with **constant** of integration on a single line. You may isw after sight of correct answer.

Question	Scheme	Marks
2.	(a) $2x\log 7 = \log 14$ or $x\log 49 = \log 14$ or $2x = \log_7 14$	M1
	$x = \dfrac{\log 14}{2\log 7}$ = awrt 0.678	M1A1
		(3)
	(b) $3x+1 = 5^{-2}$	M1
	So $x = -\dfrac{8}{25}$ or -0.32	A1
		(2)
		5 marks
	Notes	

(a)

M1: Uses logs and brings down *x* correctly

M1: Makes *x* the subject correctly. This must follow a method that did involve taking logs

A1: Accept awrt 0.678 (N.B. Correct answer with no working implies two previous marks)

(b)

M1: Uses powers correctly to undo log. Accept $3x+1=5^{-2}$ or equivalent such as $3x+1=0.04$

A1: Correct answer (Correct answer implies method mark). Accept -0.320

Question	Scheme	Marks
3 (i)	$\sqrt{45}-\dfrac{20}{\sqrt{5}}+\sqrt{6}\sqrt{30}$ $=\sqrt{9}\sqrt{5}-\dfrac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}+\sqrt{6}\sqrt{6}\sqrt{5}\quad =3\sqrt{5}-4\sqrt{5}+6\sqrt{5}$ $=5\sqrt{5}$	M1 A1* **[2]**
(ii)	$\text{LHS}=\dfrac{17\sqrt{2}(\sqrt{2}-6)}{(\sqrt{2}+6)(\sqrt{2}-6)}$ $=\dfrac{17\times 2-17\times 6\sqrt{2}}{2-36}$ oe $=\dfrac{34-102\sqrt{2}}{-34}\quad =3\sqrt{2}-1$*	M1 A1 A1* **[3]**
		5 marks
	Notes	

(i)

M1: Shows at least **one term** on LHS as multiple of $\sqrt{5}$ with a correct intermediate step

Look for $\sqrt{45}=\sqrt{9}\times\sqrt{5}$ *or* $\sqrt{3\times 3\times 5}=3\sqrt{5}$, or even $45=3\times 3\times 5$ or 9×5 followed by $\sqrt{45}=3\sqrt{5}$

$\dfrac{20}{\sqrt{5}}=\dfrac{20\sqrt{5}}{\sqrt{5}\sqrt{5}}$ or $\dfrac{20\sqrt{5}}{5}=4\sqrt{5}$ or $\dfrac{4\times 5}{\sqrt{5}}=4\sqrt{5}$

$\sqrt{6}\sqrt{30}=\sqrt{6}\sqrt{6}\sqrt{5}$ or $\sqrt{6}\sqrt{30}=\sqrt{180}=\sqrt{36\times 5}=6\sqrt{5}$

or even $180=2\times 2\times 3\times 3\times 5$ followed by $\sqrt{180}=6\sqrt{5}$

A1*: All three terms must have the intermediate step with $3\sqrt{5}-4\sqrt{5}+6\sqrt{5}$ followed by $5\sqrt{5}$

Special Case: Score M1 A0 for $\sqrt{45}-\dfrac{20}{\sqrt{5}}+\sqrt{6}\sqrt{30}=3\sqrt{5}-4\sqrt{5}+6\sqrt{5}=5\sqrt{5}$ without the intermediate steps

Alternative method:

M1: Multiplies all terms by $\sqrt{5}$ to achieve $\sqrt{45}\times\sqrt{5}-20+\sqrt{5}\sqrt{6}\sqrt{30}=5\sqrt{5}\sqrt{5}$ and simplifies any one of the above terms to 15, -20, 30 or 25 showing the intermediate step

A1: All terms simplified showing the intermediate step (see main scheme on how to apply) followed by 15 – 20 + 30 = 25, and minimal conclusion eg. hence true

(ii)

M1: Multiply numerator and denominator by $\sqrt{2}-6$ or $6-\sqrt{2}$

A1: Multiplies out to a correct (unsimplified) answer. For example allow $=\dfrac{17\times 2-17\times 6\sqrt{2}}{2-36}$

A1: The denominator must be simplified so $\dfrac{34-17\times 6\sqrt{2}}{-34}$ or similar such as $\dfrac{17\times 2-102\sqrt{2}}{-34}$ is seen before you see the given answer $3\sqrt{2}-1$. There is no need to 'split' into two separate fractions.

