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Mark Scheme (Results)
Summer 2007

GCE

GCE Mathematics

Core Mathematics C2 (6664)

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June 2007
6664 Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	$\int x^{-\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (Or equivalent, such as $2x^{\frac{1}{2}}$, or $2\sqrt{x}$)	M1 A1
	$\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_1^8 = 2\sqrt{8} - 2 = -2 + 4\sqrt{2}$ [or $4\sqrt{2} - 2$, or $2(2\sqrt{2} - 1)$, or $2(-1 + 2\sqrt{2})$]	M1 A1
		(4) **4**
	1st M: $x^{-\frac{1}{2}} \to kx^{\frac{1}{2}}$, $k \neq 0$. 2nd M: Substituting limits 8 and 1 into a 'changed' function (i.e. not $\frac{1}{\sqrt{x}}$ or $x^{-\frac{1}{2}}$), and subtracting, either way round. 2nd A: This final mark is still scored if $-2 + 4\sqrt{2}$ is reached via a decimal. N.B. Integration constant $+C$ may appear, e.g. $\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + C\right]_1^8 = (2\sqrt{8} + C) - (2 + C) = -2 + 4\sqrt{2}$ (Still full marks) <u>But</u>… a final answer such as $-2 + 4\sqrt{2} + C$ is A0. N.B. It will sometimes be necessary to 'ignore subsequent working' (isw) after a correct form is seen, e.g. $\int x^{-\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (M1 A1), followed by incorrect simplification $\int x^{-\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = \frac{1}{2}x^{\frac{1}{2}}$ (still M1 A1)…. The second M mark is still available for substituting 8 and 1 into $\frac{1}{2}x^{\frac{1}{2}}$ and subtracting.	

Question number	Scheme	Marks
2.	(a) f(2) = 24 – 20 – 32 + 12 = –16 (M: Attempt f(2) or f(–2))	M1 A1 (2)
	(If continues to say 'remainder = 16', isw)	
	Answer must be seen in part (a), not part (b).	
	(b) $(x+2)(3x^2-11x+6)$	M1 A1
	$(x+2)(3x-2)(x-3)$	M1 A1 (4)
	(If continues to 'solve an equation', isw)	
		6
	(a) Answer only (if correct) scores both marks. (16 as 'answer only' is M0 A0). Alternative (long division): Divide by (x – 2) to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. [M1] $(3x^2+x-14)$, and – 16 seen. [A1] (If continues to say 'remainder = 16', isw) (b) First M requires division by (x + 2) to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. Second M for attempt to factorise their quadratic, even if wrongly obtained, perhaps with a remainder from their division. Usual rule: $(kx^2+ax+b)=(px+c)(qx+d)$, where $\lvert pq\rvert=\lvert k\rvert$ and $\lvert cd\rvert=\lvert b\rvert$. Just solving their quadratic by the formula is M0. "Combining" all 3 factors is not required. Alternative (first 2 marks): $(x+2)(3x^2+ax+b)=3x^3+(6+a)x^2+(2a+b)x+2b=0$, then compare coefficients to find values of a and b. [M1] $a=-11,\ b=6$ [A1] Alternative: Factor theorem: Finding that $\mathrm{f}(3)=0$ $\therefore$ factor is, $(x-3)$ [M1, A1] Finding that $\mathrm{f}\left(\frac{2}{3}\right)=0$ $\therefore$ factor is, $(3x-2)$ [M1, A1] If just one of these is found, score the first 2 marks M1 A1 M0 A0. Losing a factor of 3: $(x+2)\left(x-\frac{2}{3}\right)(x-3)$ scores M1 A1 M1 A0. Answer only, one sign wrong: e.g. $(x+2)(3x-2)(x+3)$ scores M1 A1 M1 A0.	

