Pearson

Mark Scheme (Results)

October 2017

Pearson Edexcel International A-Level
In Core Mathematics C34 (WMA02)

edexcel

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PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⌞ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \quad \text{leading to } x = \ldots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \quad \text{leading to } x = \ldots$$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0, \quad$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1(a)	$x^5+x^3-12x^2-8=0 \Rightarrow x^5+x^3=12x^2+8$	M1
	$x^3\left(x^2+1\right)=12x^2+8 \Rightarrow x^3=\dfrac{12x^2+8}{(x^2+1)}$ or e.g. $x^3=\dfrac{4\left(3x^2+2\right)}{(x^2+1)}$	A1
	Note that going straight from $x^5+x^3=12x^2+8$ to $x^3=\dfrac{12x^2+8}{(x^2+1)}$ is acceptable for the first 2 marks but the final mark should be withheld for not explicitly showing the factorisation of the lhs	
	$\Rightarrow x=\sqrt[3]{\dfrac{4\left(3x^2+2\right)}{(x^2+1)}}$ or $x=\sqrt[3]{\dfrac{4\left(2+3x^2\right)}{(x^2+1)}}$	A1*
		(3)
(b)	$x_1=\sqrt[3]{\dfrac{4\left(3\times2^2+2\right)}{2^2+1}}=2.237$	M1A1
	$x_2=2.246,\quad x_3=2.247$	A1
		(3)
(c)	Interval $\left[2.2465,2.2475\right]\Rightarrow \mathrm{f}\left(2.2465\right)=...,\mathrm{f}\left(2.2475\right)=...$	M1
	$\mathrm{f}(2.2465)=-0.0057,\ \mathrm{f}(2.2475)=(+)0.083$ +Reason + Conclusion	A1
		(2)
		(8 marks)
Alt (a)	$x=\sqrt[3]{\dfrac{4\left(3x^2+2\right)}{(x^2+1)}}\Rightarrow x^3(x^2+1)=12x^2+8$	M1
	$x^5+x^3-12x^2-8=0$	A1
	Statement Hence $\mathrm{f}(x)=0$	A1*
		(3)

(a)

M1: Attempts to write equation in the form $x^5\pm x^3=12x^2\pm8$ or $x^3(x^2\pm1)=12x^2\pm8$.

A1: Intermediate line of $x^3=\dfrac{12x^2+8}{(x^2+1)}$ seen

A1*: cso with the factorisation of the lhs seen explicitly and a statement at the start that f(x) = 0 or $x^5+x^3-12x^2-8=0$ oe e.g. $x^3\left(x^2+1\right)-4\left(3x^2+2\right)=0$

Do not be overly concerned about the cube root encompassing the whole fraction but do not allow if it is only unambiguously the numerator that has the cube root e.g. $\Rightarrow x=\dfrac{\sqrt[3]{4\left(3x^2+2\right)}}{(x^2+1)}$

Beware of other algebraic methods of establishing the result in (a) – if in doubt send to review.

Alternative for part (a):

M1: Cubes the printed result and multiplies up

A1: Obtains the required equation with no errors

A1*: Makes a conclusion (may be minimal e.g. tick, QED, # etc.) and $x^3(x^2+1)=x^5+x^3$ seen explicitly in the working

(b)
M1
:
Sub
stit
ute
s x_0

= 2 into iterative equation to find x_1 which may be implied by $\sqrt[3]{\frac{4(3\times 2^2+2)}{2^2+1}}$ or awrt 2.2

A1: awrt $x_1 = 2.237$

A1: awrt $x_2 = 2.246,\quad x_3 = 2.247$ (Accept commas for decimal points)

(c)

M1: Attempts to evaluate f(x) at both ends of a suitable interval such as [2.2465, 2.2475] with evidence of substitution at least once or one correct end (1SF or 1 figure truncated). Accept a tighter interval as long as it spans the root 2.24656. (NB x = 2.246564001)

A1: $\mathrm{f}(2.2465) = -0.0057, \mathrm{f}(2.2475) = (+)0.083$ + Reason (Eg change of sign or < 0, > 0 against the appropriate value or equivalent statement) + Conclusion (E.g. a minimum "root" or α = 2.247 or "suitable" or "suitable interval" or "root lies between 2.2465 and 2.2475")

Need both values correct to 1 significant figure or truncated.

Note that candidates may use $\mathrm{g}(x) = x - \sqrt[3]{\frac{4(3x^2+2)}{x^2+1}}$ with suitable values – this gives e.g. $\mathrm{g}(2.2465) = -0.000061..., \mathrm{g}(2.2475) = (+)0.000905...$ and is an acceptable method.

If the candidate makes an attempt to compare x with $\sqrt[3]{\frac{4(3x^2+2)}{x^2+1}}$ and constructs an argument this way and you think it may be worth some credit, please send to review.

In (c) do **not** accept attempts to repeatedly apply the iterative formula to show convergence.

Question Number	Scheme	Marks
2.(a)	$y^3 + x^2 y - 6x = 0 \Rightarrow 3y^2 \frac{dy}{dx} + x^2 \frac{dy}{dx} + 2xy - 6 = 0$	B1 M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{6-2xy}{x^2+3y^2}$	M1A1
		(5)
(b)	$6 - 2xy = 0 \Rightarrow y = \frac{3}{x}$	M1
	Substitute $y = \frac{3}{x}$ into $y^3 + x^2 y - 6x = 0 \Rightarrow \frac{27}{x^3} + \frac{3x^2}{x} - 6x = 0$	**d**M1
	$\Rightarrow x^4 = 9$	**dd**M1A1
	Points $(\sqrt{3}, \sqrt{3})(-\sqrt{3}, -\sqrt{3})$	A1A1
		(6)
		(11 marks)
Alt(b)	$6 - 2xy = 0 \Rightarrow x = \frac{3}{y}$	M1
	Substitute $x = \frac{3}{y}$ into $y^3 + x^2 y - 6x = 0 \Rightarrow y^3 + \frac{9}{y^2} y - 6 \times \frac{3}{y} = 0$	**d**M1
	$\Rightarrow y^4 = 9$	**dd**M1A1
	Points $(\sqrt{3}, \sqrt{3})(-\sqrt{3}, -\sqrt{3})$	A1A1
		(6)

(a)

B1: Applies the product rule to $x^2 y$ to obtain $x^2 \frac{dy}{dx} + 2xy$

M1: Applies the chain rule to y^3 to obtain $Ay^2 \frac{dy}{dx}$

A1: $y^3 - 6x = 0 \Rightarrow 3y^2 \frac{dy}{dx} - 6 = 0$. i.e. y^3 differentiated correctly **and** $-6x \to -6$ **and** "= 0" seen or implied.

