Mark Scheme (Final)

October 2019

Pearson Edexcel International Advanced Level in Core Mathematics C12 (WMA01/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q), \text{ where} |pq| = |c|$, leading to *x* = ...

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where} |pq| = |c| \text{ and } |mn| = |a|$, leading to *x* = ...

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to *x* = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Misreading a question

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

Question Number	Scheme	Marks
1	$\int\left(\frac{1}{2x^3}+3x^{\frac{1}{2}}-6\right)\mathrm{d}x=\int\left(\frac{1}{2}x^{-3}+3x^{\frac{1}{2}}-6\right)\mathrm{d}x$	
	$=-\frac{1}{4}x^{-2}+2x^{\frac{3}{2}}-6x+c$	
	For raising any power by one. Scored for any correct index including $-6\to-6x$	M1
	For one correct term simplified **or unsimplified** including $-6x$ Unsimplified examples: $=\frac{-\frac{1}{2}x^{-2}}{-2},\ \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ Allow equivalent simplified terms e.g. $-\frac{1}{4x^2}$ for $-\frac{1}{4}x^{-2}, 2x\sqrt{x}$ or $2\sqrt{x^3}$ for $2x^{\frac{3}{2}}$	A1
	For two correct terms **simplified**	A1
	$-\frac{1}{4}x^{-2}+2x^{\frac{3}{2}}-6x+c$ or exact **simplified** equivalent all on one line including the "+ c" and apply isw once the correct answer is seen Ignore any spurious integral signs and/or dx's	A1
		[4]
		(4 marks)

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="2">2(a)</td><td colspan="2">$2^{2(2x+1)}$ or 2^{4x+2}
Accept either $2^{2(2x+1)}$ or 2^{4x+2}
but not $\left(2^2\right)^{(2x+1)}$ unless followed by $2^{2(2x+1)}$ or 2^{4x+2}
Also accept $a = 4x+2$ or equivalent e.g. $a = 2(2x+1)$
Apply isw once a correct answer is seen.</td><td>B1</td></tr>
<tr><td colspan="2"></td><td>[1]</td></tr>
<tr><td rowspan="5">(b)</td><td><u>Examples:</u>
$2^x \times 4^{2x+1} = 2^x \times 2^{'4x+2'} = 2^{x+'4x+2'}$
or
$4^{\frac{1}{2}x} \times 4^{2x+1} = 4^{\frac{1}{2}x+2x+1}$
or
$16^{\frac{1}{4}x} \times 16^{\frac{1}{2}(2x+1)} = 16^{\frac{1}{4}x+\frac{1}{2}(2x+1)}$
or
$16^{3x} = 2^{4\times 3x}$ or 2^{12x}
or
$16^{3x} = 4^{2\times 3x}$ or 4^{6x}</td><td>Either
A correct application of the addition law on the lhs. Follow through on their $4x + 2$ but if they use bases other than 2 then the powers must be correct.

Or
A correct application of the multiplication law on the rhs. As in (a) must be e.g. $2^{4\times 3x}$ not $\left(2^4\right)^{3x}$

Condone invisible brackets for this mark e.g. $4^{2x+1} = 16^{\frac{1}{2}2x+1}$</td><td>M1</td></tr>
<tr><td colspan="2">Examples:
$2^{x+4x+2} = 2^{4\times 3x}$, $4^{\frac{1}{2}x+2x+1} = 2^{12x}$, $2^{5x+2} = 16^{3x}$, $16^{\frac{5}{4}x+\frac{1}{2}} = 16^{3x}$, $2^{4x+2} = 2^{11x}$
Any correct equation or correct follow through from their answer to part (a) in the form $m^{f(x)} = n^{g(x)}$ which may be implied by their equation below
Note that it is not necessary that $m = n$
If ‘isw’ has been applied in (a), mark positively and allow this mark if possible e.g. if $2^{2(2x+1)} = 2^{4x+1}$ is seen in (a), score B1 and then allow M1A1ft in (b) if 2^{4x+1} is used in (b)</td><td>A1ft</td></tr>
<tr><td colspan="2">Examples:
$5x+2 = 12x, \frac{1}{2}x+2x+1 = 6x, \frac{1}{4}x+x+\frac{1}{2} = 3x, 4x+2 = 11x$
This is for any fully correct linear equation (no inexact decimals from logs)
(not follow through here)
<u>Note that this is an M mark on ePEN</u></td><td>A1
(M1 on ePEN)</td></tr>
<tr><td>$\Rightarrow x = \frac{2}{7}$</td><td>Correct answer and no other values. Allow equivalent exact fractions e.g. $\frac{4}{14}$ but not $\frac{-2}{-7}$.
Note that this mark is cso so cannot be ‘recovered’ once inexact decimals have been used.</td><td>A1cso</td></tr>
<tr><td colspan="2"></td><td>[4]</td></tr>
</table>

Beware this incorrect solution has been seen in (b) that gives the correct answer:

$$2^x \times 4^{2x+1} = 16^{3x} \Rightarrow 2^x \times 2^{4x+2} = 16^{3x}$$
$$\Rightarrow 4^{5x+2} = 16^{3x}$$
$$\Rightarrow (5x+2)\times 4 = 3x \times 16$$
$$\Rightarrow 20x + 8 = 48x$$
$$\Rightarrow x = \frac{2}{7}$$

(= No marks)

(b) Way 2	**Examples:** $\log_{(2)}\left(2^x \times 4^{2x+1}\right) = \log_{(2)} 2^x + \log_{(2)} 4^{2x+1}$ or $\log_{(2)} 16^{3x} = 3x\log_{(2)} 16$ or $\log_{(2)}\left(2^x \times 4^{2x+1}\right) = \log_{(2)}\left(2^x \times 2^{4x+2}\right) = \log_{(2)}\left(2^{5x+2}\right) = (5x+2)\log_{(2)} 2$ Takes log of each side **and** uses the addition law **or** the power law of logs. (Ignore presence or absence of bases and condone missing brackets)		M1
	Examples: $x\log_{(2)} 2 + (2x+1)\log_{(2)} 4 = 3x\log_{(2)} 16$ or $(5x+2)\log_{(2)} 2 = 3x\log_{(2)} 16$ Correct equation or correct follow through from their answer to part (a) with powers "brought down" (Ignore presence or absence of bases). Do not condone missing brackets unless subsequent work implies their presence. May be implied by their equation below.		A1ft
	Examples: $x + 2(2x+1) = 3x \times 4,\ 5x + 2 = 12x$ This is for any fully correct linear equation (no inexact decimals from logs) (**not follow through here**) **<u>Note that this is an M mark on ePEN</u>**		A1 **(M1 on ePEN)**
	$\Rightarrow x = \frac{2}{7}$	Correct answer and no other values. Allow equivalent exact fractions e.g. $\frac{4}{14}$ but not $\frac{-2}{-7}$. Note that this mark is cso so cannot be 'recovered' once inexact decimals have been used.	A1
			(5 marks)

Question Number	Scheme		Marks
3(a)	$\mathrm{f}(2)=4\times8-4k+2k\times2+8=$	Attempts $\mathrm{f}(\pm2)=...$ Accept sign slips in substitution.	M1
	$\mathrm{f}(2)=40\neq0\Rightarrow(x-2)$/ it is not a factor* Or **Remainder** is 40 so $(x-2)$/ it is not a factor* States $\mathrm{f}(2)=40(\text{or } 4\times8+8)\neq0\Rightarrow(x-2)$/ it is not a factor. There must be no errors or incorrect statements including $\mathrm{f}(2)=4\times8-4k+2k\times2+8\,\underline{=0}$ and **there must be a reference to** $\neq0$ **(allow e.g. 40 > 0 so not a factor)** Or states **remainder** is 40 or $4\times8+8$ so $(x-2)$/ it is not a factor.		A1*
			[2]
	Alternative by long division:		
	$$\begin{array}{r} 4x^2+(8-k)x+16 \\ (x-2)\overline{)4x^3-kx^2+2kx+8} \\ \underline{4x^3-8x^2} \\ (8-k)x^2+2kx+8 \\ \underline{(8-k)x^2-2(8-k)x} \\ 16x+8 \\ \underline{16x-32} \\ 40 \end{array}$$ Attempts long division with $(x-2)$ to obtain a 3 term quadratic expression in the numerator and a constant remainder		M1
	$40\neq0$ so $(x-2)$ is not a factor or e.g. Remainder is 40 so not a factor	There must be no errors or incorrect statements and **there must be a reference to** $\neq0$ or a reference to their being a **remainder** as above	A1
(b)	$\mathrm{f}\left(\frac{1}{2}\right)=6.25$	Attempts f(± 0.5) and sets equal to $\frac{25}{4}$. Accept sign slips in substitution.	M1
	$\frac{3}{4}k=-\frac{9}{4}\Rightarrow k=...$	Collects terms and solves a linear equation in k**. Dependent on the previous mark.**	**d**M1
	$k=-3$	Cao (only this answer)	A1
			[3]