Alternative method:

M1: Alternatively multiplies the rhs by $(\sqrt{2}+6)(3\sqrt{2}-1)$

A1: Correct unsimplified rhs Accept $3\times 2-6+18\sqrt{2}-\sqrt{2}$

A1*: Simpifies rhs to $17\sqrt{2}$ **and** gives a minimal conclusion e.g. hence true or hence $\dfrac{17\sqrt{2}}{(\sqrt{2}+6)}=3\sqrt{2}-1$

Question	Scheme	Marks
4.	$f(x) = 6x^3 - 7x^2 - 43x + 30$	
(a)(i)	Attempts $f(\pm\frac{1}{2})$ Or Use long division as far as remainder	M1
	Remainder = 49	A1
(a)(ii)	Attempts $f(\pm 3)$ Or Use long division as far as remainder	M1
	Remainder = 0	A1
		[4]
(b)	$6x^3 - 7x^2 - 43x + 30 = (x-3)(6x^2 + 11x - 10)$	M1 A1
	$(6x^2 + 11x - 10) = (ax+b)(cx+d)$ where $ac = "6"$ and $bd = "-10"$	M1
	$= (x-3)(2x+5)(3x-2)$	A1
		[4]
		8 marks
	Notes	

(a)(i)

M1: Attempts $f(\pm\frac{1}{2})$ or attempts long division $2x+1\overline{)6x^3 - 7x^2 - 43x + 30}$ with quotient $3x^2 + \ldots\ldots$ and achieves a numerical R

A1: cao Accept $f\left(-\frac{1}{2}\right) = 49$ or even just 49 for both marks

If the candidate has attempted long division they must be stating **the remainder** = 49 or R = 49

(a)(ii)

M1: Attempts $f(\pm 3)$

Or attempts long division. See above for application of this mark. This time quotient must start $6x^2$

A1: cao Accept $f(3) = 0$ or even just 0 for both marks

If the candidate has attempted long division they must be stating **the remainder** = 0 or R = 0

(b)

M1: Recognises (x - 3) is factor and obtains quadratic factor with two correct terms by any correct method.

If division is used look for a minimum of the first two terms $x-3\overline{)6x^3 - 7x^2 - 43x + 30}$ with quotient $6x^2 \pm 11x\ldots\ldots$ and first line $6x^2 - 18x$

If factorisation is used look for correct first and last terms $6x^3 - 7x^2 - 43x + 30 = (x-3)(6x^2 \ldots\ldots x \pm 10)$

A1: Correct quadratic

M1: Attempt to factorise their quadratic

A1: **cao** – need all three factors together. Do not penalise candidates who go on to state the roots.

Allow $6(x-3)\left(x+\frac{5}{2}\right)\left(x-\frac{2}{3}\right)$ following $(x-3)(6x^2+11x-10)$

Note: There may be candidates who just write down the factors from their GC. The question did state hence so we need to be careful here and see some correct work.

$6x^3 - 7x^2 - 43x + 30 = (x-3)\left(x-\frac{2}{3}\right)\left(x+\frac{5}{2}\right)$ presumably from the roots is M0A0M0A0

$6x^3 - 7x^2 - 43x + 30 = 6(x-3)\left(x-\frac{2}{3}\right)\left(x+\frac{5}{2}\right)$ with no working can score M1A0M1A0

Question	Scheme	Marks
5.	(a) $\left(3-\frac{ax}{2}\right)^5 = 3^5 + \binom{5}{1}3^4.\left(-\frac{ax}{2}\right)+\binom{5}{2}3^3.\left(-\frac{ax}{2}\right)^2+\binom{5}{3}3^2.\left(-\frac{ax}{2}\right)^3 ...$	M1
	$= 243, -\frac{405}{2}ax+\frac{135}{2}a^2x^2-\frac{45}{4}a^3x^3...$	B1, A1, A1
		[4]
	(b) $\frac{405}{2}a=\frac{45}{4}a^3$	M1
	$a^2=\frac{810}{45}=18$ or equivalent	A1
	$a=3\sqrt{2}$	A1
		[3]
		7 marks
	Notes	

(a)

M1: The **method** mark is awarded for an attempt at Binomial to get the second and/or third and/or fourth term.

You need to see the **correct** binomial coefficient combined with correct power of *x*. e.g. $\binom{5}{2}..x^2$

Condone bracket errors. Accept any notation for ${}^5C_1$, ${}^5C_2$ and ${}^5C_3$, e.g. $\binom{5}{1}$, $\binom{5}{2}$ and $\binom{5}{3}$

or 5, 10 and 10 from Pascal's triangle.

The mark can be applied in the same way if 3^5 is taken out as a factor.

B1: For the first term of 243. (writing just 3^5 is **B0).**

A1: is cao and is for **two correct and simplified terms** from $-\frac{405}{2}ax$, $+\frac{135}{2}a^2x^2$ and $-\frac{45}{4}a^3x^3...$

Allow two correct from $-\frac{405}{2}(ax)$, $+\frac{135}{2}(ax)^2$ and $-\frac{45}{4}(ax)^3...$ with the brackets.

Allow decimals. Allow lists

A1: is c.a.o and is for **all** of the terms correct and simplified.

Allow $+\frac{135}{2}(ax)^2$ and $-\frac{45}{4}(ax)^3...$ (ignore x^4 terms)

Allow decimal equivalents $-202.5ax+67.5a^2x^2-11.25a^3x^3...$ Allow listing.

