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Mark Scheme (Results)
January 2008

GCE

GCE Mathematics (6666/01)

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1. (a)	x: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π y: 0, 1.844321332…, 4.810477381…, 8.87207, 0		
		awrt 1.84432 awrt 4.81048 or 4.81047	B1 B1 **[2]**
(b) **Way 1**	Area $\approx \frac{1}{2} \times \frac{\pi}{4}$; $\times \left\{ 0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0 \right\}$ (0 can be implied)	Outside brackets awrt 0.39 or $\frac{1}{2} \times$ awrt 0.79 $\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$	B1
		For structure of trapezium rule $\{\ldots\ldots\ldots\ldots\}$;	M1 $\sqrt{}$
		Correct expression inside brackets which all must be multiplied by their "outside constant".	A1 $\sqrt{}$
	$= \frac{\pi}{8} \times 31.05374\ldots = 12.19477518\ldots = 12.1948$ (4dp)	12.1948	A1 **cao** **[4]**
Aliter (b) **Way 2**	Area $\approx \frac{\pi}{4} \times \left\{ \frac{0+1.84432}{2} + \frac{1.84432+4.81048}{2} + \frac{4.81048+8.87207}{2} + \frac{8.87207+0}{2} \right\}$	$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	M1 $\sqrt{}$
	Area $\approx \frac{1}{2} \times \frac{\pi}{4}$; $\times \left\{ 0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0 \right\}$	Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	A1 $\sqrt{}$
	$= \frac{\pi}{4} \times 15.52687\ldots = 12.19477518\ldots = 12.1948$ (4dp)	12.1948	A1 **cao** **[4]**
			6 marks

Note an expression like Area $\approx \frac{1}{2} \times \frac{\pi}{4} + 2\left(1.84432 + 4.81048 + 8.87207\right)$ would score B1M1A0A0

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
2. (a)	$(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$	Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$.	B1
	$= 2\left\{1+(\tfrac{1}{3})(**x);+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3+\ldots\right\}$ **with** $** \neq 1$	Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\frac{1}{3})(**x)$;	M1;
		A correct simplified or an un-simplified $\{\ldots\ldots\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\tfrac{3x}{8})^3+\ldots\right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3$	
	$= 2\{1-\tfrac{1}{8}x;-\tfrac{1}{64}x^2-\tfrac{5}{1536}x^3-\ldots\}$	Either $2\{1-\frac{1}{8}x\ \ldots\ldots\}$ or anything that cancels to $2-\frac{1}{4}x$;	A1;
	$= 2-\frac{1}{4}x;-\frac{1}{32}x^2-\frac{5}{768}x^3-\ldots$	Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$	A1
			[5]
(b)	$(7.7)^{\frac{1}{3}} \approx 2-\frac{1}{4}(0.1)-\frac{1}{32}(0.1)^2-\frac{5}{768}(0.1)^3-\ldots$	***Attempt to substitute*** $x=0.1$ into a candidate's binomial expansion.	M1
	$= 2-0.025-0.0003125-0.0000065104166\ldots$		
	$= 1.97468099\ldots$	awrt 1.9746810	A1
			[2]
			7 marks

You would award B1M1A0 for

$$=2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-3x)^3+\ldots\right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\ldots$

Question Number	Scheme		Marks
Aliter **2.** (a) **Way 2**	$(8-3x)^{\frac{1}{3}}$		
		2 or $(8)^{\frac{1}{3}}$ **(See note ↓)**	B1
	$= \left\{ (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3 + ... \right\}$	Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}} + (\frac{1}{3})(8)^{-\frac{2}{3}}(**x)$;	M1;
	with $** \neq 1$	A correct un-simplified or simplified $\{\underline{.........}\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= \left\{ (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(-3x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + ... \right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3$	
	$= \left\{ 2 + (\tfrac{1}{3})(\tfrac{1}{4})(-3x) + (-\tfrac{1}{9})(\tfrac{1}{32})(9x^2) + (\tfrac{5}{81})(\tfrac{1}{256})(-27x^3) + ... \right\}$		
	$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - ...$	Anything that cancels to $2 - \frac{1}{4}x$; or $2\{1 - \frac{1}{8}x ....\}$	A1;
		Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$	A1
			[5]

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

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Question Number	Scheme		Marks
3.	Volume $= \pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$	Use of $V = \pi \int y^2 \, \mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \, \mathrm{d}x$		
	$= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$		
	$= (\pi)\left[-\frac{1}{2}(2x+1)^{-1}\right]_a^b$	Integrating to give $\pm p(2x+1)^{-1}$	M1
		$-\frac{1}{2}(2x+1)^{-1}$	A1
	$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of b and a and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a - 1 + 2b + 1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\frac{\pi(b-a)}{(2a+1)(2b+1)}$	A1 aef
			[5]
			5 marks

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that π is not required for the middle three marks of this question.