Note that attempts at long division in (b) gets messy but apply the following:

M1: A full attempt to divide $4x^3-kx^2+2kx+8$ by $(2x-1)$ to give a remainder that is a linear expression in k and sets the remainder = $\frac{25}{4}$ (NB correct remainder is $\frac{17}{2}+\frac{3k}{4}$)

dM1: Solves their linear equation in k

A1: $k=-3$

(c)	$f(-2)=4(-2)^3-("-3")(-2)^2+2("-3")(-2)+8=...$ Attempts $f(\pm 2)$ with their numerical k		M1
	$f(-2)=0 \Rightarrow (x+2)$ is a factor *	Fully correct solution with conclusion	A1*
	$4(-2)^3-(-3)(-2)^2+2(-3)(-2)+8=0$ so it is a factor scores M1A1 but the A mark should be withheld for incorrect notation that is not recovered e.g. $4\times -2^3-(-3)\times -2^2+2(-3)(-2)+8=0$ therefore it is a factor scores M1A0 but $4\times -2^3-(-3)\times -2^2+2(-3)(-2)+8$ $=-32+12+12+8=0$ therefore it is a factor scores M1A1		
			[2]
	Alternative by long division:		
	$$\begin{array}{r} 4x^2-5x+4 \\ (x+2)\overline{)4x^3+3x^2-6x+8} \\ \underline{4x^3+8x^2} \\ -5x^2-6x+8 \\ \underline{-5x^2-10x} \\ 4x+8 \\ \underline{4x+8} \\ (0) \end{array}$$ Attempts long division with their k and $(x+2)$ to obtain a 3 term quadratic expression in the numerator		M1
	so $(x+2)$/it is a factor	Fully correct work and conclusion. Note that it is not necessary to see the "0" at the end of the division.	A1
			(7 marks)

Question Number	Scheme		Marks
4(a)	$y=16x\sqrt{x}-3x^2-78=16x^{\frac{3}{2}}-3x^2-78$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=24x^{\frac{1}{2}}-6x$		
	Correct index for either term in x so $16x\sqrt{x}\to\alpha x^{\frac{1}{2}}$ or $-3x^2\to\beta x$		M1
	Any one term correct and simplified e.g. $24x^{\frac{1}{2}}\left(\text{or } 24\sqrt{x}\right)$ or $-6x$		A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)24x^{\frac{1}{2}}-6x$ Correct expression with no 'extra' terms e.g. '+ c' Allow $24\sqrt{x}$ for $24x^{\frac{1}{2}}$ and allow $-6x^1$ Apply isw once a correct answer is seen		A1
			[3]
(b)	$x=4\Rightarrow y=2$	States or uses $y=2$	B1
	$x=4\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=24\times4^{\frac{1}{2}}-6\times4(=24)$	Substitutes $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$m_N=-\frac{1}{\frac{\mathrm{d}y}{\mathrm{d}x}}=\left(-\frac{1}{24}\right)$	Correct method for finding gradient of normal. **Dependent on the previous method mark.**	**d**M1
	E.g. $y-"2"="-\frac{1}{24}"(x-4)$ or $\frac{y-"2"}{x-4}="-\frac{1}{24}"$ or $y=mx+c\Rightarrow"2"="-\frac{1}{24}"\times4+c\Rightarrow c=...$ Correct method for finding the equation of the normal **with x = 4 and their y = 2, which has come from an attempt at y when x = 4, correctly placed.** **Dependent on both previous method marks.**		**dd**M1
	$x+24y-52=0$	$x+24y-52=0$ or $\pm k(x+24y-52)=0,\quad k\in\mathbb{N}$ Must see the equation not **just** values of a, b, c stated.	A1
			[5]
			(8 marks)

Question Number	Scheme		Marks
5(a)	$QR^2=(2x)^2+(2x)^2$	Attempts Pythagoras' Theorem. Condone omission of brackets e.g. $QR^2=2x^2+2x^2$	M1
	$\Rightarrow (QR=)\sqrt{8}x$ or $2\sqrt{2}x$ or $2x\sqrt{2}$	Correct expression. Do **not** allow $\sqrt{8x^2}$ or $2\sqrt{2x^2}$ or $2\sqrt{2x}$ with the vinculum clearly encompassing the x.	A1
	No working: $(QR=)2\sqrt{2}x$ or $\sqrt{8}x$ scores both marks $(QR=)2\sqrt{2x^2}$ or $\sqrt{8x^2}$ scores M1A0		
			[2]
(a) Way 2	$\sin 45=\frac{2x}{QR}\Rightarrow QR=\frac{2x}{\sin 45}$ $=\frac{2x}{1/\sqrt{2}}$	Correct trigonometry (may use cos) to find QR including use of $\sin 45$ or $\cos 45=\frac{1}{\sqrt{2}}$	M1
	$\Rightarrow (QR=)\sqrt{8}x$ or $2\sqrt{2}x$	Correct expression. Do not allow $\sqrt{8x^2}$ or $2\sqrt{2x^2}$	A1
(b)	$3(x+7)=4x+'2\sqrt{2}x'$ oe e.g. $x+7+x+7+x+7=2x+2x+'2\sqrt{2}x'$ Sets perimeters equal. **The lhs side must be correct and the rhs is 4*x* + their answer to part (a).** Follow through on an incorrect QR.		M1
	Note that if the candidate now changes to decimals, they are unlikely to score any of the subsequent marks		
	$\Rightarrow (1+2\sqrt{2})x=21$ Collects terms in x and reaches $(........)x=...$ where $(\ldots)$ is exact and contains a constant and a surd term but condone missing brackets if they are implied by subsequent work otherwise they must be present.		M1
	$x=\frac{21}{(1+2\sqrt{2})}$ or $x=\frac{21}{(1+\sqrt{8})}$ Correct intermediate answer which may be implied if both the previous marks have been awarded and a correct final answer of $6\sqrt{2}-3$ is seen later.		A1
	$\Rightarrow x=\frac{21}{(2\sqrt{2}+1)}\times\frac{\pm(2\sqrt{2}-1)}{\pm(2\sqrt{2}-1)}$ Correct method to rationalise the denominator of their expression which must be a 2-term expression **Given the wording in the question, the method must be shown but condone invisible brackets if the intention is clear.**		M1
	$\Rightarrow x=6\sqrt{2}-3$	cso $x=6\sqrt{2}-3$ (or $-3+6\sqrt{2}$)	A1
			[5]
			(7 marks)

5(b) Way 2	$3(x+7)=4x+'2\sqrt{2}x'$ Sets perimeters equal. **The lhs side must be correct and the rhs is 4*x* + their answer to part (a)**. Follow through on an incorrect *QR*.		M1
	Note that if the candidate now changes to decimals, they are unlikely to score any of the subsequent marks		
	$\Rightarrow 21-x=2\sqrt{2}x$ $\Rightarrow x^2-42x+441=8x^2$	Collects terms in x and constant to one side and squares	M1
	$\Rightarrow 7x^2+42x-441=0$	Correct 3 term quadratic	A1
	$\Rightarrow 7x^2+42x-441=0$ $\Rightarrow x=\dfrac{-42\pm\sqrt{42^2+4(7)(441)}}{2\times 7}$	Solves using the quadratic formula (usual rules). **Working must be seen**.	M1
	$\Rightarrow x=6\sqrt{2}-3$	cso $x=6\sqrt{2}-3$ **only** (or $-3+6\sqrt{2}$)	A1