(b)

M1: Puts their coefficient of *x* equal to their coefficient of x^3 (There should be no *x* terms)

A1: This is cao for obtaining a^2 or *a* correctly (may be unsimplified)

A1: This is cao for $a=3\sqrt{2}$ Condone $a=\pm3\sqrt{2}$

We will condone all 3 marks to be scored in (b) from a solution in (a) where all signs are +ve

$$= 243+\frac{405}{2}ax+\frac{135}{2}a^2x^2+\frac{45}{4}a^3x^3...$$

Question	Scheme	Marks
6.		
(a)	$u_2 = 24,\ u_3 = 16$ and $u_4 = \dfrac{32}{3}$	M1, A1 **[2]**
(b)	$r = \dfrac{2}{3}$	B1 **[1]**
(c)	$u_{11} = ar^{10} = 36 \times (r)^{10}$.	M1
	$u_{11} = ar^{10} = 36 \times \left(\dfrac{2}{3}\right)^{10} = \left(\dfrac{4096}{6561}\right)$	
	= 0.6243	A1 **[2]**
(d)	$\sum_{i=1}^{6} u_i = \dfrac{36(1-(\frac{2}{3})^6)}{1-\frac{2}{3}}$ or $\sum_{i=1}^{6} u_i = 36 + 24 + 16 + \dfrac{32}{3} + u_5 + u_6$	M1
	$= 98\frac{14}{27}$	A1cao **[2]**
(e)	$\sum_{i=1}^{\infty} u_i = \dfrac{36}{1-\frac{2}{3}} = 108$	M1 A1 **[2]**
		9 marks
	Notes	

(a)

M1: Attempt to use formula correctly at least twice. It may be seen for example in u_3 and u_4

A1: All three correct exact simplified answers. Allow $10.\dot{6}$

(b)

B1: Accept $\dfrac{2}{3}$ or equivalent such as $\dfrac{24}{36}$ Allow awrt 0.667

(c)

M1: Uses $u_{11} = ar^{10} = 36 \times (r)^{10}$ with their r

A1: Accept awrt 0.6243 or $\dfrac{4096}{6561}$

(d)

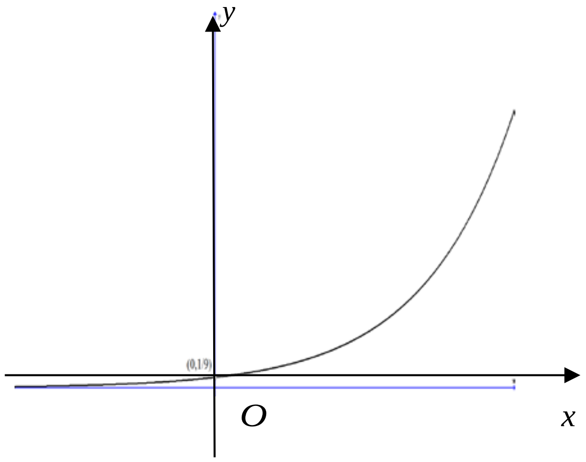
M1: Uses correct sum formula with a = 36 and their r **or** alternatively for adding their first six terms.
FYI Sight of 36, 24, 16, 10.7, 7.1, 4.7 followed by 98.5 implies this mark. (You may only see the first 4 terms in part a)

A1: Obtains $= 98\frac{14}{27}$ (must be exact). For information $\dfrac{2660}{27} = 98\frac{14}{27}$ Allow $98.\dot{5}1\dot{8}$

(e)

M1: Uses correct sum to infinity formula with a = 36 and either $r = \dfrac{2}{3}$ or their r as long as $|r| < 1$

A1: Obtains 108 (must be exact)

Question	Scheme		Marks
7. (a)		Shape and position correct (0, 1/9) correct	B1 B1 **[2]**
(b)	State h = 0.5, or use of $\frac{1}{2}\times 0.5$;		B1 aef
	$\underline{\{0.192+3+2(\ 0.333+0.577+1+1.732\ )\}}$	For structure of $\{...............\}$;	M1A1
	$\frac{1}{2}\times 0.5\times\underline{\{10.476\}}$ = awrt 2.62		A1 **[4]**
			6 marks

Notes

(a)

B1: Curve just in quadrant one and two with a gradient that is approaching zero at the lhs and increases as x increases. Curves that **just** cross the y axis into quadrant 2 may be penalised. As a rule of thumb expect it reach at least as far as x = -1.

B1: The point (0, 1/9) lies on the curve.
Accept 1/9 marked on the y axis. Accept a statement when x = 0, y = 1/9
Do not accept 3^{-2} or 0.11. Condone $\left(0, 0.\dot{1}\right)$

(b)

B1: For using $\frac{1}{2}\times 0.5$ or h = 0.5 or equivalent such as (1-0.5)

M1: Scored for the sight of the correct structure for the outer bracket.
You need to see the first y value plus the last y value plus 2 times a bracket containing the sum of the remaining y values with no additional values.
If the only mistake is a copying error or is to omit one of the remaining y values then this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however).
$\frac{1}{2}\times 0.5\times 0.192+3+2(\ 0.333+0.577+1+1.732\ )$ or awrt 8.08 implies this mark

A1: For $\{0.192+3+2(\ 0.333+0.577+1+1.732\ )\}$ **or** $\{(0.192+3)+2(\ 0.333+0.577+1+1.732\ )\}$ **oe**

A1: For answer which rounds to 2.62. Correct answer implies all 4 marks

NB: Separate trapezia may be used: B1 for 0.5, M1 for 1/2 $h(a + b)$ used 4 or 5 times followed by A1 (if it is all correct) and A1 as before.