Question Number	Scheme		Marks
***Aliter* 3. Way 2**	Volume $= \pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$	Use of $V = \pi \int y^2 \, \mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$		
	Applying substitution $u = 2x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a+1$ and $b \to 2b+1$, gives		
	$= (\pi) \int_{2a+1}^{2b+1} \frac{u^{-2}}{2} \mathrm{d}u$		
	$= (\pi) \left[\frac{u^{-1}}{(-1)(2)}\right]_{2a+1}^{2b+1}$		
	$= (\pi) \left[-\tfrac{1}{2}u^{-1}\right]_{2a+1}^{2b+1}$	Integrating to give $\pm p u^{-1}$	M1
		$-\tfrac{1}{2}u^{-1}$	A1
	$= (\pi) \left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\frac{\pi(b-a)}{(2a+1)(2b+1)}$	A1 aef
			[5]
			5 marks

Note that π is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

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Question Number	Scheme		Marks
4. (i)	$\int \ln\left(\frac{x}{2}\right) dx = \int 1.\ln\left(\frac{x}{2}\right) dx \Rightarrow \begin{cases} u = \ln\left(\frac{x}{2}\right) \Rightarrow \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{cases}$		
	$\int \ln\left(\frac{x}{2}\right) dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x} dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\, dx$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	<u>dM1</u>
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with $+ c$	A1 aef
			[4]
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, dx$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \tfrac{1}{2}(\pm 1 \pm \cos 2x)}\right]$	Consideration of double angle formula for $\cos 2x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \underline{(1-\cos 2x)}\, dx$		
	$= \frac{1}{2}\left[\underline{x - \tfrac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	<u>Integrating to give</u> <u>$\pm ax \pm b\sin 2x$</u>; $a, b \neq 0$	dM1
		Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$		
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\underline{\tfrac{1}{2}\left(\tfrac{\pi}{4} + \tfrac{1}{2}\right)}$ or $\underline{\tfrac{\pi}{8} + \tfrac{1}{4}}$ or $\underline{\tfrac{\pi}{8} + \tfrac{2}{8}}$	A1 **aef**, **cso**
		Candidate must collect their π term and constant term together for A1. No fluked answers, hence **cso**.	**[5]**
			9 marks

Note: $\int \ln\left(\frac{x}{2}\right) dx = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v).\left(\text{their } \frac{du}{dx}\right) dx$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

Question Number	Scheme		Marks
Aliter **4.** (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right)\,dx = \int \left(\ln x - \ln 2\right)\,dx = \int \ln x\,dx - \int \ln 2\,dx$		
	$\int \ln x\,dx = \int 1.\ln x\,dx \Rightarrow \left\{ \begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow v = x \end{array} \right\}$		
	$\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without + c	A1
	$\int \ln 2\,dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without + c	M1
	Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln x - x - x\ln 2 + c$	Correct integration with + c	A1 aef
			[4]

Note: $\int \ln x\,dx = (\text{their } v)\ln x - \int (\text{their } v).\left(\text{their } \frac{du}{dx}\right)\,dx$ for M1 in part (i).

Question Number	Scheme		Marks
Aliter **4.** (i) **Way 3**	$\int \ln\left(\frac{x}{2}\right)\,\mathrm{d}x$		
	$u = \frac{x}{2} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}$		
	$\int \ln\left(\frac{x}{2}\right)\,\mathrm{d}x = 2\int \ln u\,\mathrm{d}u$	Applying substitution correctly to give $\int \ln\left(\frac{x}{2}\right)\,\mathrm{d}x = 2\int \ln u\,\mathrm{d}u$ ***Decide to award 2nd M1 here!***	
	$\int \ln u\,\mathrm{d}x = \int 1.\ln u\,\mathrm{d}u$		
	$\int \ln u\,\mathrm{d}x = u\ln u - \int u.\frac{1}{u}\,\mathrm{d}u$	Use of 'integration by parts' formula in the correct direction.	M1
	$= u\ln u - u + c$	Correct integration of $\ln u$ with or without $+ c$	A1
		Decide to award 2nd M1 here!	M1
	$\int \ln\left(\frac{x}{2}\right)\mathrm{d}x = 2\left(u\ln u - u\right) + c$		
	Hence, $\int \ln\left(\frac{x}{2}\right)\,\mathrm{d}x = x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with $+ c$	A1 aef
			[4]