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="7">6(a)</td><td colspan="2">$\left(1-\frac{1}{4}x\right)^{12}=1+12\left(-\frac{1}{4}x\right)+\frac{12\times11}{2\times1}\times\left(-\frac{1}{4}x\right)^{2}+\frac{12\times11\times10}{3\times2\times1}\times\left(-\frac{1}{4}x\right)^{3}+...$
Award for a correct binomial coefficient and a correct power of $\pm\frac{1}{4}x$ for term three and/or term 4, condoning the omission of the brackets.
E.g. allow $\frac{12\times11\times10}{3!}\times\frac{1}{4}x^3$ for term 4
Accept any notation for binomial coefficients e.g. as above or:
${}^{12}C_2$, ${}^{12}C_3$, $\binom{12}{2}$, $\binom{12}{3}$ or 66 or 220 from Pascal's triangle.</td><td>M1</td></tr>
<tr><td rowspan="3">$=\underline{1-3x}+\frac{33}{8}x^2-\frac{55}{16}x^3+...$</td><td>For $1-3x$ (Allow $-\frac{3x}{1}$ for $-3x$)</td><td>B1</td></tr>
<tr><td>For either $+\frac{33}{8}x^2$ or $-\frac{55}{16}x^3$</td><td>A1</td></tr>
<tr><td>For both $+\frac{33}{8}x^2$ and $-\frac{55}{16}x^3$</td><td>A1</td></tr>
<tr><td colspan="2">Allow equivalent fractions/full decimals for $\frac{33}{8}$ and $-\frac{55}{16}$
E.g. $4\frac{1}{8}$ or 4.125 for $\frac{33}{8}$ and $-3\frac{7}{16}$ or -3.4375 for $-\frac{55}{16}$</td><td></td></tr>
<tr><td colspan="2">Note that the $+\frac{33}{8}x^2$ can score from $+\frac{1}{4}x$ used in the expansion.</td><td></td></tr>
<tr><td colspan="2"></td><td>[4]</td></tr>
</table>

(b)(i)	Coefficient of x^2 of $(2+x)\left(1-\frac{1}{4}x\right)^{12}$ is $2\times\frac{33}{8}+1\times-3=\frac{21}{4}$ For attempting $2\times \textit{their}\frac{33}{8}+1\times \textit{their}-3$ (**allow <u>one</u> sign error**) **Note that this may be seen embedded within a complete expansion provided the coefficients are combined as indicated**	M1
	$\frac{21}{4}\left(\text{or } 5\frac{1}{4},\ 5.25\right)$ oe (Allow $x^2=\frac{21}{4}$) Note that $\boxed{\frac{21}{4}}x^2$ can be taken that their coefficient is $\frac{21}{4}$ **The coefficient must be clearly "extracted" for this mark but see special case note below**	A1
(ii)	Coefficient of x^2 of $\frac{(2+x)}{2x}\left(1-\frac{1}{4}x\right)^{12}$ is $1\times-\frac{55}{16}+\frac{1}{2}\times\frac{33}{8}=-\frac{11}{8}$ For attempting $1\times \textit{their}-\frac{55}{16}+\frac{1}{2}\times \textit{their}\frac{33}{8}$ (**allow <u>one</u> sign error**) **Note that this may be seen embedded within a complete expansion provided the coefficients are combined as indicated**	M1
	A1: $-\frac{11}{8}\left(\text{or } -1\frac{3}{8},\ -1.375\right)$ oe (Allow $x^2=-\frac{11}{8}$) Note that $\boxed{-\frac{11}{8}}x^2$ can be taken that their coefficient is $-\frac{11}{8}$ **The coefficient must be clearly "extracted" for this mark but see special case note below** **In (ii), if $\frac{(2+x)}{2x}$ is "processed" incorrectly e.g. as $(2+x)2x^{-1}$, then unless there is a recovery M0A0 is very likely**	A1
	<u>Special Case:</u> **If the x^2 s are included with the coefficients then penalise this once only and at the first occurrence.**	
		[4]
		(8 marks)

Note that if $+\frac{1}{4}x$ rather than $-\frac{1}{4}x$ is consistently used in (a) then the corresponding coefficients in b(i) and (ii) are $\frac{45}{4}$ and $\frac{11}{2}$ respectively. (For reference)

Question Number	Scheme		Marks
7(a)	$\dfrac{\sin ACB}{4x}=\dfrac{\sin 30°}{3x}$	Attempts the sine rule with the sides and angles in the correct places	M1
	$\sin ACB=\dfrac{0.5\times 4x}{3x}=\dfrac{2}{3}*$	Proceeds without errors to given answer with at least one intermediate line of working.	A1*
			[2]
(a) Way 2	$\dfrac{\frac{2}{3}}{4x}=\dfrac{\sin 30°}{3x}\Rightarrow\dfrac{\frac{2}{3}}{4x}=\dfrac{\frac{1}{2}}{3x}$	Attempts the sine rule with the sides and angles in the correct places and replaces sin ACB by 2/3 and sin 30 by 1/2	M1
	$2x=2x$ so $\sin ACB=\dfrac{2}{3}$	Correct working to achieve both sides equal **and** conclusion	A1
	<u>Notes:</u> Score M1A1 for $\sin ACB=\dfrac{4\sin 30°}{3}=\dfrac{2}{3}$ Score M1A0 for $\dfrac{\sin ACB}{4x}=\dfrac{\sin 30°}{3x}\Rightarrow ACB=41.81...\Rightarrow \sin ACB=\dfrac{2}{3}$ Score M0A0 for $ACB=41.81...\Rightarrow \sin ACB=\dfrac{2}{3}$ (no sin rule used)		
			[2]
(b)	(Obtuse $ACB=$) $180-\left(\sin^{-1}\left(\dfrac{2}{3}\right)\right)$ Attempts to find obtuse ACB but ignore how it is referenced i.e. just look for an attempt at the calculation		M1
	(Angle $ABC=$) awrt $11.81°$	Awrt $11.81°$ **<u>(Must be seen in (b))</u>**	A1
			[2]
	Note that in (c) and (d), the M marks are available for using *ABC* as 41.81… if the candidate clearly thinks that this is *ABC* – this may be seen labelled on the diagram at *B* or is clearly their answer to part (b)		
(c)	$20=\dfrac{1}{2}4x\times 3x\times\sin'11.81'$	Attempts to use Area of triangle formula $\dfrac{1}{2}ab\sin C$ with $A=20$, $4x$, $3x$ and their $11.81°$	M1
	$x^2=16.29$	Proceeds using correct arithmetic and fully correct processing to $x^2=...$ **Dependent on previous mark.**	**d**M1
	$x=4.04$	Awrt 4.04	A1
			[3]

(d)	Attempts the cosine rule **to obtain a value for** AC: $AC^2 = (4\times"4.04")^2 + (3\times"4.04")^2 - 2\times(4\times"4.04")(3\times"4.04")\cos("11.81")°$ $\Rightarrow AC = ...$ Condone poor bracketing e.g. $4\times"4.04"^2$ rather than $(4\times"4.04")^2$ Or uses area **to obtain a value for** AC: Uses $\frac{1}{2}\times 4"x"\times AC\sin 30° = 20 \Rightarrow AC = ...$ Or sine rule **to obtain a value for** AC: $\frac{AC}{\sin"11.81"} = \frac{3\times"x"}{\sin 30°} \Rightarrow AC = ...$ or $\frac{AC}{\sin"11.81"} = \frac{4\times"x"}{\sin(TheirACB)} \Rightarrow AC = ...$	M1
	$\Rightarrow AC = 4.96$ Awrt 4.96 (**allow also awrt 4.95**) This comes from $\frac{1}{2}\times 4"x"\times AC\sin 30° = 20 \Rightarrow AC = \frac{20}{x} = \frac{20}{4.04} = 4.95...$	A1
		[2]
		(9 marks)

Typical responses if acute *ACB* is used:

