

Mark Scheme (Results)

June 2011

GCE Core Mathematics C4 (6666) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol \checkmark will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

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Mark Scheme

Question Number	Scheme	Marks
<p>1.</p>	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$	B1
	$x \rightarrow 1 \quad 9 = 3B \Rightarrow B = 3$	M1
	$x \rightarrow -\frac{1}{2} \quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	A1
	x^2 terms $9 = 2A + C \Rightarrow A = 4$	A1
	<p><i>Alternatives for finding A.</i></p> x terms $0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $0 = -A + B + C \Rightarrow A = 4$	Any two of A, B, C All three correct (4) [4]

Question Number	Scheme	Marks
2.	$f(x) = (\dots + \dots)^{-\frac{1}{2}}$ $= 9^{-\frac{1}{2}} (\dots + \dots)^{\dots}$ $(1+kx^2)^n = 1+nkx^2 + \dots$ $(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$ $\left(1 + \frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$ $f(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$	<p>M1</p> <p>B1 $3^{-1}, \frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$</p> <p>M1 n not a natural number, $k \neq 1$</p> <p>A1 ft ft their $k \neq 1$</p> <p>A1</p> <p>A1 (6)</p> <p>[6]</p>

Question Number	Scheme	Marks
3.	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent M1 A1
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$ M1 A1 (4)
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a) M1
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031 A1 (2) [6]

Question Number	Scheme	Marks
<p>4.</p>	<p>(a) 0.0333, 1.3596 awrt 0.0333, 1.3596</p>	<p>B1 B1 (2)</p>
	<p>(b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4} [\dots]$ $\approx \dots [0 + 2(0.0333 + 0.3240 + 1.3596) + 3.9210]$ ≈ 1.30 Accept</p> <p>1.3</p>	<p>B1 M1 A1 (3)</p>
	<p>(c) $u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x$ $\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2 + 2) dx$ $\int x^3 \ln(x^2 + 2) dx = \int x^2 \ln(x^2 + 2) x dx = \int (u - 2)(\ln u) \frac{1}{2} du$</p> <p>Hence $\text{Area}(R) = \frac{1}{2} \int_2^4 (u - 2) \ln u du$ *</p> <p>cs0</p>	<p>B1 B1 M1 A1 (4)</p>
	<p>(d) $\int (u - 2) \ln u du = \left(\frac{u^2}{2} - 2u \right) \ln u - \int \left(\frac{u^2}{2} - 2u \right) \frac{1}{u} du$ $= \left(\frac{u^2}{2} - 2u \right) \ln u - \int \left(\frac{u}{2} - 2 \right) du$ $= \left(\frac{u^2}{2} - 2u \right) \ln u - \left(\frac{u^2}{4} - 2u \right) (+C)$</p>	<p>M1 A1 M1 A1</p>
	<p>$\text{Area}(R) = \frac{1}{2} \left[\left(\frac{u^2}{2} - 2u \right) \ln u - \left(\frac{u^2}{4} - 2u \right) \right]_2^4$ $= \frac{1}{2} [(8 - 8) \ln 4 - 4 + 8 - ((2 - 4) \ln 2 - 1 + 4)]$ $= \frac{1}{2} (2 \ln 2 + 1)$ ln 2 + 1/2</p>	<p>M1 A1 (6) [15]</p>

Question Number	Scheme	Marks	
5.	$\frac{1}{y} \frac{dy}{dx} = \dots$ $\dots = 2 \ln x + 2x \left(\frac{1}{x} \right)$	B1 M1 A1	
	At $x = 2$, leading to	$\ln y = 2(2) \ln 2$ $y = 16$	M1 A1
	At (2, 16)	$\frac{1}{16} \frac{dy}{dx} = 2 \ln 2 + 2$	M1
		$\frac{dy}{dx} = 16(2 + 2 \ln 2)$	A1 (7)
	<i>Alternative</i>		[7]
		$y = e^{2x \ln x}$	B1
		$\frac{d}{dx}(2x \ln x) = 2 \ln x + 2x \left(\frac{1}{x} \right)$	M1 A1
		$\frac{dy}{dx} = \left(2 \ln x + 2x \left(\frac{1}{x} \right) \right) e^{2x \ln x}$	M1 A1
	At $x = 2$,	$\frac{dy}{dx} = (2 \ln 2 + 2) e^{4 \ln 2}$	M1
		$= 16(2 + 2 \ln 2)$	A1 (7)

