Mark Scheme (Results)

June 2013

GCE Core Mathematics 4 (6666/01)

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June 2013
Publications Code UA035676

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1. (a)	$\int x^2 e^x \, dx$, 1st Application: $\left\{ u = x^2 \Rightarrow \frac{du}{dx} = 2x, \; \frac{dv}{dx} = e^x \Rightarrow v = e^x \right\}$, 2nd Application: $\left\{ u = x \Rightarrow \frac{du}{dx} = 1, \; \frac{dv}{dx} = e^x \Rightarrow v = e^x \right\}$		
	$= x^2 e^x - \int 2xe^x dx$	$x^2 e^x - \int \lambda xe^x \{dx\}$, $\lambda > 0$	M1
		$x^2 e^x - \int 2xe^x \{dx\}$	A1 oe
	$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$	**Either** $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$	M1
	$= x^2 e^x - 2(xe^x - e^x)$ $\{+c\}$	$\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$	M1
		Correct answer, with/without $+c$	A1
			[5]
(b)	$\left\{\left[x^2 e^x - 2(xe^x - e^x)\right]_0^1\right\}$ $= \left(1^2 e^1 - 2(1e^1 - e^1)\right) - \left(0^2 e^0 - 2(0e^0 - e^0)\right)$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
	$= e - 2$	$e - 2$ **cso**	A1 oe
			[2]
			7

Notes for Question 1

(a)

M1: Integration by parts is applied in the form $x^2 e^x - \int \lambda xe^x \{dx\}$, where $\lambda > 0$. (must be in this form).

A1: $x^2 e^x - \int 2xe^x \{dx\}$ or equivalent.

M1: **Either** achieving a result in the form $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ (can be implied)

(where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$

M1: $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$)

A1: $x^2 e^x - 2(xe^x - e^x)$ or $x^2 e^x - 2xe^x + 2e^x$ or $(x^2 - 2x + 2)e^x$ or equivalent with/without $+c$.

(b)

M1: **Complete method** of applying limits of 1 and 0 to their part (a) answer in the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1.
So, just subtracting zero is M0.

A1: $e - 2$ or $e^1 - 2$ or $-2 + e$. Do not allow $e - 2e^0$ unless simplified to give $e - 2$.
Note: that 0.718... without seeing $e - 2$ or equivalent is A0.
WARNING: Please note that this A1 mark is for correct solution only.
So incorrect $[.......]_0^1$ leading to $e - 2$ is A0.

Note: If their part (a) is correct candidates can get M1A1 in part (b) for $e - 2$ from no working.

Note: 0.718... from no working is M0A0

Question Number	Scheme		Marks
2. (a)	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	$(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$= \left(1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+...\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+...\right)$	See notes	M1 A1 A1
	$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$		
	$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^2+...$	See notes	M1
	$= 1+x+\frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]
(b)	$\sqrt{\left(\frac{1+(\frac{1}{26})}{1-(\frac{1}{26})}\right)} = 1+\left(\frac{1}{26}\right)+\frac{1}{2}\left(\frac{1}{26}\right)^2$		M1
	ie: $\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$		B1
	so, $\sqrt{3} = \frac{7025}{4056}$	$\frac{7025}{4056}$	A1 **cao**
			[3] **9**

Notes for Question 2

(a)	**B1**: $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ or $\sqrt{(1+x)}(1-x)^{-\frac{1}{2}}$ seen or implied. (Also allow $\left((1+x)(1-x)^{-1}\right)^{\frac{1}{2}}$). **M1:** Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+ .... +\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ **or** expands $(1-x)^{-\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\left(-\frac{1}{2}\right)(-x)$ or $+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$ or $1+ .... +\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$ Also allow: $1+ .... +\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x)^2$ for M1. **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **Note:** Candidates can give decimal equivalents when expanding out their binomial expansions. **M1:** Multiplies out to give 1, exactly two terms in x and exactly three terms in x^2. **A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound. **Special Case**: Award SC FINAL M1A1 for ***a correct*** $\left(1+\frac{1}{2}x-\frac{1}{8}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$ multiplied out with no errors to give either $1+x+\frac{3}{8}x^2+\frac{1}{4}x^2-\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{2}x+\frac{1}{4}x^2$ or $1+\frac{1}{2}x+\frac{5}{8}x^2+\frac{1}{2}x-\frac{1}{8}x^2$ leading to the correct answer of $1+x+\frac{1}{2}x^2$.