(b):

$$ACB = \sin^{-1}\left(\frac{2}{3}\right) = 41.81... \Rightarrow ABC = 180 - (30 + 41.81..) = 108.19.. \textbf{ M0A0}$$

(c):

$$\frac{1}{2}4x\times 3x\times\sin'108.19...' = 20 \textbf{ M1}$$

$$x^2 = 3.508... \textbf{ M1}$$

$$x = 1.87... \textbf{ A0}$$

(d):

$$AC^2 = (4\times1.87...)^2 + (3\times1.87...)^2 - 2\times(4\times1.87...)(3\times1.87...)\cos(108.19...)° = 10.6... \textbf{ M1A0}$$

$$\frac{1}{2}\times 4(1.87...)\times AC\sin 30° = 20 \Rightarrow AC = 10.6... \textbf{ M1A0}$$

$$\frac{AC}{\sin"108.19..."} = \frac{3\times"x"}{\sin 30°} \Rightarrow AC = 10.6... \textbf{ M1A0}$$

$$\frac{AC}{\sin"108.19..."} = \frac{4\times"x"}{\sin 41.81...} \Rightarrow AC = 10.6... \textbf{ M1A0}$$

Question Number	Scheme		Marks
8(a)	$(x\pm 3)^2+(y\pm 7)^2 ...=...$	Attempts to complete the square. Accept $(x\pm 3)^2+(y\pm 7)^2 ...=...$ as evidence. Also score for $(\pm 3, \pm 7)$	M1
	Centre = $(3,7)$	$(3,7)$ or $x=3$, $y=7$	A1
			[2]
(b)	$(r^2=)(3)^2+(7)^2+32$	Attempts $(\pm'3')^2+(\pm'7')^2\pm 32$. Just look for an attempt at this calculation and ignore how it is referenced e.g. as r or r^2. May be implied by sight of 90 or e.g. 58 ± 32.	M1
	Radius = $3\sqrt{10}$	oe such as $\sqrt{90}$ $(\pm 3\sqrt{10}$ is A0$)$	A1
			[2]
(c)	$k=58$ **or** $k=49$	For $k=58$ or $k=49$. May be implied by their inequalities but do not award for just seeing 49 or 58 as part of a calculation unless it is stated or implied as a value for k.	M1
	$k=58$ **and** $k=49$	Both values obtained with the same conditions as the previous mark.	A1
	One correct "end" e.g. $k>49$, $k<58$, $k\geqslant 49$, $k\leqslant 58$, [49, …], […, 58] etc.		M1
	Examples: $49<k<58$ $49\leqslant k<58$ $49<k\leqslant 58$ $49\leqslant k\leqslant 58$ [49, 58], [49, 58), (49, 58], (49, 58) $k>49$, $k<58$ $k>49$ or $k<58$ $k>49$ and $k<58$	Both "ends" correct	A1
			[4]
			(8 marks)

Question Number	Scheme		Marks
9(a)	$21 = p - 2q, -9 = p - 8q$	Attempts two equations in p and q one of which is correct.	M1
	$\Rightarrow p = 31, q = 5$	Solves 2 equations in p and q simultaneously. Accept values of p and q as evidence of solving. **Dependent on the first mark.**	**d**M1
		Either $p = 31$ **or** $q = 5$	A1
		Both $p = 31$ **and** $q = 5$	A1
			[4]
(b)	$u_{100} = '31' - 100 \times '5' = \ldots$ or $u_{100} = '31' - '5' + (100-1)\times(-5) = \ldots$	Attempts to use $u_{100} = 'p' - 100 \times 'q' =$ or Attempts $a + 99d$ with $a = p - q$ and $d = \pm q$	M1
	–469	Cao	A1
	Correct answer only scores both marks		
			[2]
(c) **Way 1**	$\frac{n}{2}\{2a + (n-1)d\}$ method: Correct values $n = 25$, $a = 1$, $d = -5$		
	$\sum_{n=6}^{30} u_n = \frac{n}{2}\{2a+(n-1)d\} = \frac{25}{2}\{2\times(31-6\times5)+(25-1)\times(-5)\}$ Allow this mark for: $\sum_{n=6}^{30} u_n = \frac{n}{2}\{2a+(n-1)d\}$ with $n = 24$ or 25, $a = p - 6q$, $d = \pm q$		M1
	$\sum_{n=6}^{30} u_n = \frac{n}{2}\{2a+(n-1)d\} = \frac{25}{2}\{2\times(31-6\times5)+(25-1)\times(-5)\}$ This mark is for a fully correct method with their p and q so needs to be: $\sum_{n=6}^{30} u_n = \frac{n}{2}\{2a+(n-1)d\}$ with $n = 25, a = p - 6q, d = -q$ **Dependent on the first mark**		**d**M1
	$= -1475$		A1
			[3]

(c) Way 2	$\frac{n}{2}\{a+l\}$ method: Correct values $n = 25$, $a = 1$, $l = -119$	
	$$\sum_{n=6}^{30} u_n = \frac{n}{2}\{a+l\} = \frac{25}{2}\{31-6\times5+31-30\times5\}$$ Allow this mark for: $$\sum_{n=6}^{30} u_n = \frac{n}{2}\{a+l\} \text{ with } n = 24 \text{ or } 25, a = p-6q, l = p-30q$$	M1
	$$\sum_{n=6}^{30} u_n = \frac{n}{2}\{a+l\} = \frac{25}{2}\{31-6\times5+31-30\times5\}$$ This mark is for a fully correct method with their p and q so needs to be: $$\sum_{n=6}^{30} u_n = \frac{n}{2}\{a+l\} \text{ with } n = 25, a = p-6q, l = p-30q$$ **Dependent on the first mark**	**d**M1
	$= -1475$	A1
(c) Way 3	$\sum_{1}^{30} - \sum_{1}^{5}$ method: Correct values $a = 26$, $d = -5$ Note that there are no marks for attempting $\sum_{n=1}^{5} u_n$ in isolation	
	$$\sum_{n=1}^{30} u_n = \frac{30}{2}\{2\times(31-5)+29\times(-5)\} \quad \text{or} = \frac{30}{2}\{26+31-5\times30\}$$ Allow this mark for: $$\sum_{n=1}^{30} u_n = \frac{30}{2}\{2a+29d\} \text{ or } \frac{30}{2}\{a+l\} \text{ with } a = p \text{ or } p-q,\ d = \pm q, l = p-30q$$ **Note that** $\sum_{n=1}^{30} u_n = -1395$	M1
	This mark is for a fully correct method with their p and q so needs to be: $$\sum_{n=6}^{30} u_n = \sum_{n=1}^{30} u_n - \sum_{n=1}^{5} u_n$$ Where: $$\sum_{n=1}^{30} u_n = \frac{30}{2}\{2a+29d\} \text{ or } \frac{30}{2}\{a+l\} \text{ and } \sum_{n=1}^{5} u_n = \frac{5}{2}\{2a+4d\} \text{ or } \frac{5}{2}\{a+l\}$$ with $a = p - q$, $d = -q$, $l = p - 30q$ **Dependent on the first mark** **Note that** $\sum_{n=1}^{5} u_n = 80$ $\left(\text{from } \frac{5}{2}(2\times26+4(-5)) \text{ or } \frac{5}{2}(26+6)\right)$	**d**M1
	$= -1475$	A1

(c) **Way 4**	$\sum_{n=6}^{30} p-qn=\sum_{n=6}^{30} p-q\sum_{n=6}^{30} n=25p-q\times\frac{1}{2}25(30+6)=25p-450q=-1475$ Splits into 2 sums and attempts both with n = 24 or 25 Look for: $np-q\times\frac{1}{2}n(30+6)$ or $np-q\times\frac{1}{2}n(2\times6+(n-1)\times1)$ oe With n = 24 or 25	M1
	Fully correct work with their values and n = 25	dM1
	$=-1475$	A1
		(9 marks)

You may see candidates who recognise it is an AP from the start.
In such cases, the following should be applied:

(a)

M1 For $d/q=\pm\frac{30}{6}$ or ±5

dM1 For $21='a'\pm \mathit{their}'5'$ or $-9='a'\pm7\times \mathit{their}'6'$ leading to $a=$