Question number	Scheme	Marks	
3.	(a) $1+6kx$ [Allow unsimplified versions, e.g. $1^6+6(1^5)kx$, ${}^6C_0+{}^6C_1kx$]	B1	
	$+\frac{6\times5}{2}(kx)^2+\frac{6\times5\times4}{3\times2}(kx)^3$ [See below for acceptable versions]	M1 A1	(3)
	N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied)		
	(b) $6k=15k^2$ $k=\frac{2}{5}$ (or equiv. fraction, or 0.4) (Ignore k = 0, if seen)	M1 A1cso	(2)
	(c) $c=\frac{6\times5\times4}{3\times2}\left(\frac{2}{5}\right)^3=\frac{32}{25}$ (or equiv. fraction, or 1.28)	A1cso	(1)
	(Ignore x^3, so $\frac{32}{25}x^3$ is fine)		
			6

(a) The terms can be 'listed' rather than added.

M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x. Allow a 'slip' or 'slips' such as:

$+\frac{6\times5}{2}kx^2+\frac{6\times5\times4}{3\times2}kx^3$, $\quad +\frac{6\times5}{2}(kx)^2+\frac{6\times5}{3\times2}(kx)^3$

$+\frac{5\times4}{2}kx^2+\frac{5\times4\times3}{3\times2}kx^3$, $\quad +\frac{6\times5}{2}x^2+\frac{6\times5\times4}{3\times2}x^3$

<u>But</u>: $15+k^2x^2+20+k^3x^3$ or similar is M0.

Both x^2 and x^3 terms must be seen.

$\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable, and

even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark.

A1: Any correct (possibly unsimplified) version of these 2 terms.

$\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable.

<u>Descending powers of x:</u>
Can score the M mark if the required first 4 terms are not seen.

<u>Multiplying out</u> $(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)$:
M1: A full attempt to multiply out (power 6)
B1 and A1 as on the main scheme.

(b) M: Equating the coefficients of x and x^2 (even if trivial, e.g. $6k = 15k$).
Allow this mark also for the 'misread': equating the coefficients of x^2 and x^3.
An equation in k alone is required for this M mark, although…
…condone $6kx=15k^2x^2 \Rightarrow \left(6k=15k^2 \Rightarrow\right) k=\frac{2}{5}$.

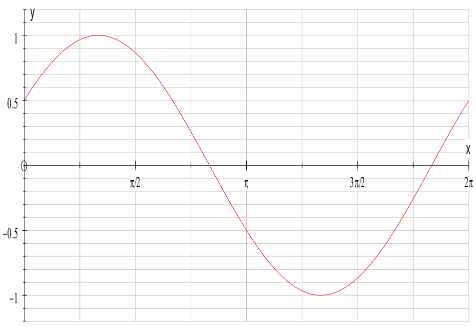
Question number	Scheme	Marks	
4.	(a) $4^2 = 5^2 + 6^2 - (2\times5\times6\cos\theta)$	M1	
	$\cos\theta = \dfrac{5^2+6^2-4^2}{2\times5\times6}$	A1	
	$\left(=\dfrac{45}{60}\right)=\dfrac{3}{4}$ (*)	A1cso	(3)
	(b) $\sin^2 A + \left(\dfrac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1	
	$\left(\sin^2 A = \dfrac{7}{16}\right)$ $\sin A = \dfrac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\dfrac{7}{16}}$, $\sqrt{0.4375}$	A1	(2)
			5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2\times4\times6\cos\theta)$ or $6^2 = 5^2 + 4^2 - (2\times5\times4\cos\theta)$ or $\cos\theta = \dfrac{4^2+6^2-5^2}{2\times4\times6}$ or $\cos\theta = \dfrac{5^2+4^2-6^2}{2\times5\times4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = ...$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). <u>Alternative</u> (verification): $4^2 = 5^2 + 6^2 - \left(2\times5\times6\times\dfrac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25+36-45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving 16 = 16 is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. <u>Correct answer without working</u> (or with unclear working or decimals): Still scores both marks.		