M1: Attempts to make $\frac{dy}{dx}$ the subject. This is dependent upon them having two $\frac{dy}{dx}$ terms in their derivative. One coming from their differentiation of $x^2 y$ and the other from their differentiation of y^3

A1: Accept $\frac{dy}{dx} = \frac{6-2xy}{x^2+3y^2}$ or equivalent.

Ignore a spurious "$\frac{dy}{dx} =$" at the start but see the note above regarding where the $\frac{dy}{dx}$'s must come from for the second method mark.

<u>**If the candidate differentiates with respect to *y*, the same scheme can be applied:**</u>

B1: $x^2 y \to x^2 + 2xy \frac{dx}{dy}$. M1: $-6x \to A\frac{dx}{dy}$ A1: $y^3 - 6x = 0 \Rightarrow 3y^2 - 6\frac{dx}{dy} = 0$

M1: Attempts to make $\frac{dx}{dy}$ the subject. This is dependent upon them having two $\frac{dx}{dy}$ terms in their derivative. One coming from their differentiation of $x^2 y$ and the other from their differentiation of $-6x$

A1: Accept $\frac{dy}{dx}=\frac{6-2xy}{x^2+3y^2}$ or equivalent.

<u>If the candidate multiplies through by dx:</u>

B1: $x^2y \to x^2dy+2xydx$. M1: $y^3 \to Ay^2dy$ A1: $y^3-6x=0 \Rightarrow 3y^2dy-6dx=0$

M1: $dy\left(3y^2+x^2\right)=(6-2xy)dx \Rightarrow \frac{dy}{dx}=...$ A1: $\frac{dy}{dx}=\frac{6-2xy}{x^2+3y^2}$

(b)

M1: Sets the numerator of their $\frac{dy}{dx}=0$ and attempts to write x in terms of y or vice versa. This means that their numerator must be a function of x **and** y.

dM1: Substitutes their answer to $\frac{dy}{dx}=0$ into $y^3+x^2y-6x=0$ to form an equation in one variable.

Dependent on the first method mark.

ddM1: Reaches an equation of the form $Ax^m=Bx^n$ OR $Cy^m=Dy^n$ or equivalent e.g. $Ax^m-Bx^n=0$ OR $Cy^m-Dy^n=0$ where $m\neq n$. **Dependent on both previous method marks.**

A1: A correct equation, either $x^4=9$ or $y^4=9$ or equivalent e.g. $x^4-9=0$ or $y^4-9=0$ (May be implied by correct coordinates below)

A1: Two correct values for x or y or a correct pair...likely to be $x=\sqrt{3}$, $y=\sqrt{3}$

Allow equivalent exact values for $\sqrt{3}$ for this mark e.g. $\sqrt[4]{9}$ or $\frac{3}{\sqrt[4]{9}}$ or $\sqrt[4]{\frac{27}{3}}$ or awrt 1.73

A1: All 4 values correct **and simplified** i.e. $x=\pm\sqrt{3}$, $y=\pm\sqrt{3}$. The points do not have to be explicitly given as coordinates so just look for values but if any extra points/coordinates are given, this mark can be withheld. Allow $3^{\frac{1}{2}}$ for $\sqrt{3}$.

Note that starting with $6-2xy=x^2+3y^2$ generally will score no marks in (b)

Note that working with $x^2+3y^2=0$ generally will score no marks in (b) and can be ignored if seen alongside work dealing with $6-2xy=0$ unless it yields extra spurious values – in which case the final mark can be withheld – see note above.

Question Number	Scheme	Marks
3(a)	3500	B1
		(1)
(b)	$3500(1.035)^t > 10000 \Rightarrow (1.035)^t > \frac{20}{7}$ (awrt 2.86)	M1A1
	$\Rightarrow t > \frac{\log\frac{20}{7}}{\log 1.035} = 30.516 =$ 30hrs 31 mins or 30 hrs 32 mins	M1A1
		(4)
(c)	$\frac{dN}{dt} = 3500(1.035)^t \ln 1.035 \Rightarrow \left.\frac{dN}{dt}\right\|_{t=8} = 3500(1.035)^8 \ln 1.035 = \text{awrt } 159$	B1M1A1
		(3)
		(8 marks)

(a)
B1: 3500
(b)
M1: For substituting N =10000 and proceeding to $(1.035)^t ... A$ where ... is $>, \geqslant, =, <$ or $\leqslant$
A1: $(1.035)^t ... \frac{20}{7}$ where ... is $>, \geqslant, =, <$ or $\leqslant$ Accept awrt 2.86 for $\frac{20}{7}$ or equivalent e.g. $\frac{10000}{3500}, \frac{100}{35}$
M1: Proceeds correctly to find a value for t.
Accept expressions such as $t ... \frac{\log\frac{20}{7}}{\log 1.035}$, $t ... \frac{\ln\frac{20}{7}}{\ln 1.035}$ or $t ... \log_{1.035}\frac{20}{7}$ or awrt 30.5 as evidence
A1: 30hrs 31 mins or 30hrs 32 mins (**Not** 1831 minutes)
Attempts and Trial and Improvement should be sent to review.

(c)
B1: For $\frac{dN}{dt} = 3500(1.035)^t \ln 1.035$ or $\frac{dN}{dt} = 3500e^{t\ln 1.035}\ln 1.035$ (Allow $\frac{dN}{dt} = N\ln 1.035$)
M1: For substituting t = 8 into their $\frac{dN}{dt}$ which is a function of t but which is **not** the original function.
A1: awrt 159 (Award as soon as a correct answer is seen and isw)

Question Number	Scheme	Marks
4 (a)	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x} = \frac{2\sin x\sin x}{2\sin x\cos x}$ Allow $\frac{1-\cos 2x}{\sin 2x} \equiv \frac{2\sin^2 x}{2\sin x\cos x}$	M1A1
	$= \frac{\sin x}{\cos x} = \tan x$	A1*
		(3)
	Examples	
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{1-1+2\sin^2 x}{2\sin x\cos x} = \frac{\sin x}{\cos x} = \tan x$	M1A1A1
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x} = \frac{\sin x}{\cos x} = \tan x$	M1A1A1
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{2\sin^2 x}{2\sin x\cos x} = \tan x$	M1A1A0
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin} x\cos x} = \tan x$	M1A1A1
		(3)
(b)	$3\sec^2\theta - 7 = \frac{1-\cos 2\theta}{\sin 2\theta} \Rightarrow 3\sec^2\theta - 7 = \tan\theta$	M1
	$\Rightarrow 3\left(1+\tan^2\theta\right) - 7 = \tan\theta$	M1
	$\Rightarrow 3\tan^2\theta - \tan\theta - 4 = 0$	A1
	$\Rightarrow (3\tan\theta - 4)(\tan\theta + 1) = 0$	
	$\Rightarrow \tan\theta = \frac{4}{3}, \tan\theta = -1$	**d**M1
	$\theta = 0.927, 4.069, \frac{3}{4}\pi\left(2.356\right), \frac{7}{4}\pi\left(5.498\right)$	A1 A1
		(6)
		(9 marks)

(a)

M1: Score for using $\cos 2x = 1 - 2\sin^2 x$ **and** $\sin 2x = 2\sin x\cos x$

If $\cos 2x = \cos^2 x - \sin^2 x$ is used first there must be an attempt to change into just $\sin^2 x$ by using the identity $\sin^2 x + \cos^2 x = 1$. Condone missing brackets for this mark.