(b)

M1 For use of $a+99d$ with their a and d

(c)

M1 Attempts S_n with $a=u_6$, $l=u_{30}$ or $d=\pm5$, and $n=24/25$

dM1 Attempts S_n with $a=u_6$, $l=u_{30}$ or $d=-5$, and $n=25$

(c) Extra Notes For Information:

1. If they use $\sum_{n=6}^{30} u_n=\sum_{n=1}^{30} u_n-\sum_{n=1}^{6} u_n$ this gives –1395 – 81 = –1476 and scores M1dM0A0

2. Listing:

M1 for attempting 24 or 25 terms of the sequence and adding them together:

Terms are:

6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	-4	-9	-14	-19	-24	-29	-34	-39	-44	-49	-54	-59	-64	-69	-74	-79	-84	-89	-94	-99	-104	-109	-114	-119

dM1 for attempting to add 25 terms
A1: –1475

3. A correct answer of –1475 with no working scores 3/3 unless you suspect malpractice (can be done on a calculator now)

Question Number	Scheme		Marks
10(a)	$s = r\theta \Rightarrow \pi = r \times \dfrac{\pi}{6} \Rightarrow r = \ldots$ (cm)	Attempts to use the formula $s = r\theta$ with $s = \pi$ and $\theta = \dfrac{\pi}{6}$ and solves for r.	M1
	$r = 6$	$r = 6$ (cm)	A1
	Correct answer only scores both marks		
			[2]
(b)	$\frac{1}{2} \times '6'^2 \times \frac{\pi}{6} = (3\pi)$ Attempts area of sector $OAEO$ by using $A = \frac{1}{2}r^2\theta$ with $r = \mathit{their}\ 6$ and $\theta = \dfrac{\pi}{6}$		M1
	$\dfrac{1}{2} \times '12'^2 \times \left(2\pi - \dfrac{\pi}{6}\right) = (132\pi)$ Attempts area sector $OBCDO$ using $A = \dfrac{1}{2}r^2\theta$ with $r = 2 \times \mathit{their}\ 6$ and $\theta = k\pi - \dfrac{\pi}{6}$, where $k = 1, \dfrac{1}{2}, 2$, or 4 **or** $\frac{1}{2} \times '12'^2 \times \left(\frac{\pi}{6}\right)(=12\pi)$ **and** $\pi \times 12^2\ (=144\pi)$ Attempts area of larger circle using πr^2 with $r = 2 \times \mathit{their}\ 6$ and the area of sector OBD with $A = \dfrac{1}{2}r^2\theta$ and $\theta = \dfrac{\pi}{6}$ with $r = 2 \times \mathit{their}\ 6$		M1
	Total area = $3\pi + 132\pi = \ldots$ **or** Total area = $144\pi - (12\pi - 3\pi) = \ldots$ Fully correct method using $k = 2$ if appropriate. Finds total area by adding their sectors or subtracting the “hole” from the area of the large circle. **Dependent upon both previous method marks.**		**d**M1
	$= 135\pi$ (cm^2)	Units not required. (Note that the exact answer is required but for reference Area = 424.11…)	A1
			[4]

(c)	Arc length of sector BCD = $'12'\times\frac{11}{6}\pi=\left(22\pi\right)$	Attempts arc length of sector BCD using the formula $s=r\theta$ with $r=2\times their\ 6$ and $\theta=k\pi-\frac{\pi}{6}$, where $k=1, \frac{1}{2}$, 2, or 4	M1
	Total perimeter = sector BCD + $2\times'6'+\pi=...$ (cm)	Fully correct method using k = 2. Attempts to find the total perimeter by adding their arc length of sector BCD to $2\times'6'+\pi$. **Dependent on the previous mark.**	**d**M1
	$23\pi+12$ (cm)	Units not required. Allow if terms not collected e.g. 22π + 6 + 6 + π (Note that the exact answer is required but for reference Perim = 84.25…)	A1
			[3]
(c) Way 2	Arc length of sector BCD = $2\times\pi\times'12'-'12'\times\frac{\pi}{6}$	Attempts arc length of sector BCD using the formula $C=2\pi r$ with $r=2\times their\ 6$ and then subtracting the arc BD using $r\theta$ with $r=2\times their\ 6$ and $\theta=\frac{\pi}{6}$	M1
	Total perimeter = sector BCD + $2\times'6'+\pi=...$ (cm)	Attempts to find the total perimeter by adding their arc length of sector BCD to $2\times'6'+\pi$. **Dependent on the previous mark.**	**d**M1
	$23\pi+12$ (cm)	Units not required. Allow if terms not collected e.g. 22π + 6 + 6 + π (Note that the exact answer is required but for reference Perim = 84.25…)	A1
			(9 marks)

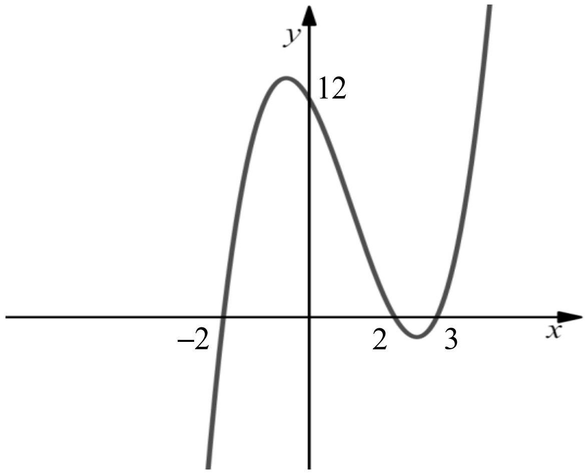
<u>**Special Case:**</u>

Some candidates having obtained“6” in part (a) think they have found OB
and then use OB = 2xOA to give OA = 3
The following can be applied but if you are unsure if this special case applies, please send to review

(a) M1A0

(b) M1M1**d**M0A0

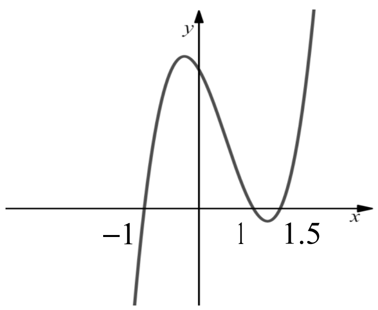
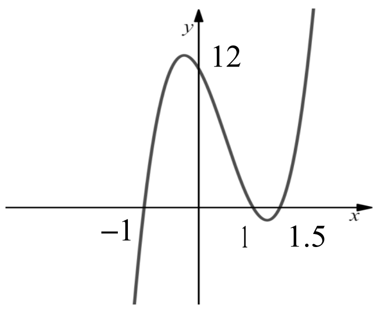
(c) M1**d**M0A0

Question Number	Scheme		Marks
11(a)		<u>Shape:</u> A positive cubic shape that crosses/touches the x-axis at least once. Allow the "ends" to turn back slightly as long as they do not tend to the horizontal or form extra turning points.	B1
		<u>Intercepts:</u> Allow for a y-intercept of 12 **or** x-intercepts of -2, 2 and 3 (See note below)	B1
		Correct shape with correct intercepts with a minimum in quadrant 4 and a maximum in quadrant 1 **or** quadrant 2 **or** at (0, 12). Allow the curve to stop at $(-2, 0)$	B1
	For the intercepts, allow them to be marked as shown in the diagram and also as e.g. (0, 12), (–2, 0), (2, 0), (3, 0) and allow the coordinates as (12, 0) etc. as long as they are marked in the correct places. If the coordinates are not on the diagram then they must be the right way round and correspond with the sketch. The sketch takes precedence if there is any ambiguity.		
	Note: If the sketch consists of 3 straight line segments but is otherwise correct award 110		
			[3]