Question	Scheme	Marks
8. (a)	$\frac{\sin D}{5} = \frac{\sin 1.1}{6}$	M1
	$\sin D = 0.74267$ so $D = 0.84$	M1, A1
	$B = \pi - (1.1 + 0.84) = 1.20$ *	A1*
		[4]
(b)	Uses angle *DBC* = $\pi - 1.2 = \text{awrt } 1.94$	B1
	Area of sector is $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times '1.94'$ $(=34.9)$ or Area of triangle ABD = $\frac{1}{2}\times 5\times 6\times \sin 1.2$ $(=14.0)$	M1
	Total area is $\frac{1}{2}\times 6^2\times '1.94' + \frac{1}{2}\times 5\times 6\times \sin 1.2$	dM1
	= 48.9cm^2	A1
		[4]
		8 marks
	Notes	

(a)

M1: Uses sine rule – the sides and angles must be in the correct positions

M1: Makes sin *D* the subject and uses inverse sine (in degrees or radians)

A1: Accept awrt 0.84 or in degrees accept answers truncating $47.9..°$ or rounding to $48.0°$

A1*: Answer is printed so should see either $\pi - (1.1 + \text{awrt } 0.84)$ **or** $\pi - 1.1 - \text{awrt } 0.84$ before you see 1.20

If the question was changed to degrees look for accuracy to one decimal places throughout the question for the final A1 mark. So $1.1 \text{ rads} = \text{awrt } 63.0°$ and $(180 - \text{awrt } 63.0 - \text{awrt } 48.0) = \text{awrt } 69.0..\times\frac{\pi}{180} = 1.20$

There are many ways to attempt this question: For example

M1: Uses cosine rule $6^2 = 5^2 + x^2 - 2\times 5\times x\cos 1.1$ (where *x* =*AD*) and attempts to solve to find *x*. For information $x \approx 6.29$

M1: Uses cosine rule $\cos B = \frac{6^2 + 5^2 - \text{their } '6.29'^2}{2\times 6\times 5}$

A1: Achieves $\cos B = \frac{6^2 + 5^2 - (\text{awrt } 6.29)^2}{2\times 6\times 5}$

A1: 1.20*

(b)

B1: Uses angles on a straight line formula. Score for $\pi - 1.2$ or allow awrt 1.94 as evidence.
If converted to degrees accept awrt $111.2°$ as evidence

M1: Uses a correct area formula for the sector **or** a correct area formula for the triangle.
You may follow through on an incorrectly found angle *DBC*
For example $2\pi - 1.2$ is acceptable but $180° - 1.2$ is not as it is using mixed units.
If the angle was found in degrees, the correct formula must be used.
For the triangle the correct combinations of sides and angle should be attempted.
e.g. You may see the area of triangle $ABD = \frac{1}{2}5\times(\text{their } 6.29)\times\sin 1.1$ or $\frac{1}{2}6\times(\text{their } 6.29)\times\sin(\text{their } ADB)$

dM1: Adds together a correct area formula for the sector **and** a correct area formula for the triangle.
You may follow through on an incorrectly found angle *DBC* or *ADB*

A1: Accept awrt 48.9 (do not need units)

Question	Scheme	Marks
9. (a)	Uses $\frac{n}{2}(2\times a+(n-1)d)$ with n = 10 to give 10a + 45d = 395 *	B1* **[1]**
(b)	Uses $\frac{n}{2}(2\times a+(n-1)d)$ with n = 18 and S=927	M1
	Obtain $18a+153d=927$ or $2a+17d=103$	A1 **[2]**
(c)	Solves simultaneous equations to find either a or d	M1
	a = 26 and d = 3	A1, A1 **[3]**
(d)	Uses $a+(n-1)d$ with n = 20	M1
	= 83	A1 **[2]**
		8 marks

Notes

Mark the whole question as one.

(a)

B1: Use the correct formula for the sum of an AP with n = 10, S =395 AND proceeds to the given answer.
It is acceptable for the 395 to appear just at the answer stage.
Could use formula with n = 10, S =395 and l = a+9d
It is OK to list but minimum would be $a+a+d+a+2d.....+a+9d=395$

(b)

M1: Obtain a correct second equation e.g. $927=\frac{18}{2}(2\times a+(18-1)d)$ or equivalent. Condone a slip on the 927.
Note that if the candidate reads 927 as 972 they will only have access to M marks in this question. This is due to the fact that with this number, the values of a and d would be fractional and this could not occur as they must be integers

A1: A simplified equation so accept either $18a+153d=927$ or $2a+17d=103$
Sight of one of these scores both marks.