Question Number	Scheme	Marks
6.	<p>(a) i: $6 - \lambda = -5 + 2\mu$ j: $-3 + 2\lambda = 15 - 3\mu$ leading to $\lambda = 3, \mu = 4$ $\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ k: LHS = $-2 + 3(3) = 7$, RHS = $3 + 4(1) = 7$ (As LHS = RHS, lines intersect)</p> <p>Alternatively for B1, showing that $\lambda = 3$ and $\mu = 4$ both give $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$</p> <p>(b) $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = -2 - 6 + 3 = \sqrt{14}\sqrt{14}\cos\theta$ ($\theta \approx 110.92^\circ$) Acute angle is 69.1°</p> <p>(c) $\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$ ($\Rightarrow B$ lies on l_1)</p> <p>(d) Let d be shortest distance from B to l_2</p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> <p>$\overline{AB} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$</p> <p>$\overline{AB} = \sqrt{(2)^2 + (-4)^2 + (-6)^2} = \sqrt{56}$</p> <p>$\frac{d}{\sqrt{56}} = \sin\theta$ $d = \sqrt{56}\sin 69.1^\circ \approx 6.99$</p> </div> <div style="flex: 1; text-align: center;"> </div> </div>	<p>Any two equations</p> <p>M1 M1 A1 M1 A1</p> <p>B1 (6)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p>M1 A1 M1 A1 (4) [14]</p>

Question Number	Scheme	Marks
7.	(a) $\tan \theta = \sqrt{3}$ or $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}$	M1 awrt 1.05 A1 (2)
	(b) $\frac{dx}{d\theta} = \sec^2 \theta, \frac{dy}{d\theta} = \cos \theta$ $\frac{dy}{dx} = \frac{\cos \theta}{\sec^2 \theta} (= \cos^3 \theta)$	M1 A1
	At P, $m = \cos^3 \left(\frac{\pi}{3} \right) = \frac{1}{8}$	Can be implied A1
	Using $mm' = -1, m' = -8$ For normal $y - \frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$	M1 M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$ leading to $x = \frac{17}{16}\sqrt{3} \quad (k = \frac{17}{16})$	1.0625 A1 (6)
	(c) $\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta = \int \sin^2 \theta \sec^2 \theta d\theta$ $= \int \tan^2 \theta d\theta$ $= \int (\sec^2 \theta - 1) d\theta$ $= \tan \theta - \theta (+C)$	M1 A1 A1 M1 A1
	$V = \pi \int_0^{\frac{\pi}{3}} y^2 dx = [\tan \theta - \theta]_0^{\frac{\pi}{3}} = \pi \left[\left(\sqrt{3} - \frac{\pi}{3} \right) - (0 - 0) \right]$ $= \sqrt{3}\pi - \frac{1}{3}\pi^2 \quad (p = 1, q = -\frac{1}{3})$	M1 A1 (7) [15]

Question Number	Scheme	Marks
8.	<p>(a) $\int (4y+3)^{-\frac{1}{2}} dx = \frac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} (+C)$ $(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C)$</p> <p>(b) $\int \frac{1}{\sqrt{4y+3}} dy = \int \frac{1}{x^2} dx$ $\int (4y+3)^{-\frac{1}{2}} dy = \int x^{-2} dx$ $\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} (+C)$</p> <p>Using $(-2, 1.5)$ $\frac{1}{2}(4 \times 1.5 + 3)^{\frac{1}{2}} = -\frac{1}{-2} + C$ leading to $C = 1$ $\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + 1$ $(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}$ $y = \frac{1}{4} \left(2 - \frac{2}{x} \right)^2 - \frac{3}{4}$</p>	<p>M1 A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p> <p>[8]</p>

or equivalent

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