edexcel
advancing learning, changing lives

Mark Scheme (Results)

Summer 2009

GCE

GCE Mathematics (6664/01)

A PEARSON COMPANY

edexcel

June 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\int\left(2x+3x^{\frac{1}{2}}\right)\mathrm{d}x=\frac{2x^2}{2}+\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$	M1 A1A1
	$\int_1^4\left(2x+3x^{\frac{1}{2}}\right)\mathrm{d}x=\left[x^2+2x^{\frac{3}{2}}\right]_1^4=(16+2\times 8)-(1+2)$	M1
	$= 29$ (29 + C scores A0)	A1 (5)
		[5]
	1st M1 for attempt to integrate $x \to kx^2$ or $x^{\frac{1}{2}} \to kx^{\frac{3}{2}}$. 1st A1 for $\frac{2x^2}{2}$ or a simplified version. 2nd A1 for $\frac{3x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ or $\frac{3x\sqrt{x}}{\left(\frac{3}{2}\right)}$ or a simplified version. Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0. 2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation). Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear. <u>No working</u>: The answer 29 with no working scores M0A0A0M1A0 (1 mark).	

Question Number	Scheme	Marks
Q2 (a)	$(7\times\ldots\times x)$ or $(21\times\ldots\times x^2)$ The 7 or 21 can be in 'unsimplified' form.	M1
	$(2+kx)^7 = 2^7 + 2^6\times 7\times kx + 2^5\times\binom{7}{2}k^2x^2$	
	$= 128;\quad +448kx,\quad +672k^2x^2$ [or $672(kx)^2$] (If $672kx^2$ follows $672(kx)^2$, isw and allow A1)	B1; A1, A1 (4)
(b)	$6\times 448k = 672k^2$	M1
	$k = 4$ (Ignore $k = 0$, if seen)	A1 (2) **[6]**
(a)	The terms can be 'listed' rather than added. Ignore any extra terms. M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{7}{1}$, $\left(\frac{7}{1}\right)$, $\binom{7}{2}$, ${}^7C_1$, ${}^7C_2$. However, $448+kx$ or similar is M0. B1, A1, A1 for the <u>simplified</u> versions seen above. <u>Alternative</u>: Note that a factor 2^7 can be taken out first: $2^7\left(1+\frac{kx}{2}\right)^7$, but the mark scheme still applies. <u>Ignoring subsequent working (isw)</u>: Isw if necessary after correct working: e.g. $128+448kx+672k^2x^2$ M1 B1 A1 A1 $= 4+14kx+21k^2x^2$ isw (Full marks are still available in part (b)).	
(b)	M1 for equating their coefficient of x^2 to 6 times that of x… to get an equation in k, … <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6\times 448k = 672k$, but beware k = 4 following from this, which is A0. <u>An equation in k alone</u> is required for this M mark, so... e.g. $6\times 448kx = 672k^2x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ... e.g. $6\times 448kx = 672k^2x^2 \Rightarrow 6\times 448k = 672k^2 \Rightarrow k = 4$ will get M1 A1 (as coefficients rather than terms have now been considered). The mistake $2\left(1+\frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1	

Question Number	Scheme	Marks
Q3 (a)	$\mathrm{f}(k) = -8$	B1 (1)
(b)	$\mathrm{f}(2) = 4 \Rightarrow \quad 4 = (6-2)(2-k) - 8$	M1
	So $\qquad k = -1$	A1 (2)
(c)	$\mathrm{f}(x) = 3x^2 - (2+3k)x + (2k-8) \quad = 3x^2 + x - 10$	M1
	$= (3x - 5)(x + 2)$	M1A1 (3)
		[6]
(b)	M1 for substituting $x = 2$ (<u>not</u> $x = -2$) and equating to 4 to form an equation in *k*. If the expression is expanded in this part, condone 'slips' for this M mark. Treat the omission of the -8 here as a 'slip' and allow the M mark. <u>Beware</u>: Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$. <u>Alternative</u>; M1 for dividing by $(x-2)$, to get $3x +$ (function of *k*), with remainder as a function of *k*, and equating the remainder to 4. [Should be $3x + (4-3k)$, remainder $-4k$]. <u>No working</u>: $k = -1$ with no working scores M0 A0.	
(c)	1st M1 for multiplying out <u>and</u> substituting their (constant) value of *k* (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the f(*x*) expression, this is M0. The 2nd M1 is still available. 2nd M1 for an attempt to factorise their three term quadratic (3TQ). A1 The correct answer, as a <u>product of factors</u>, is required. Allow $3\left(x - \frac{5}{3}\right)(x+2)$ Ignore following work (such as a solution to a quadratic equation). If the 'equation' is solved but factors are never seen, the 2nd M is not scored.	