Notes for Question 2 Continued	
2. (a) ctd	**Note:** If a candidate writes down either $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + ...$ **or** $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + ...$ with no working then you can award 1st M1, 1st A1. **Note:** If a candidate writes down both correct binomial expansions with no working, then you can award 1st M1, 1st A1, 2nd A1.
(b)	**M1:** Substitutes $x = \frac{1}{26}$ into **both** sides of $\sqrt{\left(\frac{1+x}{1-x}\right)}$ and $1 + x + \frac{1}{2}x^2$ **B1:** For sight of $\sqrt{\frac{27}{25}}$ (or better) and $\frac{1405}{1352}$ or equivalent fraction Eg: $\frac{3\sqrt{3}}{5}$ and $\frac{1405}{1352}$ or $0.6\sqrt{3}$ and $\frac{1405}{1352}$ or $\frac{3\sqrt{3}}{5}$ and $1\frac{53}{1352}$ or $\sqrt{3}$ and $\frac{5}{3}\left(\frac{1405}{1352}\right)$ are fine for B1. **A1:** $\frac{7025}{4056}$ **or any equivalent fraction**, eg: $\frac{14050}{8112}$ or $\frac{182650}{105456}$ etc. **Special Case:** Award SC: M1B1A0 for $\sqrt{3} \approx 1.732001972..$ or truncated 1.732001 or awrt 1.732002. **Note** that $\frac{7025}{4056} = 1.732001972...$ and $\sqrt{3} = 1.732050808...$

Aliter 2. (a) Way 2	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1-x)}{(1+x)(1-x)}} = \sqrt{\frac{(1-x^2)}{(1-x)^2}} = \right\} = (1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	$(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	B1
	$= \left(1 + \left(\frac{1}{2}\right)(-x^2) + ...\right) \times \left(1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + ...\right)$	See notes	M1A1A1
	$= \left(1 - \frac{1}{2}x^2 + ...\right) \times \left(1 + x + x^2 + ...\right)$		
	$= 1 + x + x^2 - \frac{1}{2}x^2$	See notes	M1
	$= 1 + x + \frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]

Aliter 2. (a) Way 2	**B1**: $(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ seen or implied. **M1:** Expands $(1-x^2)^{\frac{1}{2}}$ to give both terms simplified or un-simplified, $1 + \left(\frac{1}{2}\right)(-x^2)$ **or** expands $(1-x)^{-1}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1 + (-1)(-x)$ or $... + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2$ or $1 + ... + \frac{(-1)(-2)}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1:** Multiplies out to give 1, exactly one term in x and exactly two terms in x^2. **A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound.

Notes for Question 2 Continued		
Aliter 2. (a) Way 3	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} = \right\} = (1+x)(1-x^2)^{-\frac{1}{2}}$ $\quad (1+x)(1-x^2)^{-\frac{1}{2}}$	B1
	$= (1+x)\left(1+\frac{1}{2}x^2+...\right)$ Must follow on from above.	M1A1A1
	$= 1+x+\frac{1}{2}x^2$	dM1A1
	Note: The final M1 mark is dependent on the previous method mark for Way 3.	
Aliter 2. (a) Way 4	**Assuming the result on the Question Paper. (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).**	
	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \frac{\sqrt{(1+x)}}{\sqrt{(1-x)}} = 1+x+\frac{1}{2}x^2\right\} \Rightarrow (1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+...\ \left\{= 1+\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$, $(1-x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2+...\ \left\{= 1-\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$	M1A1A1
	$\text{RHS} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+...\right)$	
	$= 1-\frac{1}{2}x-\frac{1}{8}x^2+x-\frac{1}{2}x^2+\frac{1}{2}x^2$ See notes	M1
	$= 1+\frac{1}{2}x-\frac{1}{8}x^2$	
	So, $\text{LHS} = 1+\frac{1}{2}x-\frac{1}{8}x^2 = \text{RHS}$	A1 *
		[6]
	B1: $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$ seen or implied. **M1: For Way 4, this M1 mark is dependent on the first B1 mark.** Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+....+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ **or** expands $(1-x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(-x)$ or $+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ or $1+....+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1: For Way 4, this M1 mark is dependent on the first B1 mark.** Multiplies out RHS to give 1, exactly two terms in x and exactly three terms in x^2. **A1:** Candidate achieves the result on the exam paper. Candidate needs to have correctly processed both the LHS and RHS of $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$.	