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Question Number	Scheme		Marks
Aliter **4.** (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx$ and $I = \int \sin^2 x \, dx$		
	$\begin{Bmatrix} u = \sin x & \Rightarrow & \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x & \Rightarrow & v = -\cos x \end{Bmatrix}$		
	$\therefore I = \underline{\left\{-\sin x \cos x + \int \cos^2 x \, dx\right\}}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{-\sin x \cos x + \int (1 - \sin^2 x) dx\right\}$		
	$\int \sin^2 x \, dx = \left\{-\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx\right\}$		
	$2\int \sin^2 x \, dx = \left\{-\sin x \cos x + \int 1 \, dx\right\}$	For the LHS becoming $2I$	dM1
	$2\int \sin^2 x \, dx = \{-\sin x \cos x + x\}$		
	$\int \sin^2 x \, dx = \left\{\underline{-\tfrac{1}{2}\sin x \cos x + \tfrac{x}{2}}\right\}$	<u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[\left(-\tfrac{1}{2}\sin(\tfrac{\pi}{2})\cos(\tfrac{\pi}{2}) + \tfrac{(\frac{\pi}{2})}{2}\right) - \left(-\tfrac{1}{2}\sin(\tfrac{\pi}{4})\cos(\tfrac{\pi}{4}) + \tfrac{(\frac{\pi}{4})}{2}\right)\right]$ $= \left[(0 + \tfrac{\pi}{4}) - (-\tfrac{1}{4} + \tfrac{\pi}{8})\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \tfrac{\pi}{8} + \tfrac{1}{4}$	$\underline{\tfrac{1}{2}\left(\tfrac{\pi}{4} + \tfrac{1}{2}\right)}$ or $\underline{\tfrac{\pi}{8} + \tfrac{1}{4}}$ or $\underline{\tfrac{\pi}{8} + \tfrac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef cso** **[5]**

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

Question Number	Scheme		Marks
5. (a)	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \times\right\}$ $3x^2 - 8y\frac{dy}{dx}; = \left(\underline{12y + 12x\frac{dy}{dx}}\right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = ...$ Correct LHS equation; <u>Correct application of product rule</u>	M1 A1; (B1)
	$\left\{\dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found.	dM1 A1 A1 **cso**
			[6]
			9 marks

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 2**	$\left\{\frac{\text{d}x}{\text{d}y} \times\right\}$ $3x^2\frac{\text{d}x}{\text{d}y} - 8y;\ = \left(12y\frac{\text{d}x}{\text{d}y} + 12x\right)$	Differentiates implicitly to include either $\pm kx^2\frac{\text{d}x}{\text{d}y}$ or $12y\frac{\text{d}x}{\text{d}y}$. Ignore $\frac{\text{d}x}{\text{d}y} = \ldots$	M1
		Correct LHS equation	A1;
		Correct application of product rule	(B1)
	$\left\{\frac{\text{d}y}{\text{d}x} = \frac{3x^2 - 12y}{12x + 8y}\right\}$	*not necessarily required.*	
	$@\ (-8, 8),\quad \frac{\text{d}y}{\text{d}x} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3},$	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{\text{d}y}{\text{d}x}$ or $\frac{\text{d}x}{\text{d}y}$.	dM1
	$@\ (-8, 16),\quad \frac{\text{d}y}{\text{d}x} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}.$	One gradient found.	A1
		Both gradients of -3 and 0 ***correctly*** found.	A1 **cso**
			[6]

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$4y^2 + 12xy - x^3 = 0$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$		
	$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$		
	$y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A1
	@ $x = -8$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$	One gradient correctly found.	A1
		Both gradients of <u>-3</u> and <u>0</u> correctly found.	A1
			[6]

Question Number	Scheme		Marks
6. (a)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}$	Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. Correct answer.	M1 ± A1
			[2]
(b)	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$	An expression of the form $(\text{vector}) \pm \lambda(\text{vector})$ $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{BA})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{BA})$ ($\mathbf{r}$ is needed.)	M1 A1 $\sqrt{}$ aef
			[2]
(c)	$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left(\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$	← Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.	M1 $\sqrt{}$
	$\cos\theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}$.	$\underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}$	A1 **cao**
			[3]