A1: A correct intermediate line of e.g. $\frac{a\sin x\sin x}{a\sin x\cos x}$ or $\frac{a\sin^2 x}{a\sin x\cos x}$ or $\frac{1-1+2\sin^2 x}{2\sin x\cos x}$ or $\frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x}$

A1*: Correctly proceeds to given answer with no errors or omissions including all bracketing. There must be an intermediate line of either $\frac{\cancel{2}\cancel{\sin} x\sin x}{\cancel{2}\cancel{\sin} x\cos x}$ showing cancelling or $\frac{\sin x}{\cos x}$ or $\frac{2\sin x}{2\cos x}$ before $\tan x$ is seen and if their working necessitates the appearance of the 2's in the numerator and denominator and they are not shown, this mark can be withheld. If the candidate uses θ instead of x, the final mark should be withheld.

(b)

M1: Uses the identity from part (a) to get an equation in just $\sec^2\theta$ or $\frac{1}{\cos^2\theta}$ and $\tan\theta$

M1: Uses the identity $\sec^2\theta = \pm 1 \pm \tan^2\theta$ to get an equation in just $\tan\theta$.

A1: A correct equation in $\tan\theta$. Look for $3\tan^2\theta - \tan\theta - 4 = 0$ or equivalent.

dM1: Uses a correct method to solve a 3-term quadratic in $\tan\theta$ to obtain at least one value for $\tan\theta$.

Dependent on both previous method marks.

A1: Any two from awrt $\theta = 0.927, 4.069, \frac{3}{4}\pi(2.356), \frac{7}{4}\pi(5.498)$

A1: All four of awrt $\theta = 0.927, 4.069, \frac{3}{4}\pi(2.356), \frac{7}{4}\pi(5.498)$

If all the angles are correct but given are given to less accuracy (but at least to 2dp) then score A1A0.
The angles do not have to appear all on the same line so award marks when the correct angles are seen.
Answers in degrees: awrt 53.130, 233.130, 135, 315. Score A1 for any 2 of these but withhold the final A mark.
Ignore extra answers outside the range but deduct the final A mark for extra answers in range in an otherwise correct solution.

If the candidate starts again and you think the attempt is worth credit then please send to review.

Question Number	Scheme	Marks
5(i)	$\int\left((3x+5)^9+e^{5x}\right)dx = \frac{(3x+5)^{10}}{30}, + \frac{e^{5x}}{5}(+c)$	M1A1, B1
		(3)
(ii)	$\int \frac{x}{x^2+5}dx = \frac{1}{2}\ln\left(x^2+5\right)$	M1A1
	$\int_2^b \frac{x}{x^2+5}dx = \ln\left(\sqrt{6}\right) = \frac{1}{2}\ln\left\|b^2+5\right\| - \frac{1}{2}\ln\left\|2^2+5\right\| = \ln\left(\sqrt{6}\right)$	M1
	$\Rightarrow \ln\left(\frac{b^2+5}{9}\right) = \ln 6 \Rightarrow b = 7$	**dd**M1, A1
		(5)
		(8 marks)

(i)

M1: For an integral of the form $C(3x+5)^{10}$ or $C(3x+5)^{9+1}$ where C is a constant and no other powers of $(3x + 5)$

A1: $\frac{(3x+5)^{10}}{30}$. No need for + c. Allow un-simplified e.g. $\frac{\frac{1}{3}(3x+5)^{10}}{10}$.

B1: $e^{5x} \to \frac{e^{5x}}{5}$

Mark each integration independently i.e. there is no need to see everything all on one line.

(ii)

M1: For an answer of the form $C\ln k\left(x^2+5\right)$ where C and k are constants. Allow log for ln.

A1: $\frac{1}{2}\ln k\left(x^2+5\right)$ or $\ln k\left(x^2+5\right)^{\frac{1}{2}}$ or $\frac{1}{2}\ln k\left|x^2+5\right|$. Allow log for ln.

M1: Substitutes in both 2 and b for x correctly and subtracts either way around and sets equal to $\ln\left(\sqrt{6}\right)$.

ddM1: Removes logs correctly to obtain an equation in b. **Dependent on both previous M marks**.

A1: $b = 7$ only. $b = \pm 7$ scores A0 unless the -7 is rejected.

<u>Note: May see integration by substitution in (ii)</u>

E.g. $u = x^2 + 5$

M1: $\int \frac{x}{x^2+5}dx = \int \frac{x}{u}\frac{du}{2x} = \frac{1}{2}\ln u$

For an answer of the form $C\ln k(u)$ where C is a constant

Allow log for ln as above.

A1: $\frac{1}{2}\ln ku$

M1: $\left[\frac{1}{2}\ln u\right]_9^{b^2+5} = \frac{1}{2}\ln\left(b^2+5\right) - \frac{1}{2}\ln 9 = \ln\sqrt{6}$

Substitutes in both 9 and $b^2 + 5$ correctly and subtracts either way around and sets equal to $\ln\left(\sqrt{6}\right)$.

ddM1: Removes logs correctly to obtain an equation in b. **Dependent on both previous M marks**.

A1: $b = 7$ only. $b = \pm 7$ scores A0 unless the -7 is rejected.