Question Number	Scheme		Marks
3. (a)	1.154701		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{6}\,;\times\left[1+2\left(1.035276 + \text{their } 1.154701\right)+1.414214\right]$		B1; <u>M1</u>
	$=\frac{\pi}{12}\times 6.794168 \;=\; 1.778709023... = 1.7787$ (4 dp)	1.7787 or awrt 1.7787	A1 **[3]**
(c)	$V=\pi\int_0^{\frac{\pi}{2}}\left(\sec\left(\frac{x}{2}\right)\right)^2 \mathrm{d}x$	For $\pi\int\left(\sec\left(\frac{x}{2}\right)\right)^2$. Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$=\{\pi\}\left[2\tan\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$	$\pm\lambda\tan\left(\frac{x}{2}\right)$	M1
		$2\tan\left(\frac{x}{2}\right)$ or equivalent	A1
	$=2\pi$	2π	A1 **cao cso** **[4]** **8**

Notes for Question 3

(a)	**B1:** 1.154701 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times\frac{\pi}{6}$ or $\frac{\pi}{12}$ or awrt 0.262
	M1: <u>For structure of trapezium rule</u>[<u>............</u>]
	A1: anything that rounds to 1.7787
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 1.7787)
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 1.762747174…
	<u>Note:</u> Award B1M1A1 for $\frac{\pi}{12}(1+1.414214)+\frac{\pi}{6}\left(1.035276 + \text{their } 1.154701\right) = 1.778709023...$
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,**
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}+1+2\left(1.035276 + \text{their } 1.154701\right)+1.414214$ (nb: answer of 7.05596...).
	Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{6}\;(1+1.414214)+2\left(1.035276 + \text{their } 1.154701\right)$ (nb: answer of 5.01199...).
	<u>*Alternative method for part (b): Adding individual trapezia*</u>
	Area $\approx \frac{\pi}{6}\times\left[\frac{1+1.035276}{2}+\frac{1.035276+1.154701}{2}+\frac{1.154701+1.414214}{2}\right] = 1.778709023...$
	B1: $\frac{\pi}{6}$ and a divisor of 2 on all terms inside brackets.
	M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1: anything that rounds to 1.7787

Notes for Question 3 Continued	
3. (c)	**B1:** For a correct statement of $\pi\int\left(\sec\left(\frac{x}{2}\right)\right)^2$ or $\pi\int\sec^2\left(\frac{x}{2}\right)$ or $\pi\int\frac{1}{\left(\cos\left(\frac{x}{2}\right)\right)^2}\{dx\}$. Ignore limits and $\mathrm{d}x$. Can be implied. Note: Unless a correct expression stated $\pi\int\sec\left(\frac{x^2}{4}\right)$ would be B0. **M1:** $\pm\lambda\tan\left(\frac{x}{2}\right)$ from any working. **A1:** $2\tan\left(\frac{x}{2}\right)$ or $\frac{1}{\left(\frac{1}{2}\right)}\tan\left(\frac{x}{2}\right)$ from any working. **A1:** 2π **from a correct solution only.** **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** Decimal answer of 6.283... without correct exact answer is A0. **Note:** The B1 mark can be implied by later working – as long as it is clear that the candidate has applied $\pi\int y^2$ in their working. **Note:** Writing the correct formula of $V = \pi\int y^2\{\mathrm{d}x\}$, but incorrectly applying it is B0.