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

Question Number	Scheme		Marks
6. (d)	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 2+\lambda=\mu \quad (1)$ $\mathbf{j}: \quad 6-2\lambda=0 \quad (2)$ $\mathbf{k}: -1+2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=3$ Any two yields $\lambda=3, \quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter 6. (d) **Way 2**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \quad 3+\lambda=\mu \quad (1)$ $\mathbf{j}: \quad 4-2\lambda=0 \quad (2)$ $\mathbf{k}: \quad 1+2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=2$ Any two yields $\lambda=2, \quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}+2\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

Note: Be careful! λ and μ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

Question Number	Scheme		Marks
Aliter **6.** (d) **Way 3**	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}$: $2-\lambda=\mu$ (1) $\mathbf{j}$: $6+2\lambda=0$ (2) $\mathbf{k}$: $-1-2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=-3$ Any two yields $\lambda=-3,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	l_1 : $\mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}-3\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter **6.** (d) **Way 4**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}$: $3-\lambda=\mu$ (1) $\mathbf{j}$: $4+2\lambda=0$ (2) $\mathbf{k}$: $1-2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=-2$ Any two yields $\lambda=-2,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	l_1 : $\mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}-2\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

Question Number	Scheme		Marks
7. (a)	$\left[x = \ln(t+2),\ y = \frac{1}{t+1}\right], \quad \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$	Must state $\frac{dx}{dt} = \frac{1}{t+2}$	B1
	$\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1}\,dx ;= \int_0^2 \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right) dt$	Area $= \int \frac{1}{t+1}\,dx$. Ignore limits.	M1;
		$\int \left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right) dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x = \ln 2 \ \Rightarrow \ln 2 = \ln(t+2) \ \Rightarrow \ 2 = t+2 \ \Rightarrow t = 0$ $x = \ln 4 \ \Rightarrow \ln 4 = \ln(t+2) \ \Rightarrow \ 4 = t+2 \ \Rightarrow t = 2$	changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$	B1
	Hence, $\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)}\,dt$		**[4]**
(b)	$\left(\frac{1}{(t+1)(t+2)}\right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$	$\frac{A}{(t+1)} + \frac{B}{(t+2)}$ with A and B found	M1
	$1 = A(t+2) + B(t+1)$		
	Let $t = -1$, $1 = A(1) \ \Rightarrow \ \underline{A = 1}$ Let $t = -2$, $1 = B(-1) \ \Rightarrow \ \underline{B = -1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\int_0^2 \frac{1}{(t+1)(t+2)}\,dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)}\,dt$		
	$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$	dM1
		Both ln terms correctly ft.	A1 $\sqrt{}$
	$= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round.	ddM1
	$= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$	$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw **[6]**

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme		Marks
	$x=\ln(t+2), \quad y=\dfrac{1}{t+1}$		
7. (c)	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t = \ldots$ the subject giving $t = e^x - 2$	M1 A1
	$y=\dfrac{1}{e^x-2+1} \Rightarrow y=\dfrac{1}{e^x-1}$	Eliminates t by substituting in y giving $y=\dfrac{1}{e^x-1}$	dM1 A1
			[4]
Aliter **7.** (c) **Way 2**	$t+1=\dfrac{1}{y} \Rightarrow t=\dfrac{1}{y}-1$ or $t=\dfrac{1-y}{y}$ $y(t+1)=1 \Rightarrow yt+y=1 \Rightarrow yt=1-y \Rightarrow t=\dfrac{1-y}{y}$	Attempt to make $t = \ldots$ the subject Giving either $t=\dfrac{1}{y}-1$ or $t=\dfrac{1-y}{y}$	M1 A1
	$x=\ln\left(\dfrac{1}{y}-1+2\right)$ or $x=\ln\left(\dfrac{1-y}{y}+2\right)$	Eliminates t by substituting in x	dM1
	$x=\ln\left(\dfrac{1}{y}+1\right)$		
	$e^x=\dfrac{1}{y}+1$		
	$e^x-1=\dfrac{1}{y}$		
	$y=\dfrac{1}{e^x-1}$	giving $y=\dfrac{1}{e^x-1}$	A1
			[4]
(d)	Domain : $\underline{x>0}$	$\underline{x>0}$ or just >0	B1
			[1]
			15 marks

