# Mark Scheme (Results)

# January 2007

GCE

GCE Mathematics

Core Mathematics C3 (6665)

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# 6665 Core Mathematics C3
# Mark Scheme

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 1. | (a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$ | | B1 |
| | $= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$ | | B1 B1 |
| | $= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ | | M1 |
| | $= 3\sin\theta - 4\sin^3\theta$ ✱ | cso | A1 **(5)** |
| | (b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent | | M1 A1 **(2)** |
| | | | **[7]** |
| 2. | (a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ | | M1 A1, A1 |
| | $= \frac{x^2+4x+4-3x-6+3}{(x+2)^2} = \frac{x^2+x+1}{(x+2)^2}$ ✱ | cso | A1 **(4)** |
| | (b) $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4},\ > 0$ for all values of $x$. | | M1 A1, A1 **(3)** |
| | (c) $f(x) = \frac{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}{(x+2)^2}$<br>Numerator is positive from (b)<br>$x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive)<br>Hence $f(x) > 0$ | | B1 **(1)**<br>**[8]** |
| | *Alternative to* (b)<br>$\frac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2+x+1 = \frac{3}{4}$ | | M1 A1 |
| | A parabola with positive coefficient of $x^2$ has a minimum $\Rightarrow x^2+x+1>0$<br>Accept equivalent arguments | | A1 **(3)** |

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| 3. | (a) $y=\frac{\pi}{4} \Rightarrow x=2\sin\frac{\pi}{4}=2\times\frac{1}{\sqrt{2}}=\sqrt{2} \Rightarrow P\in C$ | B1 **(1)** |
| | Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ or exact equivalent is used. | |
| | (b) $\frac{\mathrm{d}x}{\mathrm{d}y}=2\cos y$ *or* $1=2\cos y\frac{\mathrm{d}y}{\mathrm{d}x}$ | M1 A1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\cos y}$ May be awarded after substitution | M1 |
| | $y=\frac{\pi}{4} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{2}}$ ∗ cso | A1 **(4)** |
| | (c) $m'=-\sqrt{2}$ | B1 |
| | $y-\frac{\pi}{4}=-\sqrt{2}\left(x-\sqrt{2}\right)$ | M1 A1 |
| | $y=-\sqrt{2}x+2+\frac{\pi}{4}$ | A1 **(4)** |
| | | **[9]** |
| 4. | (i) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(9+x^2\right)-x(2x)}{\left(9+x^2\right)^2}$ $\left(=\frac{9-x^2}{\left(9+x^2\right)^2}\right)$ | M1 A1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow 9-x^2=0 \Rightarrow x=\pm 3$ | M1 A1 |
| | $\left(3,\frac{1}{6}\right)$, $\left(-3,-\frac{1}{6}\right)$ Final two A marks depend on second M only | A1, A1 **(6)** |
| | (ii) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}\left(1+\mathrm{e}^{2x}\right)^{\frac{1}{2}}\times 2\mathrm{e}^{2x}$ | M1 A1 A1 |
| | $x=\frac{1}{2}\ln 3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}\left(1+\mathrm{e}^{\ln 3}\right)^{\frac{1}{2}}\times 2\mathrm{e}^{\ln 3}=3\times 4^{\frac{1}{2}}\times 3=18$ | M1 A1 **(5)** |
| | | **[11]** |