(c)

M1: Solves simultaneous equations to find either a or d.
Do not concern yourself with the process as calculators are allowed on this paper so score if they proceed to either a and/or d

A1: Obtains correct a or d (just one)

A1: Obtains correct a and d (both)

(d)

M1: Uses correct formula for n th term using their a and d but with n = 20. Look for $'a'+19'd'$

A1: Correct answer

Question	Scheme	Marks
10. (a)	Use $\frac{\sin x}{\cos x} = \tan x$ to give $8\sin x = -3\cos^2 x$	M1
	Use $\cos^2 x = 1 - \sin^2 x$ i.e. $8\sin x = -3(1-\sin^2 x)$	M1
	So $8\sin x = -3 + 3\sin^2 x$ and $3\sin^2 x - 8\sin x - 3 = 0$ *	A1 * **[3]**
(b)	Solves the three term quadratic "$3\sin^2 x - 8\sin x - 3 = 0$"	M1
	So $(\sin x) = -\frac{1}{3}$ (or 3)	A1
	$(2\theta) = -19.47$ or 199.47 or 340.53	dM1
	$\theta = 99.7, 170.3, 279.7$ or 350.3	A1, A1 **[5]**
		8 marks

Notes

(a)

M1: Use $\frac{\sin x}{\cos x} = \tan x$ to give $8\sin x = -3\cos^2 x$ or equivalent

M1: Use $\cos^2 x = 1 - \sin^2 x$ i.e. $8\sin x = -3(1-\sin^2 x)$

May also be seen $8\tan x = -3\cos x \Rightarrow 8\tan x = -3\sqrt{1-\sin^2 x}$

A1: Proceeds to given answer with no errors.

(This is a given answer so do not tolerate bracketing or notation errors such as $\cos^2 x$ written as $\cos x^2$ or $\sin x$ appearing as sin)

(b)

M1: Solving quadratic by usual methods (see notes).

If the formula is quoted it must be correct but allow solutions from calculators.

A1: You only need to see $-\frac{1}{3}$.

This is an intermediate answer so condone $-\frac{1}{3}$ appearing as awrt -0.333

Condone errors on the lhs so accept for this mark $x / a / \theta = -\frac{1}{3}, \sin x = -\frac{1}{3}, \sin 2x = -\frac{1}{3}$

dM1: Uses inverse sine to obtain an answer for 2θ.

This may appear as answers for x. The only stipulation is that $\text{invsin } k, |k| < 1$

It is dependent upon seeing a correct method of solving their quadratic

Accept answers rounding to 1 dp for 2θ e.g. awrt -19.5 or 199.5 or 340.5.

It may also be implied by a correct answer for θ e.g. awrt -9.7 or 99.7 or 170.2

A1: Two correct, awrt one dp $\theta = 99.7, 170.3, 279.7$ or 350.3

A1: All four correct, awrt one dp $\theta = 99.7, 170.3, 279.7$ or 350.3

Question	Scheme	Marks
11. **(a)**	$(13k-5)x^2-12kx-6=0$ or $(5-13k)x^2+12kx+6=0$	B1
	Uses b^2-4ac with $a=\pm13k\pm5, b=\pm12k$ and $c=\pm6$	M1
	And states $b^2-4ac>0$ with $a=\pm(13k-5), b=\pm12k$ and $c=\pm6$	A1ft
	Proceeds correctly with no errors to $6k^2+13k-5>0$ *	A1*
		[4]
(b)	Attempts to solve $6k^2+13k-5=0$ to give $k=$	M1
	$\Rightarrow$ Critical values, $k=\frac{1}{3}, \frac{-5}{2}$	A1
	$6k^2+13k-5>0$ gives $k>\frac{1}{3}$ (or) $k<\frac{-5}{2}$	M1 A1
		[4]
		8 marks
	Notes	

(a)

B1: Expresses equation as three term quadratic in *x*. $(13k-5)x^2-12kx-6=0$ **oe.**
The equals 0 may be implied by subsequent work. Allow $(5-13k)x^2+12kx+6=0$
Allow an equation of the form $13kx^2-5x^2-12kx-6(=0)$ as long as it is followed by $a=13k-5$........

M1: Attempts b^2-4ac with $a=\pm13k\pm5, b=\pm12k$ and $c=\pm6$
or uses quadratic formula to solve equation
or uses the discriminant on two sides of an equation or inequation e.g. $b^2=4ac$ or $b^2<4ac$

A1: Uses the discriminant condition, eg $b^2-4ac>0$ or $b^2>4ac$ with $a=\pm13k\pm5, b=\pm12k$ and $c=\pm6$

A1*: Proceeds to given answer with no errors. AG. Condone missing = 0 on the equation
Condone a solution where $(13k-5)x^2-12kx-6=0$ is followed by $144k^2+24(13k-5)>0$
Watch for $a=13k-5, b=+12k$ and $c=-6$ which does give the correct inequality but loses the final A1*

(b)

M1: Uses factorisation, formula, or completion of square method to find two values for k,
or finds two **correct** answers with no obvious method for **their** three term quadratic

A1: Obtains $k=\frac{1}{3}, \frac{-5}{2}$ accept -2.5, 0.333 (awrt) here but need exact answer for final A1.
Also condone $x=\frac{1}{3}, \frac{-5}{2}$ for this mark .