Question Number	Scheme	Marks
4.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$	
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$ — At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
	Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$ — Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$ — and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	
	Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
		[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$	M1
	$= 2\sin^2 t$	
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ — $y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$	B1
		[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$ — See notes	B1 B1
		[2]
		9

Notes for Question 4

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 ***by correct solution only.***

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$ which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Notes for Question 4 Continued	
4. (b)	**M1:** Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. **A1:** Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form y = f(x)**. For example: $y = \frac{2x^2}{4}$ or $y = 2\left(\frac{x}{2}\right)^2$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ or $y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. **IMPORTANT: Please check working as this result can be fluked from an incorrect method.** **Award A0** if there is a $+c$ added to their answer. **B1:** Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.
(c)	**Note: The values of 0 and/or 2 need to be evaluated in this part** **B1:** Achieves an inclusive upper **or** lower limit, using acceptable notation. Eg: $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ **B1:** $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$ **Special Case: SC: B1B0** for either $0 < \mathrm{f}(x) < 2$ or $0 < \mathrm{f} < 2$ or $0 < y < 2$ or $(0, 2)$ **Special Case: SC: B1B0** for $0 \leqslant x \leqslant 2$. **IMPORTANT: Note that:** Therefore candidates can use either y or f in place of $\mathrm{f}(x)$ **Examples:** $0 \leqslant x \leqslant 2$ is SC: B1B0 — $0 < x < 2$ is B0B0 $x \geqslant 0$ is B0B0 — $x \leqslant 2$ is B0B0 $\mathrm{f}(x) > 0$ is B0B0 — $\mathrm{f}(x) < 2$ is B0B0 $x > 0$ is B0B0 — $x < 2$ is B0B0 $0 \geqslant \mathrm{f}(x) \geqslant 2$ is B0B0 — $0 < \mathrm{f}(x) \leqslant 2$ is B1B0 $0 \leqslant \mathrm{f}(x) < 2$ is B1B0. — $\mathrm{f}(x) \geqslant 0$ is B1B0 $\mathrm{f}(x) \leqslant 2$ is B1B0 — $\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$ $2 \leqslant \mathrm{f}(x) \leqslant 2$ is B0B0 — $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ is B1B0. $\lvert \mathrm{f}(x) \rvert \leqslant 2$ is B1B0 — $\lvert \mathrm{f}(x) \rvert \geqslant 2$ is B0B0 $1 \leqslant \mathrm{f}(x) \leqslant 2$ is B1B0 — $1 < \mathrm{f}(x) < 2$ is B0B0 $0 \leqslant \mathrm{f}(x) \leqslant 4$ is B1B0 — $0 < \mathrm{f}(x) < 4$ is B0B0 $0 \leqslant$ Range $\leqslant 2$ is B1B0 — Range is in between 0 and 2 is B1B0 $0 <$ Range < 2 is B0B0. — Range $\geqslant 0$ is B1B0 Range $\leqslant 2$ is B1B0 — Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0. $[0, 2]$ is B1B1 — $(0, 2)$ is SC B1B0

Aliter 4. (a) Way 2	$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = 2\sin 2t$, At $t = \frac{\pi}{6}$, $\frac{dx}{dt} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{dy}{dt} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{dy}{dx} = 1$	So B1, B1. So implied M1, A1.