(b)	$(x^2-4)(x-3)=x^3-3x^2-4x+12$	Attempts to multiply out. To score this mark must obtain a cubic with a least 3 (different) terms. **Allow this mark to score anywhere.**	M1
	$\int x^3-3x^2-4x+12\,\mathrm{d}x=\frac{1}{4}x^4-x^3-2x^2+12x$ M1: Integrates with at least three terms having their powers raised by 1 **Dependent on the first method mark** A1: Fully correct integration (allow unsimplified)		**d**M1 A1
	$\left[\frac{1}{4}x^4-x^3-2x^2+12x\right]_{-2}^{2}=(\ )-(\ )$	Uses limits 2 and −2 in their integrated (changed) function and subtracts either way round. May be implied – see note below.	M1
	$=32$ Note that some candidates calculate other areas in addition to *R*. In such cases, this final mark should be withheld if it is not clear that the area of *R* has been identified as 32 e.g. area under *x*-axis = 0.75 so area of *R* is 32 + 0.75 = 32.75		A1
			[5]

(b) Notes:

Correct integration followed by a correct answer scores **full marks** e.g. $\int_{-2}^{2}(x^3-3x^2-4x+12)\,\mathrm{d}x=\left[\frac{1}{4}x^4-x^3-2x^2+12x\right]_{-2}^{2}=32$ So that the substitution can be implied in such cases But Values to look for when substituting if needed: $\left[\frac{1}{4}x^4-x^3-2x^2+12x\right]_{-2}^{2}=(4-8-8+24)-(4+8-8-24)=12-(-20)=32$
If there is no integration then only the first mark for expanding is available e.g. $\int_{-2}^{2}(x^3-3x^2-4x+12)\,\mathrm{d}x=32$ Scores M1dM0A0M0A0

<table>
<tr><td>(c)(i)</td><td colspan="3">$(y=)(4x^2-4)(2x-3)$
Allow any equivalent correct expressions
e.g. $(2x)^3-3(2x)^2-4(2x)+12,\ (2x-2)(2x+2)(2x-3)$
and “y =” not required.
isw once a correct expression is seen</td><td>B1</td></tr>
<tr><td>(ii)</td><td colspan="3">In (c) part (ii), mark positively where possible
Note that strictly speaking, a stretch requires an invariant line but we are not insisting that candidates refer to an invariant line here</td><td></td></tr>
<tr><td></td><td>1. Examples
Stretch/Contract/Shrink/ Compress/Enlarge/ Smaller/Thinner/ Contracted
(Any idea of size change)</td><td>2. Examples
Scale factor 0.5/Divides by 2</td><td>3. Examples
Parallel to/on/at the x-axis/ Horizontally</td><td>M1A1</td></tr>
<tr><td></td><td colspan="3">M1: For any 2 of the above
A1: For all of the above</td><td></td></tr>
<tr><td></td><td colspan="3"><u>Special Case: Covers 2 & 3 above</u>
x (values) divided by 2 (halved) on its own scores M1A0 - must reference x and halving e.g. accept for this special case
• x halved
• multiply x by ½</td><td></td></tr>
<tr><td></td><td colspan="3"></td><td>[3]</td></tr>
<tr><td></td><td colspan="3"><u>Examples:</u>
Enlarge scale factor 2 parallel to the x-axis = M1A0
The x values are divided by 2 and no change in the y values = M1A1
The x values change to $-1, 1$ and $\frac{3}{2}$ = M1A0
New coordinates are (0, 12), (-1, 0), (1, 0), (1.5, 0) = M1A0</td><td></td></tr>
<tr><td></td><td></td><td colspan="2">scores M1A0</td><td></td></tr>
<tr><td></td><td></td><td colspan="2">scores M1A1</td><td></td></tr>
<tr><td></td><td colspan="3"></td><td>(11 marks)</td></tr>
</table>

Question Number	Scheme		Marks
12(i)	**Examples:** $\log_p 2x - \log_p 5 = \log_p\left(\frac{2x}{5}\right)$, $\log_p 8 + \log_p 5 = \log_p 40$ $3 = \log_p p^3$, $\log_p 8 + 3 = \log_p 8 + \log_p "y" = \log_p 8"y"$ This mark is to be awarded for evidence of the use of a correct log law. Allow slips when rearranging as long as a correct law is used e.g. $\log_p 2x - \log_p 5 = 3 + \log_p 8 \Rightarrow \log_p 2x = 3 + \log_p 8 - \log_p 5 = \log_p \frac{8}{5}$		M1
	Examples: $\log_p\left(\frac{2x}{5}\right) = \log_p 8p^3$, $\log_p\left(\frac{2x}{40}\right) = \log_p p^3$, $\log_p\left(\frac{2x}{40}\right) = 3$, $\log_p\left(\frac{\frac{2x}{5}}{8}\right) = 3$ This mark is for a correct equation of the form log p = log q or log p = q		A1
	Examples: $\frac{2x}{5} = 8p^3 \Rightarrow x = ...$, $\frac{2x}{40} = p^3 \Rightarrow x = ...$ This mark is for removing the logs correctly and reaches $x = \ldots$ **Dependent on the first method mark**		**d**M1
	$x = 20p^3$	($x = \frac{40p^3}{2}$ or $\frac{p^3}{0.05}$ or $\frac{p^3}{\frac{1}{20}}$ is A0)	A1cso
			[4]

(i) Special case – incorrect use of logs:

$$\log_p 2x - \log_p 5 = 3 + \log_p 8$$

$$\frac{\log_p 2x}{\log_p 5} = \log_p p^3 + \log_p 8$$

$$\frac{\log_p 2x}{\log_p 5} = \log_p 8p^3$$

$$\frac{2x}{5} = 8p^3$$

$$x = 20p^3$$

Scores M1A0M1A0

(This can be applied in similar cases but if in doubt send to review)

(ii)	$2\left(\log_2 y\right)^2+7\log_2 y-15=0 \Rightarrow \left(2\log_2 y-3\right)\left(\log_2 y+5\right)=0$ or e.g. $2x^2+7x-15=0 \Rightarrow \left(2x-3\right)\left(x+5\right)=0$ Attempts to solve the correct quadratic equation – see General Guidance		M1
	$\Rightarrow \left(\log_2 y\right)=\frac{3}{2},-5$	Correct values (ignore lhs)	A1
	$\log_2 y=C \Rightarrow y=2^C$	Undoes the log correctly at least once. May be implied by e.g. $\log_2 y=1.5 \Rightarrow y=2.82...$ **Dependent on the first method mark.**	**d**M1
	$y=2\sqrt{2}$ **or** $y=\dfrac{1}{32}$	One correct. Must be $2\sqrt{2}$ but allow $2^{-5},\frac{1}{2^5}$, 0.03125 for $\frac{1}{32}$	A1
	$y=2\sqrt{2}$ **and** $y=\dfrac{1}{32}$	Both correct. Must be $2\sqrt{2}$ but allow 2^{-5}, $\frac{1}{2^5}$, 0.03125 for $\frac{1}{32}$ and no other values.	A1
			[5]
			(9 marks)

Beware wrong working leading to $y=2^{-5}$

$$2\left(\log_2 y\right)^2+7\log_2 y=15 \Rightarrow \log_2 y^4+\log_2 y^7=15 \Rightarrow \log_2 \frac{y^4}{y^7}$$

$$y^{-3}=2^{15} \Rightarrow y=\left(2^{15}\right)^{-\frac{1}{3}}=2^{-5}$$

(= No marks)

Question Number	Scheme		Marks
13(i)	$7\sin 2\theta = 5\cos 2\theta \Rightarrow (2\theta =)\arctan\left(\frac{5}{7}\right)$ Score for $\tan ... = \frac{5}{7}$ or $\tan ... = \frac{7}{5}$		M1
	$(2\theta =)\arctan\left(\frac{5}{7}\right)$	For sight of $\arctan\left(\frac{5}{7}\right)$. This may be implied by awrt 35…° or 215…° or a value for θ of 17…° or 107…° Or the equivalent in radians (0.62, 3.8, 0.31, 1.9)	A1
	$(\theta =)$ awrt $17.8°$, $107.8°$	Proceeds to find at least one value for θ using correct order of operations. May be implied by one correct value or truncated e.g. $17.7°, 107.7°$. **Dependent on the first method mark.**	**d**M1
		Both correct. Allow awrt $17.8°$, $107.8°$ and no other values in range. Ignore answers outside the range.	A1
			[4]