Question Number	Scheme	Marks
6 (a)	0.41576	B1
		(1)
(b)	Strip width = $\frac{\pi}{4}$	B1
	Area $\approx \frac{1}{2}\times\frac{\pi}{4}\times\left\{0+2\left(0.76679+0.41576+0.15940\right)+0\right\}$ Or separate trapezia: $\frac{1}{2}\times\frac{\pi}{4}\times\{0+0.766792\}+\frac{1}{2}\times\frac{\pi}{4}\times\{0.766792+0.41576\}+$ $\frac{1}{2}\times\frac{\pi}{4}\times\{0.41576+0.15940\}+\frac{1}{2}\times\frac{\pi}{4}\times\{0.15940+0\}$	M1
	1.0540	A1
		(3)
(c)	Uses $vu'+uv'$: $\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{-x}\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{-x}(\sin x)^{\frac{1}{2}}$ **or** Uses $\frac{vu'-uv'}{v^2}$: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^{x}\times 2\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{x}(\sin x)^{\frac{1}{2}}}{\mathrm{e}^{2x}}$	M1A1A1
		(3)
(d)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 2\mathrm{e}^{-x}\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{-x}(\sin x)^{\frac{1}{2}}=0$	
	$\cos x=2\sin x$	M1
	$\tan x=\frac{1}{2}\Rightarrow x=0.464$	dM1A1
		(3)
		(10 marks)

(a)
B1: awrt 0.41576
(Note that degrees gives 0.068835….and scores B0)
(b)
B1: Strip width = $\frac{\pi}{4}$ or awrt 0.785. This may be implied by seeing $\frac{1}{2}\times\frac{\pi}{4}\times\{...\}$ or $\frac{\pi}{8}\times\{...\}$ within the trapezium formula
M1: Correct structure for the trapezium formula. Do not condone missing brackets unless they are implied by subsequent work. (Allow the 0's to be omitted in the brackets)
A1: awrt 1.0540 (Not 1.054) (note that this mark is still available even if (a) is not given to the required accuracy)
(Note that degrees gives 0.78149…)
(c)
M1: Uses *vu*'+*uv*' with $u/v=2\mathrm{e}^{-x}, u/v=(\sin x)^{0.5}$ If the rule is quoted it must be correct.
It may be implied by, for example, $u=2\mathrm{e}^{-x}, v=\sqrt{\sin x}$ followed by their $u'=..,v'=..$ and $vu'+uv'$
If it is not quoted nor implied then look for an expression of the form f(x) ± g(x) where f(x) or g(x) is of the form $A\mathrm{e}^{-x}\sqrt{\sin x}$ or $A\mathrm{e}^{-x}(\sin x)^{-0.5}\cos x$ with A non-zero.
A1: Either term of the derivative correct
A1: Completely correct derivative $\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{-x}\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2\mathrm{e}^{-x}(\sin x)^{\frac{1}{2}}$. Allow un-simplified and allow …+–… for … – …

Penalise poor use of powers once only e.g. $(\sin x)^{-\frac{1}{2}}$ written as $\sin x^{-\frac{1}{2}}$ unless corrected later.

Quotient rule on $\frac{2\sqrt{\sin x}}{e^x}$: M1: Uses $\frac{vu' - uv'}{v^2}$ with $u = 2(\sin x)^{0.5}, v = e^x$ If the rule is quoted it must be correct. It may be implied by, for example, $u = 2\sqrt{\sin x}, v = e^x$ followed by their $, u' = .., v' = ..$ and $\frac{vu' - uv'}{v^2}$

If it is not quoted nor implied then look for an expression of the form $\frac{dy}{dx} = \frac{f(x) \pm g(x)}{(e^x)^2}$ where f(x) or g(x) is of the form $Ae^x\sqrt{\sin x}$ or $Ae^x(\sin x)^{-0.5}\cos x$ with A non-zero.

A1: Either term of the numerator correct, including the correct denominator.

A1: Completely correct derivative $\frac{dy}{dx} = \frac{e^x \times 2 \times \frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x) - 2e^x(\sin x)^{\frac{1}{2}}}{e^{2x}}$. Allow un-simplified and allow …+–… for … – …

Penalise poor use of powers once only e.g. $(\sin x)^{-\frac{1}{2}}$ written as $\sin x^{-\frac{1}{2}}$ unless corrected later.

Attempts at $ye^x = 2\sqrt{\sin x}$ followed by implicit differentiation should be sent to review.

(d)

M1: Sets $\frac{dy}{dx} = 0$ and proceeds, using correct algebra and allowing arithmetic slips only, to $A\cos x = B\sin x$ or $A\cos x - B\sin x = 0$

dM1: Divides by cos$x$ to reach tan$x$ = α where $\alpha \neq \pm 1$

A1: cso awrt 0.464 (Do not allow 0.148π)

Note that in (d), some candidates may square once they reach $A\cos x = B\sin x$

E.g.

$$\cos x = 2\sin x \Rightarrow \cos^2 x = 4\sin^2 x \Rightarrow 5\sin^2 x = 1 \Rightarrow \sin x = \frac{1}{\sqrt{5}}$$

Or

$$\cos x = 2\sin x \Rightarrow \cos^2 x = 4\sin^2 x \Rightarrow 5\cos^2 x = 4 \Rightarrow \cos x = \frac{2}{\sqrt{5}}$$

In such cases, score dM1 for reaching cos x = α or cos x = β $\alpha, \beta \neq \frac{1}{\sqrt{2}}$ and A1 for x = awrt 0.464 but withhold the A1 if there are any extra solutions in range.

Candidates who attempt to square both sides of $\cos x - 2\sin x = 0$ are unlikely to progress further but if you see work that you think deserves credit, send to review.

Question Number	Scheme	Marks
7(a)	2^{-3} or $\frac{1}{2^3}$ or 0.125	B1
	$\frac{1}{(2-3x)^3}=(2-3x)^{-3}=\frac{1}{2^3}\left(1-\frac{3x}{2}\right)^{-3}$	
	$=\frac{1}{8}\left(1+(-3)\times\left(-\frac{3x}{2}\right)+\frac{-3\times-4}{2!}\times\left(-\frac{3x}{2}\right)^2+...\right)$	M1A1
	$=\frac{1}{8}+\frac{9}{16}x\,;+\frac{27}{16}x^2+...$	A1; A1
		(5)
(b)	$\frac{4+kx}{(2-3x)^3}=(4+kx)\left(\frac{1}{8}+\frac{9}{16}x+\frac{27}{16}x^2+...\right)$	
	Compares x^2 terms $\frac{27}{4}+\frac{9k}{16}=\frac{81}{16}\Rightarrow k=...$	M1
	$k=-3$	A1
		(2)
(c)	Compares x terms $\frac{9}{4}+\frac{1}{8}\times'-3'=A\Rightarrow A=...$	M1
	$A=\frac{15}{8}$	A1
		(2)
		(9 marks)
7(a) ALT	2^{-3} or $\frac{1}{2^3}$	B1
	$(2-3x)^{-3}=2^{-3}+(-3)2^{-4}(-3x)+\frac{(-3)(-4)}{2}2^{-5}(-3x)^2$ M1: For 2^{-3} and the structure of at least one of the other terms correct A1: Fully correct	M1A1
	$=\frac{1}{8}+\frac{9}{16}x\,;+\frac{27}{16}x^2+...$	A1; A1
		(5)

(a)

B1: For taking out a factor of 2^{-3} or $\frac{1}{2^3}$ or $\frac{1}{8}$ or 0.125

M1: Score for the form of the binomial expansion with index -3

Eg $=\left\{\frac{1}{8}\right\}\left[1+(-3)(**x)+\frac{(-3)(-4)}{2!}(**x)^2+...\right]$ where $**\neq 1$ or -1

Requires 1 + … with the structure of at least one of the other terms correct as shown above.