edexcel

Question Number	Scheme	Marks
Q4 (a)	$x = 2$ gives 2.236 (allow AWRT) Accept $\sqrt{5}$	B1
	$x = 2.5$ gives 2.580 (allow AWRT) Accept 2.58	B1 (2)
(b)	$\left(\frac{1}{2}\times\frac{1}{2}\right)$, $\left[(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)\right]$	B1,[M1A1ft]
	$= 6.133$ (AWRT 6.13, even following minor slips)	A1 (4)
(c)		
	Overestimate	B1
	'Since the trapezia lie <u>above the curve</u>', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	dB1 (2) **[8]**
(b)	B1 for $\frac{1}{2}\times\frac{1}{2}$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\left(\frac{1}{2}\times\frac{1}{2}\right)(1.414+3)+2(1.554+1.732+1.957+2.236+2.580)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative</u>: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.414+1.554)+\frac{1}{4}(1.554+1.732)+\ldots\ldots\ldots\ldots\ldots+\frac{1}{4}(2.580+3)\right]$ 1^{st} A1ft for correct expression, ft their 2.236 and their 2.580	
(c)	1^{st} B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2^{nd} B1 is dependent upon the 1^{st} B1 (overestimate).	

Question Number	Scheme	Marks
Q5 (a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \ldots$, 729	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1-\left[\frac{2}{3}\right]^{15}\right)}{1-\frac{2}{3}}$, $= 2182.00\ldots$ (AWRT 2180)	M1A1ft, (3)
(d)	$S_\infty = \frac{729}{1-\frac{2}{3}}$, $= 2187$	M1, A1 (2)
		[9]

(a)

M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction.

A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp and the final answer 2/3 is seen.

Alternative: (verification)

M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times).

A1 Obtaining 96 (cso). (A conclusion is not required).

$324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.

(b)

M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96).

Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)… but no other exceptions are allowed.

(c)

M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated.

1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$.

2nd A1 for awrt 2180, even following 'minor inaccuracies'.

Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c).

Alternative:

M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.

(d)

M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $|r| < 1$.