	Alternative by squaring:		
	$7\sin 2\theta = 5\cos 2\theta \Rightarrow 49\sin^2 2\theta = 25\cos^2 2\theta$ $\Rightarrow 49\left(1-\cos^2 2\theta\right) = 25\cos^2 2\theta$ or $\Rightarrow 49\sin^2 2\theta = 25\left(1-\sin^2 2\theta\right)$ Squares both sides and uses $\cos^2 2\theta = \pm 1 \pm \sin^2 2\theta$ or $\sin^2 2\theta = \pm 1 \pm \cos^2 2\theta$		M1
	$(2\theta =)\arccos\left((\pm)\frac{7}{\sqrt{74}}\right)$ or $(2\theta =)\arcsin\left((\pm)\frac{5}{\sqrt{74}}\right)$	For sight of $\arccos\left((\pm)\frac{7}{\sqrt{74}}\right)$ or $\arcsin\left((\pm)\frac{5}{\sqrt{74}}\right)$. This may be implied by awrt 35…° or 215…° or a value for θ of 17…° or 107…° Or the equivalent in radians (0.62, 3.8, 0.31, 1.9)	A1
	$\theta =$ awrt $17.8°$, $107.8°$	Proceeds to find at least one value for θ using correct order of operations. May be implied by one correct value or truncated e.g. $17.7°, 107.7°$. **Dependent on the first method mark.**	dM1
		Both correct. Allow awrt $17.8°$, $107.8°$ and no other values in range. Ignore answers outside the range.	A1

Any attempts in (i) that use double angle formulae that you think may deserve any credit should be sent to review

(ii)	$24\tan x = 5\cos x \Rightarrow 24\sin x = 5\cos^2 x$	Uses the identity $\tan x = \dfrac{\sin x}{\cos x}$ and moves to an equation of the type $A\sin x = B\cos^2 x$ or equivalent.	M1
	$\Rightarrow 24\sin x = 5\left(1-\sin^2 x\right)$	Uses the identity $\cos^2 x = 1-\sin^2 x$ to produce a quadratic equation in $\sin x$ **Depends on the first method mark**	**d**M1
	$\Rightarrow 5\sin^2 x + 24\sin x - 5 = 0$	Correct 3 term quadratic **with terms all on one side**.	A1
	$\Rightarrow \sin x = \frac{1}{5}$	Attempts to solve 3TQ in $\sin x$ – see general guidance. Must be $\sin x = \ldots$ but may be implied by their attempt to solve.	M1
	$\Rightarrow x = \text{awrt}\, 0.201,\ 2.940$ Or $x = \text{awrt}\, 0.064\pi,\ 0.936\pi$ *or* $\dfrac{23}{360}\pi, \dfrac{337}{360}\pi$ Both (awrt) $x = 0.201,\ 2.940$ or $0.064\pi, 0.936\pi$ or $\dfrac{23}{360}\pi, \dfrac{337}{360}\pi$ and no other values in range. Ignore answers outside the range. Allow 2.94 as the second angle **but not awrt 2.94 e.g. <u>do not accept 2.941</u>**		A1
	<u>Note:</u> The final mark in (ii) depends on having a correct 3TQ in $\sin x$ i.e. must follow the previous A1, but if the 3TQ is factorised incorrectly e.g. $\left(5\sin x - 1\right)\left(\sin x - 5\right) = 0 \Rightarrow \sin x = \dfrac{1}{5}, \left(5\right) \Rightarrow x = 0.201,\ 2.940$ then allow full recovery. **Mark their final answers and do not apply isw for the final mark.**		
			[5]
			(9 marks)

Possible alternative in (ii):		
$24\tan x = 5\cos x \Rightarrow 576\tan^2 x = 25\cos^2 x$ $\Rightarrow 576\left(\sec^2 x - 1\right) = 25\cos^2 x$	Squares both sides and uses the identity $1+\tan^2 x = \sec^2 x$ to reach $\alpha\left(\sec^2 x - 1\right) = \beta\cos^2 x$	M1
$\Rightarrow 576\left(\dfrac{1}{\cos^2 x} - 1\right) = 25\cos^2 x$ $\Rightarrow 576\left(1-\cos^2 x\right) = 25\cos^4 x$	Uses the identity $\sec^2 x = \dfrac{1}{\cos^2 x}$ to produce a quadratic equation in $\cos^2 x$ **Depends on the first method mark**	**d**M1
$\Rightarrow 25\cos^4 x + 576\cos^2 x - 576 = 0$	Correct 3 term quadratic (not necessarily all on one side e.g. allow $25\cos^4 x + 576\cos^2 x = 576$)	A1
$\Rightarrow \left(25\cos^2 x - 24\right)\left(\cos^2 x + 24\right) = 0$ $\Rightarrow \cos^2 x = \dfrac{24}{25} \Rightarrow \cos x = \dfrac{2\sqrt{6}}{5}$	Attempts to solve 3TQ in $\cos^2 x$ – see general guidance and reaches cos x = … but may be implied by their attempt to solve.	M1
$\Rightarrow x = 0.201,\ 0.940$	See above	A1

Question Number	Scheme		Marks
14(a)	$140\,000 \times r^2 = 150\,000$	For sight of $140\,000 \times r^2 = 150\,000$ (r may be called p or even $1 + p$)	M1
	$r^2 = \frac{15}{14} \Rightarrow r = 1.0351$	For awrt 1.03 or 1.04 or exact $\sqrt{\frac{15}{14}}, \frac{\sqrt{210}}{14}$. (It may be called p and ignore any % symbols)	A1
	$\Rightarrow p = 3.51$	Correct value only	B1
			[3]
(a) Way 2	$140\,000 \times \left(1 + \frac{p}{100}\right)^2 = 150\,000$	For sight of $140\,000 \times \left(1 + \frac{p}{100}\right)^2 = 150\,000$ or e.g. $140\,000 \times \left(\frac{100+p}{100}\right)^2 = 150\,000$	M1
	$\left(1 + \frac{p}{100}\right) = 1.0351$	For awrt 1.03 or 1.04 or exact $\sqrt{\frac{15}{14}}, \frac{\sqrt{210}}{14}$.	A1
	$\Rightarrow p = 3.51$	Correct value only	B1
(a) Way 3	$\frac{150\,000}{u_2} = \frac{u_2}{140000} \Rightarrow u_2 = \sqrt{150000 \times 140000} \Rightarrow r = \frac{\sqrt{150000 \times 140000}}{140000}$ Sight of $\frac{150\,000}{u_2} = \frac{u_2}{140000}$ (oe) and attempts to find r		M1
	$r = 1.0351$	For awrt 1.03 or 1.04 or exact $\sqrt{\frac{15}{14}}, \frac{\sqrt{210}}{14}$. (It may be called p)	A1
	$\Rightarrow p = 3.51$	Correct value only	B1
(a) Way 4	$140\,000 \times \left(1 + \frac{p}{100}\right)^2 = 150\,000$ or $140\,000 \times \left(\frac{100+p}{100}\right)^2 = 150\,000$ Sight of the above		M1
	$140\,000 \times \left(1 + \frac{p}{50} + \frac{p^2}{10000}\right) = 150\,000$ $\Rightarrow 7p^2 + 1400p - 5000 = 0$	Correct 3TQ	A1
	$\Rightarrow p = 3.51$	Correct value only	B1