A1: Correct un-simplified form $=\left\{\frac{1}{8}\right\}\left(1+(-3)\times\left(-\frac{3x}{2}\right)+\frac{-3\times-4}{2!}\times\left(-\frac{3x}{2}\right)^2+...\right)$

Condone missing brackets around the $**x$ provided they are recovered later.

A1: First two terms correct and simplified $\frac{1}{8}+\frac{9}{16}x$

A1: Third term correct and simplified $+\frac{27}{16}x^2$.

(Allow this mark from an expansion using $\frac{3x}{2}$ rather than $-\frac{3x}{2}$)

(b)

M1: Finds the sum of the coefficients of their two x^2 terms and sets equal to $\frac{81}{16}$ and proceeds to find a value for k. E.g. $4\times"\frac{27}{16}"+"\frac{9}{16}"k=\frac{81}{16}\Rightarrow k=...$

A1: cso $k=-3$ (Must come from correct work)

(c)

M1: Finds the sum of the coefficients of their two x terms using their value of k and proceeds to find a value for A. E.g. $A=4\times"\frac{9}{16}"+"\frac{1}{8}"k$

A1: $A=\frac{15}{8}$ oe e.g. 1.875 (If $k = -3$ is obtained fortuitously in (b) allow $A=\frac{15}{8}$ here)

Question Number	Scheme	Marks
8	$2+...$	B1
	Obtains $\frac{A}{x}+\frac{B}{x-1}$ where A and B are constants	M1
	$\frac{3}{x}$ or $-\frac{1}{x-1}$ or $A = 3$ or $B = -1$	A1
	$\frac{3}{x}-\frac{1}{x-1}$	A1 **(B1 on Epen)**
	$\int_3^4 \frac{2x^2-3}{x(x-1)}\,dx = \int_3^4 \left(2+\frac{3}{x}-\frac{1}{x-1}\right)dx$	
	$=\left[2x+3\ln x-\ln(x-1)\right]_3^4$	M1 A1ft
	$=\left(8+3\ln 4-\ln 3\right)-\left(6+3\ln 3-\ln 2\right)=2+\ln\left(\frac{128}{81}\right)$	M1 A1cso
		(8 marks)

B1: 2 +…

M1: Obtains $\frac{A}{x}+\frac{B}{x-1}$ where A and B are constants

A1: $\frac{3}{x}$ or $-\frac{1}{x-1}$ or one correct constant

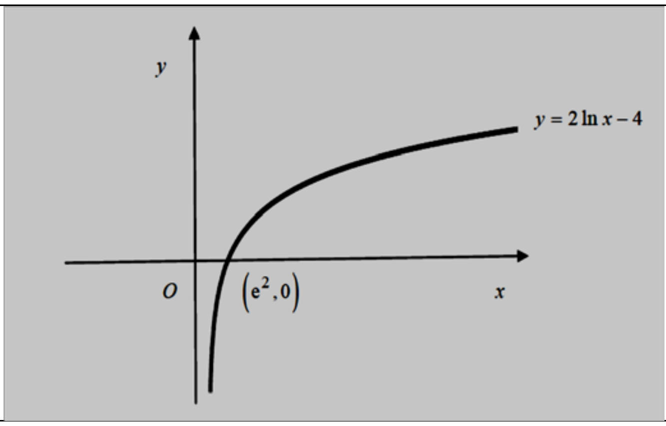
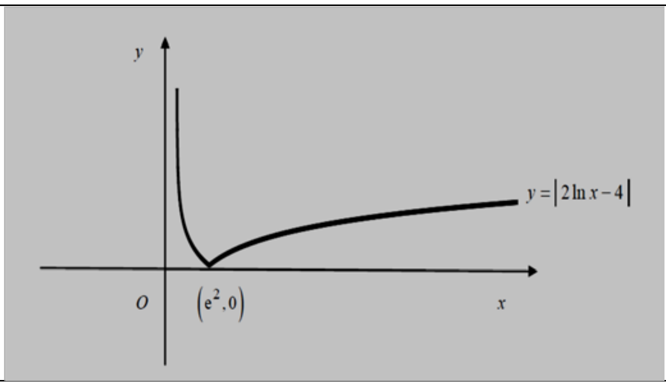
B1: $\frac{3}{x}-\frac{1}{x-1}$

M1: For $\int \frac{*}{x}+\frac{*}{x-1}\,dx \rightarrow p\ln mx+q\ln n(x-1)$ where *, p, q, m and n are constants.

A1ft: $2x+3\ln x-\ln(x-1)$. Follow through their "2", A and B so look for "2"x+Aln$x$+Bln(x – 1). This mark can be withheld if the brackets are missing unless subsequent work suggests their intended presence.

M1: For substituting in 3 and 4, subtracting either way around and using correct addition or subtraction log laws at least once.

A1: cso $2+\ln\left(\frac{128}{81}\right)$ or $2+\ln\left(1\frac{47}{81}\right)$ (Do not allow $2+\ln\left(\frac{2^7}{3^4}\right)$) $2+\ln\left(\frac{128}{81}\right)+c$ is also A0

Question Number	Scheme		Marks
9(a)(i)		Shape	B1
		$\left(e^2, 0\right)$	B1
		Asymptote $x = 0$	B1
			(3)
(a)(ii)		Shape	B1ft
		Asymptote and coordinate	B1ft
			(2)
(b)	$2\ln x - 4 = 4 \Rightarrow \ln x = 4 \Rightarrow x = e^4$		M1A1
	$2\ln x - 4 = -4 \Rightarrow \ln x = 0 \Rightarrow x = 1$		M1A1
			(4)
(c)	$\text{gf}\,(x) = e^{2\ln x - 4 + 5} - 2, = e^1 \times e^{2\ln x} - 2 = e\,x^2 - 2$		M1,dM1A1
			(3)
(d)	$\text{gf}(x) > -2$		B1
			(1)
			(13 marks)

(a)(i)

B1: For a logarithmic shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1: Intersection with the *x* axis at $\left(e^2, 0\right)$.

Allow e^2 marked on the *x* axis. Condone $\left(0, e^2\right)$ being marked on the positive *x* axis.

Do not allow e^2 appearing as 7.39 for this mark unless e^2 is seen in the body of the script.
Allow if the coordinate is given in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then the ones on the curve take precedence.