M1: Chooses outside region **(** $k<$ Their Lower Limit $k>$ Their Upper Limit **)** for appropriate 3 term quadratic inequality . Do not award simply for diagram or table.
Award if final answer is $k\geqslant\frac{1}{3}$ (or) $k\leqslant\frac{-5}{2}$ or $\frac{1}{3}<k<\frac{-5}{2}$
Condone *x* appearing instead of *k*

A1: $k>\frac{1}{3}$ (or) $k<\frac{-5}{2}$ $\left(k\neq\frac{5}{13}\right)$ must be exact and must be *k*.
Must be two separate inequalities and not be $k>\frac{1}{3}$ **and** $k<\frac{-5}{2}$

Question	Scheme	Marks
12 (a)	$\mathrm{f}(x)=\frac{x^3-9x^2-81x}{27}=0 \Rightarrow x(x^2-9x-81)=0$	M1
	$x=\frac{9\pm\sqrt{81+324}}{2}$	dM1
	$x=\frac{9\pm\sqrt{405}}{2}$ or $x=\frac{9\pm9\sqrt{5}}{2}$	A1 A1 **[4]**
(b)	Differentiates (usual rules), correctly and sets = 0 $\mathrm{f}'(x)=3x^2-18x-81=0$	M1, A1
	Solves $\mathrm{f}'(x)=0$ (or multiple) $\Rightarrow$ x = 9 and -3	dM1 A1
	Substitutes one of their values for x into f(x)	ddM1
	x = 9 y = -27 and x =-3 y = 5	A1 **[6]**
(c)	a = 9	B1 **[1]**
		11 marks

Notes

(a)

M1: Attempts to solve f(x) = 0, by taking out a factor of (/cancelling by) x and obtaining a quadratic factor.

Allow on $x\left(\frac{x^2}{27}-\frac{9x}{27}-\frac{81}{27}\right)=0$ or just the numerator $x(x^2-9x-81)=0$

This is implied by sight of $x^2-9x-81=0$

dM1: Uses formula or completion of square method to find at least one value for x , for **their** three term quadratic. Factorisation is M0. Note that their 3 term quadratic equation may be $\frac{1}{27}x^2-\frac{1}{3}x-3=0$

A1: One correct solution – need not be fully simplified. So allow $x=\frac{9+\sqrt{405}}{2}$ but not $x=\frac{9+\sqrt{81+324}}{2}$

A1: Two correct solutions – need not be simplified or attributed correctly to *A* or *B*.

Special case: If a candidate takes out a common factor of x and uses a calculator to write down the exact surd answers to the quadratic they have used (a limited) amount of algebra. Decimals would not be awarded for this SC. We will therefore score this SC M1 M1 A0 A0 for 2 out of 4. $x(x^2-9x-81)=0\Rightarrow x=\frac{9\pm9\sqrt{5}}{2}$ Just writing down the answers with no working scores 0 marks

(b)

M1: Differentiates f(x) to a 3 term quadratic

You may see confusion over the 27 but score for $\mathrm{f}'(x)$ being a 3 term quadratic

A1: Differentiates correctly and sets correct derivative = 0

$3x^2-18x-81=0$ or any multiple thereof. For example it may be common to see $\frac{3x^2}{27}-\frac{18x}{27}-\frac{81}{27}=0$

dM1: Solves quadratic to give two solutions. It is dependent upon the previous M.

Allow any appropriate method including the use of a calculator.

Condone $\frac{x^2}{9}-\frac{2x}{3}-3=0\Rightarrow(x-9)(x+3)=0$

A1 : Gives **both** 9 and −3

ddM1: Substitute at least one of their values of x (obtained from a solution of $\mathrm{f}'(x)=0$) into f(x) to give y = .

A1: Gives both −27 and 5 (arising from x values of 9 and −3) (Do not require coordinates).

Again they do not need to be attributed correctly to *C* or *D*

(c)

B1: For a = 9 only (no ft)

Question	Scheme		Marks
13 (a)	See $(x\pm1)^2+(y\pm3)^2=r^2$	Or see $x^2+y^2\pm2x\pm6y+c=0$	M1
	Attempt $\sqrt{(8-1)^2+(-2-(-3))^2}$ **or** $(8-1)^2+(-2-(-3))^2$	Substitute $(8,-2)$ into equation	M1
	$(x-1)^2+(y+3)^2,=50$	$x^2+y^2-2x+6y-40=0$	A1, A1 **[4]**
(b)	Gradient of AP = $\frac{1}{7}$		B1
	So gradient of tangent is -7		M1
	Equation of tangent is $(y+2)=-7(x-8)$		dM1
	$y=-7x+54$ or m = -7, c =54		A1 **[4]**
	Way 1	Way 2	
(c)	$y = x + 6$ meets circle when $(x-1)^2+(x+9)^2=50$ or when $(y-7)^2+(y+3)^2=50$	As tangent has gradient 1 AQ has gradient -1 and $\frac{y-(-3)}{x-1}=-1$	M1
	i.e. $2x^2+16x+32=0$ **or** when $2y^2-8y+8=0$	$y+x=-2$	A1
	Solve to give x or y =	Solve $y+x=-2$ with $y = x + 6$ or alternatively solve $y+x=-2$ with the equation of the circle to give x or y =	M1
	Substitute to give y = (**or** x =)		dM1
	$(-4, 2)$ only		A1 **[5]**
			13 marks