(b)	In (b) the marks are available for solving an equation or an inequality so allow "=", ">", "<" etc. but the final mark must be a value not a range so e.g. $N > 37$ scores B0		
	$140\,000\times(1.0351)^{"N"}=500\,000$	States or uses $140\,000\times("1.0351")^{"N"}=500\,000$ or $140\,000\times("1.0351")^{"N-1"}=500\,000$ Condone poor notation e.g. if their r is $\frac{15}{14}$, allow $140\,000\times\frac{15}{14}^{"N"}=500\,000$ **Requires $r > 1$**	M1
	$("1.0351")^{"N"}=\frac{25}{7}$	"Correct" intermediate statement $("1.0351")^{"N"}=\frac{25}{7}$ or $("1.0351")^{"N-1"}=\frac{25}{7}$	A1
	Examples: $"N"=\frac{\log\left(\frac{25}{7}\right)}{\log"1.0351"}=...,\quad "N"=\log_{"1.0351"}\left(\frac{25}{7}\right)=...$ Uses logs correctly to find N or $N-1$ **Dependent on the first method mark**		**d**M1
	Note that the first 3 marks in (b) can score for <u>their r</u> (which may be p) but r must be greater than 1 for the dM1 mark		
	"N" = awrt 36.9 or "$N-1$" = awrt 36.9	Correct value for N or $N-1$. May be implied by a final answer of 37 and can be implied by e.g. $"N-1"=\log_{"1.0351"}\left(\frac{25}{7}\right)\Rightarrow N=37.9$	A1
	$N = 37$	Cao	B1
	Note that if e.g. $("1.0351")^{"N"}=\frac{25}{7}$ is followed by $N = 37$ without the intermediate log work, this scores M1A1M0A0B1		
			[5]
			(8 marks)

Note that some may work with ar^{N-1} in (b) completely correctly if they take "*a*" as the second term:

E.g.

$$140\,000\times\sqrt{\frac{15}{14}}("1.0351")^{"N-1"}=500\,000$$

$$\left(\sqrt{\frac{15}{14}}\right)^{"N-1"}=\frac{500\,000}{140000}\sqrt{\frac{14}{15}}$$

$$"N-1"=\log_{\sqrt{\frac{15}{14}}}\frac{500\,000}{140000}\sqrt{\frac{14}{15}}=35.9...$$

$$N=37$$

Question Number	Scheme	Marks
15(a)	**NB Allow *H* for *h* throughout**	
	$5 = \pi r^2 h + \frac{4}{3}\pi r^3 \Rightarrow h = \frac{5 - \frac{4}{3}\pi r^3}{\pi r^2}$ Uses $5 = \pi r^2 h + \frac{4}{3}\pi r^3$ or $5 = \pi r^2 h + \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$ and attempts to make h, rh or πrh the subject. **Must use a correct volume formula**	M1
	$h = \frac{5 - \frac{4}{3}\pi r^3}{\pi r^2}$ or $h = \frac{5}{\pi r^2} - \frac{4}{3}r$ or $rh = \frac{5 - \frac{4}{3}\pi r^3}{\pi r}$ or $hr = \frac{5}{\pi r} - \frac{4}{3}r^2$ or $\pi rh = \frac{5 - \frac{4}{3}\pi r^3}{r}$ Correct expression for h, rh or πrh **Award this mark once a correct expression is seen and ignore subsequent attempts to "simplify"**	A1
	$A = 4\pi r^2 + 2\pi rh \Rightarrow A = 4\pi r^2 + 2\pi r \times \frac{5 - \frac{4}{3}\pi r^3}{\pi r^2}$ Subs $h = ..$ or $rh = ..$ or $\pi rh = ...$ into $A = 4\pi r^2 + 2\pi rh$ to get A in terms of r **Must use a correct area formula**	M1
	$\Rightarrow A = \frac{10}{r} + \frac{4}{3}\pi r^2 *$ Completes proof with no errors or omissions. Allow $A = 4\pi r^2 + 2\pi r \times \frac{5 - \frac{4}{3}\pi r^3}{\pi r^2} = \frac{10}{r} + \frac{4}{3}\pi r^2$	A1*
		[4]

(b)	$\left(\frac{\mathrm{d}A}{\mathrm{d}r}=\right)-\frac{10}{r^2}+\frac{8}{3}\pi r$	Differentiates and gets one term correct (unsimplified)	M1
		$\frac{\mathrm{d}A}{\mathrm{d}r}=-\frac{10}{r^2}+\frac{8}{3}\pi r$ (may be unsimplified)	A1
	$\Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r}=0 \Rightarrow r=1.06(\mathrm{m})$	Sets $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ and proceeds to $r^3=C$ where C is a positive constant. This is implied by $r=$... **Dependent on first method mark.**	**d**M1
		$r=$ awrt $1.06(\mathrm{m})$ or exact $r=\sqrt[3]{\frac{15}{4\pi}}$ oe May be implied.	A1
	$\Rightarrow A=\frac{10}{1.06}+\frac{4}{3}\pi\times1.06^2=14.14(\mathrm{m}^2)$	Substitutes their 1.06 (must be positive) into $A=\frac{10}{r}+\frac{4}{3}\pi r^2$ **Dependent on both previous method marks**	**dd**M1
		awrt $14.1(\mathrm{m}^2)$	A1
			[6]
(c)	$\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=\frac{8}{3}\pi+\frac{20}{r^3}\Big\vert_{r=1.06}=...$	Obtains $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=A\pm\frac{B}{r^3}\,(A,B\neq0)$ and substitutes in their positive r from (b) and considers sign **or** makes reference to the sign of the second derivative **provided they have a positive *r*.**	M1
	$\left(\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=\right)\frac{8}{3}\pi+\frac{20}{1.06^3}\Big\vert \Rightarrow \frac{\mathrm{d}^2A}{\mathrm{d}r^2}>0 \therefore$ minimum Requires a **correct second derivative** and the **correct value of *r*.** There must be a reference to the sign of the second derivative. If r is substituted and then $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ is evaluated incorrectly allow this mark if the other conditions are met. If r is not substituted then the reference to $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ being positive must also include a reference to the fact that r is positive. NB $\left(\frac{\mathrm{d}^2A}{\mathrm{d}r^2}\right)_{r=\sqrt[3]{\frac{15}{4\pi}}}=8\pi=25.13...$ If there are any incorrect statements this mark should be withheld E.g. "$r>0$ therefore minimum" rather than $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}>0 \therefore$ minimum		A1
			[2]

(d)	$r = 1.06 \Rightarrow h = \dfrac{5 - \frac{4}{3}\pi r^3}{\pi r^2}$	Substitutes their positive $r = 1.06$ into a **correct** expression for h **or** their (possibly incorrect) h from part (a). **Must obtain a value.**	M1
	$h = 0$	Cao	A1
			[2]
(d) Way 2	$4\pi r^2 + 2\pi rh = \dfrac{10}{r} + \dfrac{4}{3}\pi r^2$ $4\pi(1.06)^2 + 2\pi(1.06)h = 14.1...$ $\Rightarrow h = ...$	Uses the given A in terms of r and sets equal to a **correct** expression for A **or** their (possibly incorrect) A from part (a) and uses their r to find h **Must obtain a value.**	M1
	$h = 0$	Cao	A1
(d) Way 3	$\dfrac{4}{3}\pi r^3 + \pi r^2 h = 5$ $\Rightarrow \pi\left(\dfrac{15}{4\pi}\right)^{\frac{2}{3}} h + \dfrac{4}{3}\pi\left(\dfrac{15}{4\pi}\right) = 5 \Rightarrow h = ...$	Uses $V = 5$ with a **correct** V **or** their (possibly incorrect) V from part (a) and their r to find h. **Must obtain a value.**	M1
	$h = 0$	Cao	A1
			(14 marks)

Note regarding a correct value for *r* fortuitously:

Example – (this has been seen):

$$\left(\frac{dA}{dr} = \right)\frac{10}{r^2} + \frac{8}{3}\pi r = 0 \text{ (Sign error)}$$

$$\left(\frac{dA}{dr} = \right)\frac{10}{r^2} = \frac{8}{3}\pi r \Rightarrow r^3 = \frac{15}{4\pi} \text{ (Another sign error)}$$

$$\Rightarrow r = \sqrt[3]{\frac{15}{4\pi}}$$

$$\Rightarrow A = \frac{10}{1.06} + \frac{4}{3}\pi \times 1.06^2 = 14.14\left(\text{m}^2\right)$$

Can score M1A0M1A0M1A1 and then allow a full recovery in (c) and (d)

Also, if e.g. *r* = - 1.06 is obtained in (b) then a similar "recovery" approach can be taken with the marking so that the final M1A1 can be awarded in (b) if *r* = +1.06 is used to obtain 14.1 and allow a full recovery in (c) and (d) if *r* = +1.06 is also used