B1: **Equation** of asymptote is $x = 0$ (do not allow " *y*-axis"). Note that the curve must appear to have an asymptote at $x = 0$

(a)(ii)

B1ft: For either the correct shape or a reflection of their "negative" curve in (a) in the *x*-axis. For this to be scored it must have appeared both above and below the *x*-axis. The curve to the lhs of the intercept must appear to have the correct curvature

B1ft: Score for the correct coordinates and asymptote. Alternatively follow through on the coordinates and asymptote given in part (a) as long as the curve appeared both above and below the *x*-axis and the curve approaches the same asymptote stated in (a)(i). Do not penalise "*y*-axis" given as the asymptote twice – i.e. penalise in (a)(i) only.

If the curves are sketched on the same axes – it must be clear which curve is which – if in doubt use review.

(b)
M1: Sets $2\ln x - 4 = 4$ and proceeds to $x = e^{\cdots}$. This may be implied by an answer of awrt 55
A1: $x = e^4$
A correct answer only of $x = e^4$ implies both marks.
M1: Sets $-2\ln x + 4 = 4$ and proceeds to $x = e^{\cdots}$ or sets $2\ln x + 4 = -4$ and proceeds to $x = e^{\cdots}$
May be implied by an answer of e°
A1: $x = 1$
Note that $x = 1$ may be found by symmetry if (1, -4) is identified as a point on the original curve.
Allow M1A1 if $x = 1$ is found by this approach.

Alternative by squaring:

M1: $(2\ln x - 4)^2 = 16 \Rightarrow 4(\ln x)^2 - 16\ln x + 16 = 16 \Rightarrow \ln x = \ldots$
Squares both sides including expanding lhs and proceeds to solve for $\ln x$
M1: Proceeds from $\ln x = \ldots$ to find at least one value for x
A1: $x = e^4$
A1: $x = 1$

(c)
M1: Attempts $gf(x)$ the correct way around. Evidence is $gf(x) = e^{2\ln x - 4 + 5} - 2$

Look for $gf(x) = e^{2\ln x \pm ..}....$

dM1: Correct processing leading to an expression of the form $e^k x^2 - 2,\ k \neq 0$
(Only allow slips on the "– 4 + 5")
A1: cso $ex^2 - 2$ (Allow $e^1x^2 - 2$)

(d)
B1: Acceptable answers are: " > -2", $gf(x) > -2$, range > -2, $y > -2$, $-2 < gf(x) < \infty, (-2, \infty)$ but not $x > -2$
Allow in words e.g. gf is greater than – 2 or y is bigger than –2 etc.

Question Number	Scheme	Marks
10 (a)	$t(t-4)=0 \Rightarrow t=4$ Hence $x=\frac{20\times 4}{2\times 4+1}=\frac{80}{9}$	M1A1
		(2)
(b)	$x=\frac{20t}{2t+1} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=\frac{20(2t+1)-20t\times 2}{(2t+1)^2}=\left(\frac{20}{(2t+1)^2}\right)$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}=\frac{(2t-4)}{20/(2t+1)^2},=\frac{(t-2)(2t+1)^2}{10}$	M1A1,A1
		(5)
	Mark c(i) and (ii) together:	
(c)(i)	$x=\frac{20t}{2t+1} \Rightarrow 2tx+x=20t \Rightarrow t(20-2x)=x \Rightarrow t=\frac{x}{20-2x}$ or $\frac{-x}{2x-20}$	M1A1
		(2)
(ii)	Sub $t=\frac{x}{20-2x}$ into $y=t(t-4) \Rightarrow y=\frac{x}{20-2x}\left(\frac{x}{20-2x}-4\right)$	M1
	$\Rightarrow y=\frac{x}{20-2x}\times\left(\frac{x}{20-2x}-\frac{4(20-2x)}{20-2x}\right)$	**d**M1
	$\Rightarrow y=\frac{x}{20-2x}\times\left(\frac{9x-80}{20-2x}\right)$	
	$\Rightarrow y=\frac{x(9x-80)}{(20-2x)^2}$, oe $0<x<10$ or $k=10$	A1, B1
		(4)
		(13 marks)

(a)

M1: Attempts to find x when $t=4$

A1: $\frac{80}{9}$ (Not 8.88… but isw if $\frac{80}{9}$ is seen)

(Ignore any attempts to find x when $t = 0$)

(b)

M1: Attempts to apply the quotient rule on $\frac{20t}{2t+1}$ with $u=20t, v=2t+1$

Alternatively applies the product rule on $20t(2t+1)^{-1}$ OR writes $\frac{20t}{2t+1}$ as $A-\frac{B}{2t+1}$ and uses the chain rule

A1: $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{20(2t+1)-20t\times 2}{(2t+1)^2}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}=20(2t+1)^{-1}+20t\times -2(2t+1)^{-2}$

M1: Attempts to use $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Need to see correct chain rule seen or implied with their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ which is not y.

A1: $\frac{dy}{dx}$ correct and un-simplified. Score for $\frac{dy}{dx} = \frac{(2t-4)}{\frac{20}{(2t+1)^2}}$

Allow "invisible" brackets to be recovered if a correct answer appears later.

A1: $\frac{dy}{dx} = \frac{(t-2)(2t+1)^2}{10}$

(c)(i)

M1: A full attempt to make t the subject of $x = \frac{20t}{2t+1}$. Need to see a correct method here with sign slips only so must reach $t = \frac{\pm x}{\pm 20 \pm 2x}$.

A1: $t = \frac{x}{20-2x}$ or equivalent e.g. $t = \frac{-x}{2x-20}$

(c)(ii)

M1: Substitutes THEIR $t = \frac{x}{20-2x}$ into $y = t(t-4)$ to find y in terms of x

dM1: Uses a correct common denominator, adapting the numerator of the second fraction. Condone sign errors only when combining **their** fractions either inside the brackets or once the brackets have been expanded. **Dependent on the first method mark.**

A1: $y = \frac{x(9x-80)}{(20-2x)^2}$ Accept exact alternatives such as $y = \frac{x(9x-80)}{4(10-x)^2}$, $y = \frac{9x^2-80x}{4(10-x)^2}$, $y = \frac{9x^2-80x}{400-80x+4x^2}$

Note that it is possible to find y from integrating dy/dx as a function of x – send such cases to review.