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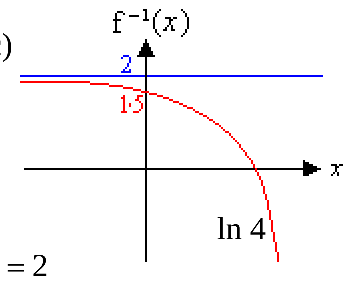
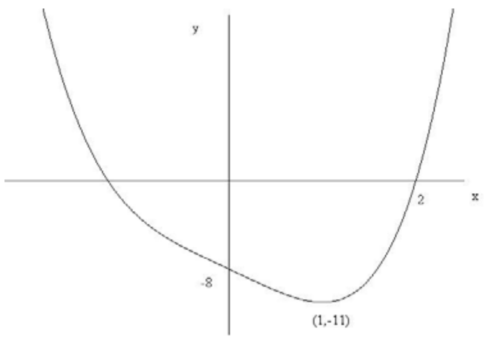
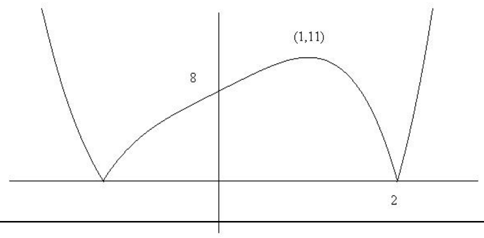
| 5. | (a) $R^2 = \left(\sqrt{3}\right)^2 + 1^2 \Rightarrow R = 2$ | M1 A1 |
| | $\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ accept awrt 1.05 | M1 A1 **(4)** |
| | (b) $\sin\left(x + \text{ their } \alpha\right) = \frac{1}{2}$ | M1 |
| | $x + \text{ their } \alpha = \frac{\pi}{6} \left(\frac{5\pi}{6}, \frac{13\pi}{6}\right)$ | A1 |
| | $x = \frac{\pi}{2}, \frac{11\pi}{6}$ accept awrt 1.57, 5.76 | M1 A1 **(4)** |
| | | **[8]** |
| | The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore. | |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| 6. | (a) $y=\ln(4-2x)$ | |
| | $e^y=4-2x$ leading to $x=2-\frac{1}{2}e^y$ Changing subject and removing ln | M1 A1 |
| | $y=2-\frac{1}{2}e^x \quad \Rightarrow \quad f^{-1}\mapsto 2-\frac{1}{2}e^x$ ✱ cso | A1 |
| | Domain of $f^{-1}$ is | B1 **(4)** |
| | (b) Range of $f^{-1}$ is $f^{-1}(x)<2$ (and $f^{-1}(x)\in$ ) | B1 **(1)** |
| | (c)  | |
| | Shape<br>1.5<br>ln 4 | B1<br>B1<br>B1 |
| | $y=2$ | B1 **(4)** |
| | (d) $x_1\approx-0.3704$, $x_2\approx-0.3452$ cao<br>If more than 4 dp given in this part a maximum on one mark is lost.<br>Penalise on the first occasion. | B1, B1 **(2)** |
| | (e) $x_3=-0.354\,030\,19\ \ldots$<br>$x_4=-0.350\,926\,88\ \ldots$<br>$x_5=-0.352\,017\,61\ \ldots$ | |
| | $x_6=-0.351\,633\,86\ \ldots$ Calculating to at least $x_6$ to at least four dp | M1 |
| | $k\approx-0.352$ cao | A1 **(2)**<br>**[13]** |
| | Alternative to (e)<br>$k\approx-0.352$ Found in any way<br>Let $g(x)=x+\frac{1}{2}e^x$ | |
| | $g(-0.3515)\approx+0.0003$, $g(-0.3525)\approx-0.001$ | M1 |
| | Change of sign (and continuity) $\Rightarrow k\in(-0.3525,-0.3515)$<br>$\Rightarrow k=-0.352$ (to 3 dp) | A1 **(2)** |

| Question Number | Scheme | Marks |
|---|---|---|
| 7. | (a) $\mathrm{f}(-2) = 16 + 8 - 8\ (= 16) > 0$ | B1 |
| | $\mathrm{f}(-1) = 1 + 4 - 8\ (= -3) < 0$ | B1 |
| | Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$ | B1ft **(3)** |
| | ft their calculation as long as there is a sign change | |
| | (b) $\dfrac{dy}{dx} = 4x^3 - 4 = 0 \quad \Rightarrow \quad x = 1$ | M1 A1 |
| | Turning point is $(1, -11)$ | A1 **(3)** |
| | (c) $a = 2, b = 4, c = 4$ | B1 B1 B1 **(3)** |
| | (d)   | |
| | Shape | B1 |
| | ft their turning point in correct quadrant only | B1 ft |
| | 2 and $-8$ | B1 **(3)** |
| | (e)   | |
| | Shape | B1 **(1)** |
| | | **[13]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 8. | (i) $\sec^2 x - \operatorname{cosec}^2 x = \left(1+\tan^2 x\right) - \left(1+\cot^2 x\right)$ | | M1 A1 |
| | $= \tan^2 x - \cot^2 x$ ✱ | cso | A1 **(3)** |
| | (ii)(a) $y = \arccos x \Rightarrow x = \cos y$ | | B1 |
| | $x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$ | | B1 **(2)** |
| | Accept $\arcsin x = \arcsin \cos y$ | | |
| | (b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$ | | B1 **(1)** |
| | | | **[6]** |
| | *Alternatives for* (i) | | |
| | $\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$ | | M1 A1 |
| | Rearranging $\sec^2 x - \operatorname{cosec}^2 x = \tan^2 x - \cot^2 x$ ✱ cso | | A1 **(3)** |
| | $\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}\right)$ | | |
| | RHS $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{\left(\sin^2 x - \cos^2 x\right)\left(\sin^2 x + \cos^2 x\right)}{\cos^2 x \sin^2 x}$ | | M1 |
| | $= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$ | | A1 |
| | $=$ LHS ✱ | or equivalent | A1 **(3)** |