Question number	Scheme	Marks
5.	(a) 1.414 (allow also exact answer √2), 3.137 Allow awrt	B1, B1 (2)
	(b) $\frac{1}{2}(0.5)$ ….	B1
	…. $\{0+6+2(0.530+1.414+3.137)\}$	M1 A1ft
	= 4.04 (Must be 3 s.f.)	A1 (4)
	(c) Area of triangle = $\frac{1}{2}(2\times 6)$	B1
	(Could also be found by integration, or even by the trapezium rule on $y = 3x$)	
	Area required = Area of triangle – Answer to (b) (Subtract either way round)	M1
	6 – 4.04 = 1.96 Allow awrt	A1ft (3)
	(ft from (b), dependent on the B1, and on answer to (b) less than 6)	
		9
	(a) If answers are given to only 2 d.p. (1.41 and 3.14), this is B0 B0, but full marks can be given in part (b) if 4.04 is achieved. (b) Bracketing mistake: i.e. $\frac{1}{2}(0.5)(0+6)+2(0.530+1.414+3.137)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). Alternative (finding and adding separate areas): $\frac{1}{2}\times\frac{1}{2}$ (Triangle/trapezium formulae, and height of triangle/trapezium)[B1] Fully correct method for total area, with values from table. [M1, A1ft] 4.04 [A1] (c) B1: Can be given for 6 with no working, but should not be given for 6 obtained from wrong working. A1ft: This is a dependent follow-through: the B1 for 6 must have been scored, and the answer to (b) must be less than 6.	

Question number	Scheme	Marks
6.	(a) $x = \dfrac{\log 0.8}{\log 8}$ or $\log_8 0.8$, $= -0.107$ Allow awrt	M1, A1 (2)
	(b) $2\log x = \log x^2$	B1
	$\log x^2 - \log 7x = \log \dfrac{x^2}{7x}$	M1
	"Remove logs" to form equation in x, using the base correctly: $\dfrac{x^2}{7x} = 3$	M1
	$x = 21$ (Ignore $x = 0$, if seen)	A1cso (4)
		6
	(a) Allow also the 'implicit' answer $8^{-0.107}$ (M1 A1). Answer only: –0.107 or awrt: Full marks. Answer only: –0.11 or awrt (insufficient accuracy): M1 A0 Trial and improvement: Award marks as for "answer only". (b) <u>Alternative:</u> $2\log x = \log x^2$ B1 $\log 7x + 1 = \log 7x + \log 3 = \log 21x$ M1 "Remove logs" to form equation in x: $x^2 = 21x$ M1 $x = 21$ (Ignore $x = 0$, if seen) A1 <u>Alternative:</u> $\log 7x = \log 7 + \log x$ B1 $2\log x - (\log 7 + \log x) = 1$ $\log_3 x = 1 + \log_3 7$ M1 $x = 3^{(1+\log_3 7)}$ $\left(= 3^{2.771...}\right)$ or $\log_3 x = \log_3 3 + \log_3 7$ M1 $x = 21$ A1 Attempts using change of base will usually require the same steps as in the main scheme or alternatives, so can be marked equivalently. <u>A common mistake:</u> $\log x^2 - \log 7x = \dfrac{\log x^2}{\log 7x}$ B1 M0 $\dfrac{x^2}{7x} = 3$ $x = 21$ M1('Recovery'), but A0	

Question number	Scheme	Marks	
7.	(a) Gradient of AM: $\frac{1-(-2)}{3-1}=\frac{3}{2}$ or $\frac{-3}{-2}$	B1	
	Gradient of l: $=-\frac{2}{3}$ M: use of $m_1 m_2=-1$, or equiv.	M1	
	$y-1=-\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3}=-\frac{2}{3}$ $[3y=-2x+9]$ (Any equiv. form)	M1 A1	(4)
	(b) $x = 6$: $3y = -12 + 9 = -3$ $y = -1$ (or show that for $y = -1$, $x = 6$) (*) (A conclusion is <u>not</u> required).	B1	(1)
	(c) $(r^2 =)$ $(6-1)^2+(-1-(-2))^2$ M: Attempt r^2 or r	M1 A1	
	N.B. Simplification is <u>not</u> required to score M1 A1		
	$(x\pm 6)^2+(y\pm 1)^2=k$, $k\neq 0$ (<u>Value</u> for k not needed, could be r^2 or r)	M1	
	$(x-6)^2+(y+1)^2=26$ (or equiv.)	A1	(4)
	Allow $\left(\sqrt{26}\right)^2$ or other exact equivalents for 26. (But… $(x-6)^2+(y--1)^2=26$ scores M1 A0)		
	(Correct answer with no working scores full marks)		
			9
	(a) 2^{nd} M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞. <u>Alternative:</u> Using (3, 1) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen. Having coords the <u>wrong way round</u>, e.g. $y-3=-\frac{2}{3}(x-1)$, loses the 2^{nd} M mark <u>unless</u> a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$. If the point $P(6,-1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0. (c) 1^{st} M1: Condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket. Must be attempting to use points $P(6, -1)$ and $A(1, -2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)). 1^{st} M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP=MA=\sqrt{13}$. <u>Special case:</u> If candidate persists in using <u>their</u> value for the y-coordinate of P instead of the given -1, allow the M marks in part (c) if earned.		