Notes

(a)

M1 : Scored for centre at $(1,-3)\Rightarrow(x\pm1)^2+(y\pm3)^2=...$ or $x^2+y^2\pm2x\pm6y+...=0$

M1: Scored for an attempt at finding the radius or the radius ^{2} (see scheme).
It need not be in the equation It can be <u>implied</u> by $\sqrt{50}$ *or* $5\sqrt{2}$ or 50
If the form $x^2+y^2\pm2x\pm6y+c=0$ is used it is for substituting $(8,-2)$ into the equation

A1: LHS or RHS correct $(x-1)^2+(y+3)^2,=...$ or $(x\pm a)^2+(y\pm b)^2,=50$ $x^2+y^2-2x+6y...=0$

A**1:** Correct equation. Accept $(x-1)^2+(y+3)^2=50$ or $x^2+y^2-2x+6y-40=0$ or $x^2+y^2-2x+6y=40$

(b)

B1 : Obtain 1/7 . Implied by use of – 7 in their tangent

M1: Uses negative reciprocal

dM1: Linear equation through point (8, -2) with their negative reciprocal gradient

A1: cao

(c)

M1: Eliminates x or y from two relevant equations, that is whose intersection is Q.

A1: Correct quadratic in x or in y

M1: Solves (with usual rules) to give first variable. The first M must have been scored

dM1: Substitute in either (relevant) equation to give second coordinate, dependent upon both previous M's

A1: Correct answer accept $x=-4$, $y = 2$. Withhold this if two answers given

Question	Scheme	Marks
14.	$y=-x^2+6x-8$	
(a)	$\frac{dy}{dx}=-2x+6$ and substitutes $x = 5$ to give gradient = m = -4	M1 A1
	Normal has gradient $\frac{-1}{m}=\left(\frac{1}{4}\right)$	M1
	Equation of normal is $(y+3)="\frac{1}{4}"(x-5)$ so $x-4y-17=0$	dM1 A1 **[5]**
(b)	$\int -x^2+6x-8\,dx=-\frac{x^3}{3}+6\frac{x^2}{2}-8x$	M1
	The Line meets the x-axis at 17	B1
	The Curve meets the x-axis at 4	B1
	Uses correct limits correctly for their integral i.e. $\left[-\frac{x^3}{3}+6\frac{x^2}{2}-8x\right]_4^5=-\frac{5^3}{3}+6\frac{5^2}{2}-8\times5-(-\frac{4^3}{3}+6\frac{4^2}{2}-8\times4)$	M1
	Finds area above line, using area of triangle or integration $=\frac{1}{2}\times3\times("17"-5)$	M1
	Area of R = $18+1\frac{1}{3}=19\frac{1}{3}$	A1
		[6]
		11 marks

	Notes	

(a)

M1: Differentiates to give $\frac{dy}{dx} = \pm 2x \pm 6$ and substitutes $x = 5$

A1: Obtains answer -4.

M1: Uses negative reciprocal of their numerical $\frac{dy}{dx}$ (follow through). M1 must have been awarded

dM1: Linear equation through point $(5, -3)$ with their **changed** gradient.

Dependent upon the first M, so you would allow for $(y+3) = 4(x-5)$ following an answer of -4

A1: cao accept $k(x-4y-17)=0$ where k is a positive or negative integer

Candidates who work with a gradient of ± 2 from their $\frac{dy}{dx} = \pm 2x \pm 6$ will score 0 marks in this part of the question.

(b)

M1: Integrates a quadratic expression correctly.
If they integrate (line -curve) follow through on their new quadratic
The terms including the coefficients must be correct for their quadratic

B1: Obtains 17 for the point where the line meets the x - axis

B1: Finds that the curve meets the x axis at 4.
You may score this for $y = 0 \Rightarrow x = 2, 4$ ignoring even an incorrect 2
Also allow for a limit in the integral.
You may even score this if 4 appears (in the correct place) on the diagram

M1: Uses the limits 4 and 5 in their integrated function

If a candidate writes down $\int_4^5 \pm\left(-x^2+6x-8\right)dx = \pm\frac{4}{3}$ (from a GC) we will allow them to score this mark.