B1: Accept either the domain is $0 < x < 10$ or $k = 10$

Question Number	Scheme	Marks
11 (a)	$\frac{du}{dh}=-\frac{1}{2}h^{-\frac{1}{2}}$ OR $\frac{dh}{du}=-2(5-u)$ or $du=-\frac{1}{2}h^{-\frac{1}{2}}dh$ etc.	B1
	$\int\frac{dh}{5-\sqrt{h}}=\int\frac{-2(5-u)du}{u}=\int\left(\frac{-10}{u}+2\right)du$	M1dM1A1
	$=-10\ln u+2u+c$	
	$=-10\ln\left(5-\sqrt{h}\right)+2\left(5-\sqrt{h}\right)+c$	M1
	$=-10\ln\left(5-\sqrt{h}\right)-2\sqrt{h}+k$	A1*
		(6)
(b)	$\frac{dh}{dt}=\frac{t^{0.2}\left(5-\sqrt{h}\right)}{5}\Rightarrow\int\frac{dh}{\left(5-\sqrt{h}\right)}=\int\frac{t^{0.2}\,dt}{5}$	B1
	$\Rightarrow-10\ln\left(5-\sqrt{h}\right)-2\sqrt{h}+k=\frac{t^{1.2}}{6}$ or equivalent	M1A1
	Substitute $t=0, h=2\Rightarrow\ k=10\ln\left(5-\sqrt{2}\right)+2\sqrt{2}=(\text{awrt } 15.6)$	M1
	Substitute $h=15\ \Rightarrow\frac{t^{1.2}}{6}=-10\ln\left(5-\sqrt{15}\right)-2\sqrt{15}+10\ln\left(5-\sqrt{2}\right)+2\sqrt{2}$	**d**M1
	$\Rightarrow t^{1.2}=39.94\Rightarrow t=21.6$ (or 21.7)	**d**M1A1
		(7)
(c)	$\frac{dh}{dt}=\frac{21.6^{0.2}\left(5-\sqrt{15}\right)}{5}=0.42$ = 42 (cm per year)	B1
		(1)
		(14 marks)
Note this is where "5" is brought to lhs	$\frac{dh}{dt}=\frac{t^{0.2}\left(5-\sqrt{h}\right)}{5}\Rightarrow\int\frac{5\,dh}{\left(5-\sqrt{h}\right)}=\int t^{0.2}\,dt$	B1
	$\Rightarrow-50\ln\left(5-\sqrt{h}\right)-10\sqrt{h}+'k'=\frac{t^{1.2}}{1.2}$ or equivalent	M1A1
	Substitute $t=0, h=2\Rightarrow\ k=50\ln\left(5-\sqrt{2}\right)+10\sqrt{2}=(\text{awrt } 78)$	M1
	Substitute $h=15$ $\Rightarrow\frac{t^{1.2}}{1.2}=-50\ln\left(5-\sqrt{15}\right)-10\sqrt{15}+50\ln\left(5-\sqrt{2}\right)+10\sqrt{2}$	M1
	$\Rightarrow t^{1.2}=39.94\Rightarrow t=21.6$ (or 21.7)	dM1A1
		(7)

(a)

B1: For $\frac{\mathrm{d}u}{\mathrm{d}h}=-\frac{1}{2}h^{-\frac{1}{2}}$ OR $\frac{\mathrm{d}h}{\mathrm{d}u}=-2(5-u)$ or equivalent. For example accept versions such as $\mathrm{d}u=-\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h$ and $\mathrm{d}h=-2\sqrt{h}\,\mathrm{d}u$

M1: Attempts to rewrite the integral in terms of *h* to an integral in terms of *u*. Expect to see both $\mathrm{d}h$ and $5-\sqrt{h}$ written in terms of *u* but $\mathrm{d}h\neq\mathrm{d}u$

dM1: Divides by '*u*' to reach a form $\int\left(\frac{A}{u}+B\right)\mathrm{d}u$ where *A* and *B* are constants. **Dependent on the first method mark**.

A1: $\int\left(\frac{-10}{u}+2\right)\mathrm{d}u$ or e.g. $2\int\left(\frac{-5}{u}+1\right)\mathrm{d}u$

M1: $\int\left(\frac{A}{u}+B\right)\mathrm{d}u\to A\ln u+Bu\to A\ln\left(5-\sqrt{h}\right)+B\left(5-\sqrt{h}\right)+C$

Reaches $A\ln\left(5-\sqrt{h}\right)+B\left(5-\sqrt{h}\right)$ **with or without** a constant of integration

A1*: CSO. There must have been a constant at the point of integration above and evidence that $2\left(5-\sqrt{h}\right)+c\to-2\sqrt{h}+k$ but do not accept $2\left(5-\sqrt{h}\right)+k\to-2\sqrt{h}+k$ unless it is accompanied by an explanation that 10 + *k* is a constant.

May see $-10\ln\left(5-\sqrt{h}\right)+2\left(5-\sqrt{h}\right)+c=-10\ln\left(5-\sqrt{h}\right)+10-2\sqrt{h}+c$ where 10 + *c* = *k* which is acceptable.

(b)

B1: Separates the variables.

Accept $\int\frac{\mathrm{d}h}{\left(5-\sqrt{h}\right)}=\int\frac{t^{0.2}\,\mathrm{d}t}{5}$ or equivalent, even without the integral signs.

M1: Attempts to integrate both sides. **Must see:**

$$\int\frac{\mathrm{d}h}{\left(5-\sqrt{h}\right)}\to A\ln\left(5-\sqrt{h}\right)+B\sqrt{h}\,(+k)\ \textbf{and}\ \int t^{0.2}\,\mathrm{d}t\to Ct^{1.2}\ \text{(Allow } C=1\text{)}$$

A1: $-10\ln\left(5-\sqrt{h}\right)-2\sqrt{h}+k=\frac{t^{1.2}}{6}$ or $-10\ln\left(5-\sqrt{h}\right)-2\sqrt{h}=\frac{t^{1.2}}{6}+c$

$-50\ln\left(5-\sqrt{h}\right)-10\sqrt{h}+'k'=\frac{t^{1.2}}{1.2}$ or $-50\ln\left(5-\sqrt{h}\right)-10\sqrt{h}=\frac{t^{1.2}}{1.2}+c$

All correct with the constant appearing one side or the other (or both)

Note that some candidates think the *k* in part (a) is 10 – in these cases, provided all the work is correct, allow all the marks in (b) and (c) but a constant of integration must be found.

M1: Substitutes *t* = 0 and *h* = 2 to find a value for their constant

dM1: Substitute $h=15$ in an equation for *t* involving a numerical constant. **Dependent on the previous method mark.**

dM1: **All previous method marks must have been scored**. It is for obtaining a value for *t* (even if the processing is poor).

A1: cso $t=21.6$ (years) or $t=21.7$ (years)

Alternative:

B1: Separates the variables.

Accept $\int \frac{\mathrm{d}h}{\left(5-\sqrt{h}\right)} = \int \frac{t^{0.2}\,\mathrm{d}t}{5}$ or equivalent, even without the integral signs.