Question number	Scheme	Marks	
8.	(a) $50\,000r^{n-1}$ (or equiv.) (Allow ar^{n-1} if $50\,000r^{n-1}$ is seen in (b))	B1	(1)
	(b) $50\,000r^{n-1} > 200000$	M1	
	(Using answer to (a), which must include r and n, and 200 000) (Allow equals sign or the wrong inequality sign) (Condone 'slips' such as omitting a zero)		
	$r^{n-1} > 4 \quad \Rightarrow \quad (n-1)\log r > \log 4$	M1	
	(Introducing logs and dealing correctly with the power) (Allow equals sign or the wrong inequality sign)		
	$n > \dfrac{\log 4}{\log r} + 1$ (*)	A1cso	(3)
	(c) r = 1.09: $n > \dfrac{\log 4}{\log 1.09} + 1$ or $n - 1 > \dfrac{\log 4}{\log 1.09}$ ($n > 17.086...$) (Allow equality)	M1	
	Year 18 or 2023 (If one of these is correct, ignore the other)	A1	(2)
	(d) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{50000(1-1.09^{10})}{1-1.09}$	M1 A1	
	£760 000 (Must be this answer… nearest £10000)	A1	(3)
			9
	(b) Incorrect inequality sign at any stage loses the A mark. Condone missing brackets if otherwise correct, e.g $n-1\log r > \log 4$. A common mistake: $50\,000r^{n-1} > 200000$ M1 $(n-1)\log 50\,000r > \log 200\,000$ M0 ('Recovery' from here is not possible). (c) Correct answer with no working scores full marks. Year 17 (or 2022) with no working scores M1 A0. Treat other methods (e.g. "year by year" calculation) as if there is no working. (d) M1: Use of the correct formula with a = 50000, 5000 or 500000, and n = 9, 10, 11 or 15. M1 can also be scored by a "year by year" method, with terms added. (Allow the M mark if there is evidence of adding 9, 10, 11 or 15 terms). 1st A1 is scored if 10 correct terms have been added (allow "nearest £100"). (50000, 54500, 59405, 64751, 70579, 76931, 83855, 91402, 99628, 108595) No working shown: Special case: 760 000 scores 1 mark, scored as 1, 0, 0. (Other answers with no working score no marks).		