Notes for Question 4 Continued		
Aliter 4. (a) Way 3	$y=\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=x$ Correct differentiation of their Cartesian equation. Finds $\frac{dy}{dx}=x$, using the correct Cartesian equation only.	B1ft B1
	At $t=\frac{\pi}{6}$, $\frac{dy}{dx}=2\sin\left(\frac{\pi}{6}\right)$ Finds the value of "x" when $t=\frac{\pi}{6}$ and substitutes this into their $\frac{dy}{dx}$	M1
	$= 1$ Correct value for $\frac{dy}{dx}$ of 1	A1
Aliter 4. (b) Way 2	$y=1-\cos 2t = 1-(2\cos^2 t-1)$	M1
	$y=2-2\cos^2 t \Rightarrow \cos^2 t=\frac{2-y}{2} \Rightarrow 1-\sin^2 t=\frac{2-y}{2}$ $1-\left(\frac{x}{2}\right)^2=\frac{2-y}{2}$	(Must be in the form $y=f(x)$).
	$y=2-2\left(1-\left(\frac{x}{2}\right)^2\right)$	A1
Aliter 4. (b) Way 3	$x=2\sin t \Rightarrow t=\sin^{-1}\left(\frac{x}{2}\right)$ So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ Rearranges to make t the subject and substitutes the result into y.	M1
	$y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 **oe**
Aliter 4. (b) Way 4	$y=1-\cos 2t \Rightarrow \cos 2t=1-y \Rightarrow t=\frac{1}{2}\cos^{-1}(1-y)$ So, $x=\pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$ Rearranges to make t the subject and substitutes the result into y.	M1
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 **oe**
Aliter 4. (b) Way 5	$\frac{dy}{dx}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2+c$ $\frac{dy}{dx}=x \Rightarrow y=\frac{1}{2}x^2+c$	M1
	Eg: when eg: $t=0$ (nb: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$), $x=0,\ y=1-1=0 \Rightarrow c=0 \Rightarrow y=\frac{1}{2}x^2$ Full method of finding $y=\frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$	A1
	Note: $\frac{dy}{dx}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2$, with no attempt to find c is M1A0.	

Question Number	Scheme	Marks
5. (a)	$\left\{x = u^2 \Rightarrow\right\} \dfrac{dx}{du} = 2u$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$	B1
	$\left\{\int \dfrac{1}{x(2\sqrt{x}-1)}\,dx\right\} = \int \dfrac{1}{u^2(2u-1)}\,2u\,du$	M1
	$= \int \dfrac{2}{u(2u-1)}\,du$	A1 * **cso**
		[3]
(b)	$\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$	
	$u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$ $u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$ — See notes	M1 A1
	So $\int \dfrac{2}{u(2u-1)}\,du = \int \dfrac{-2}{u} + \dfrac{4}{(2u-1)}\,du$ — Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to obtain any one of $\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$	M1
	$= -2\ln u + 2\ln(2u-1)$ — At least one term correctly followed through	A1 ft
	$-2\ln u + 2\ln(2u-1)$.	A1 **cao**
	So, $\left[-2\ln u + 2\ln(2u-1)\right]_1^3$	
	$= \left(-2\ln 3 + 2\ln(2(3)-1)\right) - \left(-2\ln 1 + 2\ln(2(1)-1)\right)$ — Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$	
	$= 2\ln\left(\dfrac{5}{3}\right)$ — $2\ln\left(\dfrac{5}{3}\right)$	A1 **cso cao**
		[7] **10**

Notes for Question 5

(a)

B1: $\dfrac{dx}{du} = 2u$ or $dx = 2u\,du$ or $\dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$ or $du = \dfrac{dx}{2\sqrt{x}}$

M1: A full substitution producing an integral in u only (including the du) (Integral sign not necessary). The candidate needs to deal with the "x", the "$(2\sqrt{x}-1)$" and the "dx" and converts from an integral term in x to an integral in u. (Remember the integral sign is not necessary for M1).

A1*: leading to the result printed on the question paper (including the du). (Integral sign is needed).

(b)

M1: Writing $\dfrac{2}{u(2u-1)} \equiv \dfrac{A}{u} + \dfrac{B}{(2u-1)}$ or writing $\dfrac{1}{u(2u-1)} \equiv \dfrac{P}{u} + \dfrac{Q}{(2u-1)}$ and a complete method for finding the value of at least one of their A or their B (or their P or their Q).

A1: Both their $A = -2$ **and** their $B = 4$. (Or their $P = -1$ **and** their $Q = 2$ with the multiplying factor of 2 in front of the integral sign).

M1: Integrates $\dfrac{M}{u} + \dfrac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. ***a two term partial fraction***) to obtain any one of

$\pm\lambda\ln u$ or $\pm\mu\ln(2u-1)$ or $\pm\mu\ln\left(u - \frac{1}{2}\right)$

A1ft: At least one term correctly followed through from their A or from their B (or their P and their Q).