Question Number	Scheme		Marks
Aliter **7.** (c) **Way 3**	$e^x = t+2 \Rightarrow t+1 = e^x - 1$	Attempt to make $t+1 = \ldots$ the subject	M1
		giving $t+1 = e^x - 1$	A1
	$y = \dfrac{1}{t+1} \Rightarrow y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y	dM1
		giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]
Aliter **7.** (c) **Way 4**	$t+1 = \dfrac{1}{y} \Rightarrow t+2 = \dfrac{1}{y}+1$ or $t+2 = \dfrac{1+y}{y}$	Attempt to make $t+2 = \ldots$ the subject	M1
		Either $t+2 = \dfrac{1}{y}+1$ or $t+2 = \dfrac{1+y}{y}$	A1
	$x = \ln\left(\dfrac{1}{y}+1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y}+1\right)$		
	$e^x = \dfrac{1}{y}+1 \Rightarrow e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]

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Question Number	Scheme		Marks
8. (a)	$\frac{dV}{dt}=1600-c\sqrt{h}$ or $\frac{dV}{dt}=1600-k\sqrt{h}$,	Either of these statements	M1
	$(V=4000h \Rightarrow)\ \frac{dV}{dh}=4000$	$\frac{dV}{dh}=4000$ or $\frac{dh}{dV}=\frac{1}{4000}$	M1
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}=\frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt}=\frac{1600-c\sqrt{h}}{4000}=\frac{1600}{4000}-\frac{c\sqrt{h}}{4000}=0.4-k\sqrt{h}$ or $\frac{dh}{dt}=\frac{1600-k\sqrt{h}}{4000}=\frac{1600}{4000}-\frac{k\sqrt{h}}{4000}=0.4-k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h=25$ water ***leaks out such that*** $\frac{dV}{dt}=400$		
	$400=c\sqrt{h} \Rightarrow 400=c\sqrt{25} \Rightarrow 400=c(5) \Rightarrow c=80$		
	From above; $k=\frac{c}{4000}=\frac{80}{4000}=0.02$ as required	Proof that $k=0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400=4000k\sqrt{h}$		
	$\Rightarrow 400=4000k\sqrt{25}$		
	$\Rightarrow 400=k(20000) \Rightarrow k=\frac{400}{20000}=0.02$	Using 400, 4000 and $h=25$ or $\sqrt{h}=5$. Proof that $k=0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt}=0.4-k\sqrt{h} \Rightarrow \int\frac{dh}{0.4-k\sqrt{h}}=\int dt$	***Separates the variables*** with $\int\frac{dh}{0.4-k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $=\int_0^{100}\frac{1}{0.4-0.02\sqrt{h}}\,dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $=\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh$	Correct proof	A1 **AG**
			[2]

Question Number	Scheme		Marks
8. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\, dh$ with substitution $h=(20-x)^2$		
	$\frac{dh}{dx} = 2(20-x)(-1)$ or $\frac{dh}{dx} = -2(20-x)$	Correct $\frac{dh}{dx}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h} = 20-x \Rightarrow x = 20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\, dh = \int \frac{50}{x}.-2(20-x)\, dx$	$\pm\lambda\int \frac{20-x}{x}\, dx$ or $\pm\lambda\int \frac{20-x}{20-(20-x)}\, dx$ where λ is a constant	M1
	$= 100\int \frac{x-20}{x}\, dx$		
	$= 100\int \left(1-\frac{20}{x}\right) dx$		
	$= 100\left(x-20\ln x\right)\ (+c)$	$\pm\alpha x \pm \beta\ln x$; $\alpha,\beta \neq 0$ $100x - 2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100} \frac{50}{20-\sqrt{h}}\, dh = \left[100x - 2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100} \frac{50}{20-\sqrt{h}}\, dh = \left[100\left(20-\sqrt{h}\right) - 2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$= \left(1000-2000\ln 10\right) - \left(2000-2000\ln 20\right)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$= 2000\ln 20 - 2000\ln 10 - 1000$	Combining logs to give...	
	$= 2000\ln 2 - 1000$	$2000\ln 2 - 1000$ or $-2000\ln\left(\frac{1}{2}\right) - 1000$	A1 **aef**
			[6]
(e)	Time required $= 2000\ln 2 - 1000 = 386.2943611...$ sec		
	$= 386$ seconds (nearest second)		
	$= 6$ minutes and 26 seconds (nearest second)	6 minutes, 26 seconds	B1
			[1]
			13 marks