M1: Attempts to integrate both sides. **Must see:**

$$\int \frac{\mathrm{d}h}{\left(5-\sqrt{h}\right)} \to A\ln\left(5-\sqrt{h}\right)+B\sqrt{h}\left(+k\right) \textbf{ and } \int t^{0.2}\,\mathrm{d}t \to Ct^{1.2}$$

A1: $\left[-10\ln\left(5-\sqrt{h}\right)-2\sqrt{h}\right]_2^{15} = \left[\frac{t^{1.2}}{6}\right]_0^T$

All correct with the correct limits attached (constants not needed but may be present)

M1: Substitutes h = 15 and h = 2 and subtracts

dM1: Substitutes t = "T" and t = 0 and subtracts – may be implied by just $\frac{t^{1.2}}{6}$. **Dependent on the previous method mark.**

dM1: **All previous method marks must have been scored**. It is for finding t.

A1: cso $t = 21.6$ (years) or $t = 21.7$ (years)

(c)

B1: 42 only

Question Number	Scheme	Marks
12(a)	$(2,0,7)$	B1
		(1)
(b)	$\begin{pmatrix}2\\-2\\1\end{pmatrix}.\begin{pmatrix}8\\4\\1\end{pmatrix}=16-8+1=3\times9\cos\theta$	M1A1
	$\cos\theta=\frac{1}{3}$	A1
	Uses $\sin^2\theta+\cos^2\theta=1\Rightarrow\sin\theta=\sqrt{1-\left(\frac{1}{3}\right)^2}=\sqrt{\frac{8}{9}}=\frac{2}{3}\sqrt{2}$	M1A1*
		(5)
(c)	$AB=\sqrt{8^2+8^2+4^2}$ OR $AB=4\times'3'$	M1
	Area = $'\frac{1}{2}ab\sin C'=\frac{1}{2}\times12\times24\times\frac{2}{3}\sqrt{2}=96\sqrt{2}$	M1A1
		(3)
(d)	Attempts to find value of μ by $\frac{\text{length } AC}{\lvert(8,4,1)\rvert}=\frac{24}{9}$	M1A1
	Attempts $\begin{pmatrix}2\\0\\7\end{pmatrix}\pm\frac{8}{3}\begin{pmatrix}8\\4\\1\end{pmatrix}$	dM1
	$\left(\frac{70}{3},\frac{32}{3},\frac{29}{3}\right),\left(-\frac{58}{3},-\frac{32}{3},\frac{13}{3}\right)$	A1A1
		(5)
		(14 marks)

(a)

B1: Accept $(2,0,7)$ or the vector equivalent

(b)

M1: Correct full method for the scalar product of the direction vectors or any multiple of the direction vectors.

A1: $\begin{pmatrix}2\\-2\\1\end{pmatrix}.\begin{pmatrix}8\\4\\1\end{pmatrix}=16-8+1=\sqrt{2^2+2^2+1^2}\times\sqrt{8^2+4^2+1^2}\cos\theta$

A correct numerical statement involving $\cos\theta$

May see the cosine rule e.g. $72=3^2+9^2-2\sqrt{2^2+2^2+1^2}\times\sqrt{8^2+4^2+1^2}\cos\theta$ or equivalent

A1: $\cos\theta=\frac{1}{3}$ oe (may be implied)

M1: Uses $\sin^2\theta+\cos^2\theta=1\Rightarrow\sin\theta=\sqrt{1-their\left(\frac{1}{3}\right)^2}$. Allow methods using a right angled triangle but must see correct work e.g. $\cos\theta=\frac{1}{3}\Rightarrow\sin\theta=\frac{\sqrt{3^2-1}}{3}$. Also this mark may be implied by a correct answer of

$\frac{2}{3}\sqrt{2}$. So allow e.g. $\cos\theta = \frac{1}{3} \Rightarrow \sin\theta = \sin\left(\cos^{-1}\frac{1}{3}\right) = \frac{2}{3}\sqrt{2}$ or

$\cos\theta = \frac{1}{3} \Rightarrow \theta = 70.52...\sin\theta = \sin(70.52...) = \frac{2}{3}\sqrt{2}$ or just $\cos\theta = \frac{1}{3} \Rightarrow \sin\theta = \frac{2}{3}\sqrt{2}$

A1*: $\sin\theta = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}$ with no need to state the value of *k*.

(c)

M1: A correct method of finding $|AB| = |\boldsymbol{b} - \boldsymbol{a}| = \sqrt{8^2 + 8^2 + 4^2}$ or alternatively uses $|AB| = 4 \times '3'$

M1: Uses Area = $'\frac{1}{2}|AB| \times 2|AB|\sin\theta'$ with their $\sin\theta$ but not $'\frac{1}{2}|AB| \times \frac{1}{2}|AB|\sin\theta'$

A1: $96\sqrt{2}$

(d)

M1: Attempts to find value of μ by $\frac{\text{length } AC}{|(8,4,1)|}$ or e.g. $\sqrt{(8\mu)^2 + (4\mu)^2 + \mu^2} = "24"$ or

$(8\mu)^2 + (4\mu)^2 + \mu^2 = "24"^2$

but not $\sqrt{(8\mu)^2 + (4\mu)^2 + \mu^2} = 24^2$ i.e. both sides must be consistent.

A1: $\mu = (\pm)\frac{24}{9}$

dM1: Attempts to find at least one position for *C* by using $\begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} \pm$ **their** $'\frac{8}{3}'\begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}$

A1: Either of $\left(\frac{70}{3}, \frac{32}{3}, \frac{29}{3}\right), \left(-\frac{58}{3}, -\frac{32}{3}, \frac{13}{3}\right)$.

Allow in vector form as $\begin{pmatrix} \frac{70}{3} \\ \frac{32}{3} \\ \frac{29}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 70 \\ 32 \\ 29 \end{pmatrix}$ or $\begin{pmatrix} -\frac{58}{3} \\ -\frac{32}{3} \\ \frac{13}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} -58 \\ -32 \\ 13 \end{pmatrix}$ but not e.g. $\frac{1}{3}(70, 32, 29)$

A1: Both of $\left(\frac{70}{3}, \frac{32}{3}, \frac{29}{3}\right), \left(-\frac{58}{3}, -\frac{32}{3}, \frac{13}{3}\right)$.

Allow in vector form as $\begin{pmatrix} \frac{70}{3} \\ \frac{32}{3} \\ \frac{29}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} 70 \\ 32 \\ 29 \end{pmatrix}$ or $\begin{pmatrix} -\frac{58}{3} \\ -\frac{32}{3} \\ \frac{13}{3} \end{pmatrix}$ or $\frac{1}{3}\begin{pmatrix} -58 \\ -32 \\ 13 \end{pmatrix}$ but not e.g. $\frac{1}{3}(70, 32, 29)$

Note that using $\overrightarrow{OC} = \overrightarrow{OA} \pm 2\overrightarrow{AB}$ is common and gives $(18, -16, 15), (-14, 16, -1)$ and generally scores **no** marks in (d).