M1: Finds appropriate area above the line for their attempted integral, so

if they integrate just curve look for area of triangle $= \frac{1}{2} \times 3 \times \text{"}their\ 17-5\text{"}$ or $\int_5^{\text{"}17\text{"}} \text{"}\left(\frac{1}{4}x - \frac{17}{4}\right)\text{"}dx = \left[\frac{1}{8}x^2 - \frac{17}{4}x\right]_5^{\text{"}17\text{"}}$

if they integrate (line - curve) from 4 to 5, then the triangle would be $= \frac{1}{2} \times their\ \text{"}\frac{13}{4}\text{"} \times \text{"}their\ 17-4\text{"}$

A1: correct work leading to $19\frac{1}{3}$

A candidate who does the integration on a GC can potentially score M0 B1 B1 M1 M1 A0

Question	Scheme	Marks
15 (a)	$200 = \pi r^2 + \pi rh + 2rh$	M1 A1
	$(h =)\dfrac{200-\pi r^2}{\pi r + 2r}$ or $(rh =)\dfrac{200-\pi r^2}{\pi + 2}$	dM1
	$V = \dfrac{1}{2}\pi r^2 h =$	M1
	$\Rightarrow V = \dfrac{\pi r^2(200-\pi r^2)}{2(2r+\pi r)} = \dfrac{\pi r(200-\pi r^2)}{4+2\pi}$ *	A1 cso * **[5]**
(b)	$\dfrac{dV}{dr} = \dfrac{200\pi - 3\pi^2 r^2}{4+2\pi}$ Accept awrt $\dfrac{dV}{dr} = 61.1 - 2.9r^2$	M1 A1
	$\dfrac{200\pi - 3\pi^2 r^2}{4+2\pi} = 0$ or $200\pi - 3\pi^2 r^2 = 0$ leading to $r^2 =$	dM1
	$r = \sqrt{\dfrac{200}{3\pi}}$ or answers which round to 4.6	dM1 A1
	$V = 188$	B1 **[6]**
(c)	$\dfrac{d^2V}{dr^2} = \dfrac{-6\pi^2 r}{4+2\pi}$, and sign considered Accept $\dfrac{d^2V}{dr^2} =$ awrt $-5.8r$	M1
	$\left.\dfrac{d^2V}{dr^2}\right\rvert_{r=..} = -27 < 0$ and therefore maximum	A1 **[2]**
		13 marks

	Notes	

(a)

M1: Sets total surface area equal to 200 with at least two correct terms.
Note that $200 = 2\pi r^2 + \pi rh$ or even $200 = \pi r^2 + \pi rh + \pi r^2$ does not mean that two terms are correct.

A1: Completely correct $200 = \pi r^2 + \pi rh + 2rh$

dM1: Makes *h* or *rh* the subject of their formula which must have had two terms in *h*
This is dependent upon the previous M1

M1: Gives formula for volume. This may be implied by sight of $V = \frac{1}{2}\pi r^2 \times$ their *h*

A1*: cso – substitutes for *r* or for *rh* correctly and proceeds correctly to $V = \dfrac{\pi r(200 - \pi r^2)}{4 + 2\pi}$

(b) **Parts b and c can be scored together**

M1: Attempts to differentiate *V* or numerator of *V* Accept $\frac{dV}{dr} = A \pm Br^2$
You may see $(4 + 2\pi)\frac{dV}{dr} = A \pm Br^2$ if candidates multiply by $(4 + 2\pi)$ first

A1: Accept any equivalent correct answer or correct numerator if only this was considered.
Also accept decimals.

dM1: Setting $\frac{dV}{dr}$=0 and finding a value for r^2 using correct mathematics (May be implied by answer).
Note that you may not see r^2. It is acceptable to go straight to *r*. Allow $\frac{dy}{dx} = 0$

dM1: Using square root to find *r*. Dependent upon all previous M's.
An answer of 5 for *r* following a correct derivative may imply this mark as some candidates find r to the nearest cm rather than V to the nearest cm^3.
If you don't see incorrect work you may award this mark.

A1 : For any equivalent correct answer. Accept $r = \sqrt{\frac{200}{3\pi}}$ or awrt 4.6
Correct answer implies previous two M marks

B1 : Obtain *V*= 188 Exact answer only. Do not accept, for example, 187.8

(c)

M1: Score for either a second derivative of $\frac{d^2V}{dr^2} = \pm Cr$ and considers the sign.
It can be implied by $\dfrac{\pi r(200 - \pi r^2)}{4 + 2\pi} \to A \pm Br^2 \to \pm Cr$ and a consideration of the sign
Or a second derivative of $\frac{d^2V}{dr^2} = \pm Cr$ and substitutes in their value of '*r*' from (b)
Or a completely correct second derivative $\frac{d^2V}{dr^2} = \frac{-6\pi^2 r}{4 + 2\pi}$ accept $\frac{d^2V}{dr^2} = \text{awrt} -5.76r$

A1: Clear statements and conclusion. For both marks

(1) $\frac{d^2V}{dr^2}$ must be correct (see above), not just the numerator.

(2) A statement (which could be implied) that when their *r* (which does not need to be correct) is substituted into $\frac{d^2V}{dr^2}$ then $\frac{d^2V}{dr^2}$ is either negative or < 0

(3) and a minimal conclusion such as hence maximum

For example, accept for both marks $\frac{d^2V}{dr^2} = -5.76r$ When $r = 4.5 \Rightarrow \frac{d^2V}{dr^2} < 0$, hence max

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