edexcel

Question Number	Scheme	Marks
Q6 (a)	$(x-3)^2-9+(y+2)^2-4=12$ Centre is (3, –2)	M1 A1, A1
	$(x-3)^2+(y+2)^2=12+"9"+"4"$ $r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$)	M1 A1 (5)
(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	= 10 = 2×radius, ∴ diam. (N.B. For A1, need a comment or conclusion) [ALT: midpt. of *PQ* $\left(\frac{7+(-1)}{2},\frac{1+(-5)}{2}\right)$: M1, = (3, –2) = centre: A1] [ALT: eqn. of *PQ* $3x+4y-1=0$: M1, verify (3, –2) lies on this: A1] [ALT: find two grads, e.g. *PQ* and *P* to centre: M1, equal ∴ diameter: A1] [ALT: show that point $S(-1,-5)$ or (7, 1) lies on circle: M1 because ∠*PSQ* = 90°, semicircle ∴ diameter: A1]	A1 (2)
(c)	*R* must lie on the circle (angle in a semicircle theorem)… often <u>implied</u> by <u>a diagram with *R* on the circle</u> or by subsequent working)	B1
	$x=0\Rightarrow$ $y^2+4y-12=0$	M1
	$(y-2)(y+6)=0$ $y=.....$ (M is dependent on previous M)	dM1
	$y=-6$ or 2 (Ignore $y=-6$ if seen, and 'coordinates' are not required))	A1 (4) **[11]**
(a)	1st M1 for attempt to complete square. Allow $(x\pm3)^2\pm k$, or $(y\pm2)^2\pm k$, $k\neq0$. 1st A1 *x*-coordinate 3, 2nd A1 *y*-coordinate –2 2nd M1 for a full method leading to *r* = …, with their 9 and their 4, 3rd A1 5 or $\sqrt{25}$ The 1st M can be <u>implied</u> by $(\pm3,\pm2)$ but a full method must be seen for the 2nd M. Where the 'diameter' in part (b) has <u>clearly</u> been used to answer part (a), no marks in (a), but in this case the M1 (<u>not</u> the A1) for part (b) can be given for work seen in (a). <u>Alternative</u> 1st M1 for comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. 2nd M1 for using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this M mark.	
(c)	1st M1 for setting *x* = 0 and getting a 3TQ in *y* by using eqn. of circle. 2nd M1 (dep.) for attempt to solve a 3TQ leading to <u>at least one</u> solution for *y*. <u>Alternative 1</u>: (Requires the B mark as in the main scheme) 1st M for using (3, 4, 5) triangle with vertices $(3,-2),(0,-2),(0,y)$ to get a linear or quadratic equation in *y* (e.g. $3^2+(y+2)^2=25$). 2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. <u>Alternative 2</u>: (Not requiring realisation that *R* is on the circle) B1 for attempt at $m_{PR}\times m_{QR}=-1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle *PQR*. 1st M1 for setting *x* = 0, i.e. (0, *y*), and proceeding to get a 3TQ in *y*. Then main scheme. <u>Alternative 2 by 'verification'</u>: B1 for attempt at $m_{PR}\times m_{QR}=-1$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle *PQR*. 1st M1 for trying (0, 2). 2nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Question Number	Scheme	Marks
Q7 (i)	$\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$	B1, B1ft
	$\sin\theta = \frac{2}{5} \Rightarrow \quad \theta = 23.6, \quad 156.4$ (AWRT: 24, 156)	B1, B1ft (4)
(ii)	$4\sin x = \dfrac{3\sin x}{\cos x}$	M1
	$4\sin x\cos x = 3\sin x \quad \Rightarrow \quad \sin x(4\cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16\sin^2 x - 7) = 0$, $(16\cos^2 x - 9)(\cos^2 x - 1) = 0$	M1
	$x = 0, \ 180$ <u>seen</u>	B1, B1
	$x = 41.4, \quad 318.6$ (AWRT: 41, 319)	B1, B1ft (6)
		[10]

(i) 1st B1 for -45 seen (α, where $|\alpha| < 90$)

2nd B1 for 135 seen, <u>or ft</u> $(180 + \alpha)$ if α is negative, or $(\alpha - 180)$ if α is positive.
If $\tan\theta = k$ is obtained from <u>wrong working</u>, 2nd B1ft is still available.

3rd B1 for awrt 24 (β, where $|\beta| < 90$)

4th B1 for awrt 156, <u>or ft</u> $(180 - \beta)$ if β is positive, or $-(180 + \beta)$ if β is negative.
If $\sin\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.

(ii) 1st M1 for use of $\tan x = \dfrac{\sin x}{\cos x}$. Condone $\dfrac{3\sin x}{3\cos x}$.

2nd M1 for correct work leading to 2 factors (may be implied).

1st B1 for 0, 2nd B1 for 180.

3rd B1 for awrt 41 (γ, where $|\gamma| < 180$)

4th B1 for awrt 319, <u>or ft</u> $(360 - \gamma)$.
If $\cos\theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.