A1: $-2\ln u + 2\ln(2u-1)$

Notes for Question 5 Continued

5. (b) ctd **M1:** Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subtracts the

	correct way round. **Note:** If a candidate just writes $\left(-2\ln 3 + 2\ln(2(3)-1)\right)$ oe , this is ok for M1. **A1:** $2\ln\left(\frac{5}{3}\right)$ **correct answer only.** (Note: $a=5,\ b=3$). **Important note: Award M0A0M1A1A0** for a candidate who writes $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = \int \frac{2}{u} + \frac{2}{(2u-1)}\,\mathrm{d}u \ = \ 2\ln u + \ln(2u-1)$ **AS EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ **AS PARTIAL FRACTIONS IS GIVEN.** **Important note: Award M0A0M0A0A0** for a candidate who writes down either $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + 2\ln(2u-1)$ **or** $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = 2\ln u + \ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Important note: Award M1A1M1A1A1** for a candidate who writes down $\int \frac{2}{u(2u-1)}\,\mathrm{d}u = -2\ln u + 2\ln(2u-1)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{2}{u(2u-1)}$ as partial fractions. **Note:** In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)}\,\mathrm{d}u$ we can allow a maximum of M1A0 M1A1ftA0 M1A0.

Question Number	Scheme		Marks
6.	$\dfrac{d\theta}{dt} = \lambda(120-\theta), \quad \theta \leqslant 100$		
(a)	$\displaystyle\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$ or $\displaystyle\int \frac{1}{\lambda(120-\theta)}\,d\theta = \int dt$		B1
	$-\ln(120-\theta); = \lambda t + c$ or $-\dfrac{1}{\lambda}\ln(120-\theta); = t + c$	See notes	M1 A1; M1 A1
	$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$	See notes	M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either... $-\lambda t = \ln(120-\theta) - \ln 100$; $-\lambda t = \ln\left(\dfrac{120-\theta}{100}\right)$; $e^{-\lambda t} = \dfrac{120-\theta}{100}$ ***or...*** $\lambda t = \ln 100 - \ln(120-\theta)$; $\lambda t = \ln\left(\dfrac{100}{120-\theta}\right)$; $e^{\lambda t} = \dfrac{100}{120-\theta}$		dddM1
	then either... $100e^{-\lambda t} = 120-\theta$ ***or...*** $(120-\theta)e^{\lambda t} = 100 \Rightarrow 120-\theta = 100e^{-\lambda t}$; leading to $\theta = 120 - 100e^{-\lambda t}$		A1 *
			[8]
(b)	$\{\lambda = 0.01,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100e^{-0.01t}$		M1
	$\Rightarrow 100e^{-0.01t} = 120-100 \Rightarrow -0.01t = \ln\left(\dfrac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = ...$ and $t = A\ln B$, where $B > 0$	
	$t = \dfrac{1}{-0.01}\ln\left(\dfrac{120-100}{100}\right)$		dM1
	$\left\{t = \dfrac{1}{-0.01}\ln\left(\dfrac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379... =$ 161 (s) (nearest second)	awrt 161	A1
			[3]
			11