Question number	Scheme	Marks	
9.	(a)  Sine wave (anywhere) with at least 2 turning points.	M1	
	Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the y-axis… (e.g. 1, −1 or 0.5)	A1	(2)
	Ignore parts of graph outside 0 to 2π. n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed).		
	(b) $\left(0, \frac{1}{2}\right)$, $\left(\frac{5\pi}{6}, 0\right)$, $\left(\frac{11\pi}{6}, 0\right)$ (Ignore any extra solutions) (<u>Not</u> 150°, 330°)	B1, B1, B1	(3)
	$\left(\pi - \frac{\pi}{6}\right)$ and $\left(2\pi - \frac{\pi}{6}\right)$ are insufficient, but if <u>both</u> are seen allow B1 B0.		
	(c) awrt 0.71 radians (0.70758…), or awrt 40.5° (40.5416…) (α)	B1	
	$(\pi - \alpha)$ (2.43…) or $(180 - \alpha)$ <u>if α is in degrees</u>. $\left[\underline{\text{NOT}}\ \pi - \left(\alpha - \frac{\pi}{6}\right)\right]$	M1	
	Subtract $\frac{\pi}{6}$ from α (or from $(\pi - \alpha)$)… or subtract 30 <u>if α is in degrees</u>	M1	
	0.18 (or 0.06π), 1.91 (or 0.61π) Allow awrt (The 1st A mark is dependent on just the 2nd M mark)	A1, A1	(5)
			10
	(b) The zeros are not required, i.e. allow 0.5, etc. (and also allow coordinates the wrong way round). These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence. (c) B1: If the required value of α is <u>not seen</u>, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90) is achieved. (Also see premature approx. note*). <u>Special case:</u> $\sin\left(x + \frac{\pi}{6}\right) = 0.65 \Rightarrow \sin x + \sin\frac{\pi}{6} = 0.65 \Rightarrow \sin x = 0.15$ $x = \arcsin 0.15 = 0.15056...$ and $x = \pi - 0.15056 = 2.99$ (B0 M1 M0 A0 A0) (This special case mark is also available for degrees… 180 – 8.62…) Extra solutions outside 0 to 2π : Ignore. Extra solutions between 0 and 2π : Loses the final A mark. *<u>Premature approximation</u> in part (c): e.g. α = 41°, 180 – 41 = 139, 41 – 30 = 11 and 139 – 30 = 109 Changing to radians: 0.19 and 1.90 This would score B1 (required value of α not seen, but there is a final answer 0.19 (or 1.90)), M1 M1 A0 A0.		

Question number	Scheme	Marks
10.	(a) $4x^2 + 6xy = 600$	M1 A1
	$V = 2x^2 y = 2x^2\left(\dfrac{600-4x^2}{6x}\right)$ $\quad V = 200x - \dfrac{4x^3}{3}$ (*)	M1 A1cso (4)
	(b) $\dfrac{dV}{dx} = 200 - 4x^2$	B1
	Equate their $\dfrac{dV}{dx}$ to 0 and solve for x^2 or x : $x^2 = 50$ or $x = \sqrt{50}$ (7.07…)	M1 A1
	Evaluate V: $V = 200(\sqrt{50}) - \dfrac{4}{3}(50\sqrt{50}) = 943\ \text{cm}^3$ Allow awrt	M1 A1 (5)
	(c) $\dfrac{d^2V}{dx^2} = -8x$ Negative, $\therefore$ Maximum	M1, A1ft (2)
		11
	(a) 1st M: Attempting an expression in terms of x and y for the total surface area (the expression should be dimensionally correct). 1st A: Correct expression (not necessarily simplified), equated to 600. 2nd M: Substituting their y into $2x^2y$ to form an expression in terms of x only. (Or substituting y from $2x^2y$ into their area equation). (b) 1st A: Ignore $x = -\sqrt{50}$, if seen. The 2nd M mark (for substituting their x value into the given expression for V) is dependent on the 1st M. Final A: Allow also exact value $\dfrac{400\sqrt{50}}{3}$ or $\dfrac{2000\sqrt{2}}{3}$ or equiv. <u>single term</u>. (c) Allow marks if the work for (c) is seen in (b) (or vice-versa). M: Find second derivative <u>and consider its sign</u>. A: Second derivative following through correctly from their $\dfrac{dV}{dx}$, and correct reason/conclusion (it must be a maximum, not a minimum). An actual value of x does not have to be used… this mark can still be awarded if no x value has been found or if a wrong x value is used. <u>Alternative</u>: M: Find <u>value</u> of $\dfrac{dV}{dx}$ on each side of "$x = \sqrt{50}$" and consider sign. A: Indicate sign change of positive to negative for $\dfrac{dV}{dx}$, and conclude max. <u>Alternative</u>: M: Find <u>value</u> of V on each side of "$x = \sqrt{50}$" and compare with "943". A: Indicate that both values are less than 943, and conclude max.	