N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1

<u>Alternative:</u> (squaring both sides)

1st M1 for squaring both sides and using a 'quadratic' identity.
e.g. $16\sin^2\theta = 9(\sec^2\theta - 1)$

2nd M1 for reaching a factorised form.
e.g. $(16\cos^2\theta - 9)(\cos^2\theta - 1) = 0$

Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme.

<u>For both parts of the question</u>:

<u>Extra solutions outside required range</u>: Ignore

<u>Extra solutions inside required range</u>:
For each <u>pair</u> of B marks, the 2nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan\theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0

<u>Answers in radians</u>:
Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).

Question Number	Scheme	Marks
Q8 (a)	$\log_2 y = -3 \quad \Rightarrow \quad y = 2^{-3}$	M1
	$y = \frac{1}{8}$ or 0.125	A1 (2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$	M1
	[or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$]	
	[or $\log_2 32 = \dfrac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \dfrac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \dfrac{\log_{10} 512}{\log_{10} 2}$]	
	$\log_2 32 + \log_2 16 = 9$	A1
	$(\log x)^2 = ...$ or $(\log x)(\log x) = ...$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1
	$\log_2 x = 3 \quad \Rightarrow \quad x = 2^3 = 8$	A1
	$\log_2 x = -3 \quad \Rightarrow \quad x = 2^{-3} = \dfrac{1}{8}$	A1ft (5) **[7]**

(a)

M1 for <u>getting out of logs</u> correctly.
If done by change of base, $\log_{10} y = -0.903...$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1.

A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502..$ scores M1 (implied) A0.

<u>Correct answer</u> with no working scores both marks.

<u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$.

(b)

1st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively).

1st A1 for 9 (exact).

2nd M1 for getting $(\log_2 x)^2$ = constant. The constant can be a log or a sum of logs.
If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation.

2nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0.

3rd A1ft for an answer of $\dfrac{1}{\text{their } 8}$. An ft answer may be non-exact.

<u>Possible mistakes</u>:

$\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = ...$ scores M1A1(implied by 9)M0A0A0

$\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = ...$ scores M0A0(9 never seen)M1A0A0

$\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145,\ x = 0.194$ scores M0A0M1A0A1ft

<u>No working</u> (or 'trial and improvement'):
x = 8 scores M0 A0 M1 A1 A0

Question Number	Scheme	Marks
Q9 (a)	(Arc length =) $r\theta = r\times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).	B1
	(Sector area =) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2\times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).	B1
	Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)	M1
	Volume = 300 = $\frac{1}{2}r^2h$	B1
	Sub for h: $S = r^2 + 3\times\frac{600}{r} = r^2 + \frac{1800}{r}$ (*)	A1cso (5)
(b)	$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\frac{dS}{dr} = 0 \Rightarrow r^3 = \ldots$, $r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)	M1, A1 (4)
(c)	$\frac{d^2S}{dr^2} = \ldots.$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} >$ 0 so point is a minimum	M1, A1ft (2)
(d)	$S_{\min} = (9.65\ldots)^2 + \frac{1800}{9.65\ldots}$ (Using their value of r, however found, in the <u>given</u> S formula)	M1
	$= 279.65\ldots$ (AWRT: 280) (Dependent on full marks in part (b))	A1 (2) **[13]**

(a) M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.

(b) <u>In parts (b), (c) and (d), ignore labelling of parts</u>

1st M1 for attempt at differentiation (one term is sufficient) $r^n \to kr^{n-1}$

2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = \ldots$ (depending upon their 'changed function', this could be $r = \ldots$ or $r^2 = \ldots$, etc., but the algebra <u>must deal with a negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = \ldots$ is consistent with their derivative).

(c) M1 for attempting second derivative (one term is sufficient) $r^n \to kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>).

A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.

<u>Alternative</u>:

M1: Find <u>value</u> of $\frac{dS}{dr}$ on each side of their value of r and consider sign.

A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum.

<u>Alternative</u>:

M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65.

A1ft: Indicate that both values are more than 279.65, and conclude minimum.