Notes for Question 6		
(a)	**B1:** Separates variables as shown. $\mathrm{d}\theta$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. ***Either*** / ***or*** **M1:** $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$ or $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to \pm A\ln(120-\theta)$, A is a constant. **A1:** $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln(120-\theta)$ or $\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120-\theta)$ or $-\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$, **M1:** $\int \lambda\,\mathrm{d}t \to \lambda t$ or $\int 1\,\mathrm{d}t \to t$ **A1:** $\int \lambda\,\mathrm{d}t \to \lambda t + c$ or $\int 1\,\mathrm{d}t \to t + c$ The $+c$ can appear on either side of the equation. **IMPORTANT:** $+c$ can be on either side of their equation for the 2nd A1 mark. **M1:** Substitutes $t=0$ AND $\theta=20$ in an integrated or changed equation containing c (or A or $\ln A$). **Note** that this mark can be implied by the correct value of c. { Note that $-\ln 100 = -4.60517...$ }. **dddM1:** Uses their value of c which must be a ln term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on all three previous method marks being awarded. **A1*:** This is a given answer. All previous marks must have been scored and there must not be any errors in the candidate's working. Do not accept huge leaps in working at the end. So a minimum of either: (1): $\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100} \Rightarrow 100\mathrm{e}^{-\lambda t} = 120-\theta \Rightarrow \theta = 120-100\mathrm{e}^{-\lambda t}$ or (2): $\mathrm{e}^{\lambda t} = \frac{100}{120-\theta} \Rightarrow (120-\theta)\mathrm{e}^{\lambda t} = 100 \Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t} \Rightarrow \theta = 120-100\mathrm{e}^{-\lambda t}$ is required for A1. **Note:** $\int \frac{1}{(120\lambda-\lambda\theta)}\,\mathrm{d}\theta \to -\frac{1}{\lambda}\ln(120\lambda-\lambda\theta)$ is ok for the first M1A1 in part (a).	
(b)	**M1:** Substitutes $\lambda = 0.01$ and $\theta = 100$ into the printed equation or one of their earlier equations connecting θ and t. This mark can be implied by subsequent working. **dM1:** Candidate uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to $t = ...$ **Note:** that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (b). **A1:** awrt 161 or "awrt" 2 minutes 41 seconds. (Ignore incorrect units).	
***Aliter* 6. (a) Way 2**	$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$	B1
	$-\ln(120-\theta) = \lambda t + c$ See notes	M1 A1; M1 A1
	$-\ln(120-\theta) = \lambda t + c$ $\ln(120-\theta) = -\lambda t + c$ $120-\theta = A\mathrm{e}^{-\lambda t}$ $\theta = 120 - A\mathrm{e}^{-\lambda t}$	
	$\{t=0,\ \theta=20 \Rightarrow\}\ 20 = 120 - A\mathrm{e}^{0}$ $A = 120-20 = 100$	M1
	So, $\theta = 120 - 100\mathrm{e}^{-\lambda t}$	dddM1 A1 *
		[8]

<table>
<tr><th colspan="3">Notes for Question 6 Continued</th></tr>
<tr><td>(a)</td><td colspan="2">B1M1A1M1A1: Mark as in the original scheme.
M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Note that this mark can be implied by the correct value of c or A.
dddM1: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.
Note: This mark is dependent on all three previous method marks being awarded.
Note: $\ln(120-\theta) = -\lambda t + c$ leading to $120-\theta = \mathrm{e}^{-\lambda t} + \mathrm{e}^{c}$ or $120-\theta = \mathrm{e}^{-\lambda t} + A$, would be dddM0.
A1*: Same as the original scheme.
Note: The jump from $\ln(120-\theta) = -\lambda t + c$ to $120-\theta = A\mathrm{e}^{-\lambda t}$ with no incorrect working is condoned in part (a).</td></tr>
<tr><td>Aliter
6. (a)
Way 3</td><td>$\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t \quad \left\{\Rightarrow \int \frac{-1}{\theta-120}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t\right\}$

$-\ln\lvert\theta-120\rvert = \lambda t + c$ Modulus required for 1st A1.

$\{t=0,\ \theta=20 \Rightarrow\}\ -\ln\lvert 20-120\rvert = \lambda(0) + c$ Modulus not required here!

$\Rightarrow c = -\ln 100 \Rightarrow -\ln\lvert\theta-120\rvert = \lambda t - \ln 100$

<table><tr><td>then either...</td><td>or...</td></tr><tr><td>$-\lambda t = \ln\lvert\theta-120\rvert - \ln 100$
$-\lambda t = \ln\left\lvert\frac{\theta-120}{100}\right\rvert$</td><td>$\lambda t = \ln 100 - \ln\lvert\theta-120\rvert$
$\lambda t = \ln\left\lvert\frac{100}{\theta-120}\right\rvert$</td></tr><tr><td colspan="2">As $\theta \leqslant 100$</td></tr><tr><td>$-\lambda t = \ln\left(\frac{120-\theta}{100}\right)$
$\mathrm{e}^{-\lambda t} = \frac{120-\theta}{100}$</td><td>$\lambda t = \ln\left(\frac{100}{120-\theta}\right)$
$\mathrm{e}^{\lambda t} = \frac{100}{120-\theta}$</td></tr><tr><td>$100\mathrm{e}^{-\lambda t} = 120-\theta$</td><td>$(120-\theta)\mathrm{e}^{\lambda t} = 100$
$\Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t}$</td></tr><tr><td colspan="2">leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$</td></tr></table>
Understanding of modulus is required here!</td><td>B1

M1 A1
M1 A1
M1

dddM1

A1 *

[8]</td></tr>
<tr><td></td><td colspan="2">B1: Mark as in the original scheme.
M1: Mark as in the original scheme ignoring the modulus.
A1: $\int \frac{1}{120-\theta}\,\mathrm{d}\theta \to -\ln\lvert\theta-120\rvert$. (The modulus is required here).
M1A1: Mark as in the original scheme.
M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation containing their constant of integration which could be c or A. Mark as in the original scheme ignoring the modulus.
dddM1: Mark as in the original scheme AND the candidate must demonstrate that they have converted $\ln\lvert\theta-120\rvert$ to $\ln(120-\theta)$ in their working. Note: This mark is dependent on all three previous method marks being awarded.
A1: Mark as in the original scheme.</td></tr>
</table>

Notes for Question 6 Continued		
Aliter 6. (a) Way 4	***Use of an integrating factor***	
	$\dfrac{d\theta}{dt} = \lambda(120-\theta) \Rightarrow \dfrac{d\theta}{dt} + \lambda\theta = 120\lambda$	
	$\text{IF} = e^{\lambda t}$	B1
	$\dfrac{d}{dt}\left(e^{\lambda t}\theta\right) = 120\lambda e^{\lambda t}$,	M1A1
	$e^{\lambda t}\theta = 120\lambda e^{\lambda t} + k$	M1A1
	$\theta = 120 + Ke^{-\lambda t}$	M1
	$\{t = 0, \theta = 20 \Rightarrow\}\ -100 = K$	
	$\theta = 120 - 100e^{-\lambda t}$	M1A1

Question Number	Scheme	Marks
7.	$x^2 + 4xy + y^2 + 27 = 0$	
(a)	$\left\{\frac{dy}{dx} = \right\}$ $\underline{2x} + \left(4y + 4x\frac{dy}{dx}\right) + \underline{2y\frac{dy}{dx} = 0}$	M1 A1 B1
	$2x + 4y + (4x + 2y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$ $\left\{= \frac{-x - 2y}{2x + y}\right\}$	A1 **cso oe**
		[5]
(b)	$4x + 2y = 0$	M1
	$y = -2x$ | $x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$ | $\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$ | $-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$ | $y^2 = 36$	dM1*
	$x = -3$ | $y = 6$	A1
	When $x = -3$, $y = -2(-3)$ | When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$ | $x = -3$	A1 **cso**
		[7]
		12

Notes for Question 7

(a)

M1: Differentiates implicitly to include either $4x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).

A1: $(x^2) \to (\underline{2x})$ and $\left(... + y^2 + 27 = 0 \to \underline{+ 2y\frac{dy}{dx} = 0}\right)$.

Note: If an extra term appears then award A0.
Note: The "= 0" can be implied by rearrangement of their equation.

i.e.: $2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx}$ leading to $4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$ will get A1 (implied).

B1: $4y + 4x\frac{dy}{dx}$ or $4\left(y + x\frac{dy}{dx}\right)$ or equivalent

dM1: An attempt to factorise out $\frac{dy}{dx}$ as long as there are at least two terms in $\frac{dy}{dx}$.

ie. $... + (4x + 2y)\frac{dy}{dx} = ...$ or $... + 2(2x + y)\frac{dy}{dx} = ...$

Note: This mark is dependent on the previous method mark being awarded.

A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x - 2y}{2x + y}$

cso: If the candidate's solution is not completely correct, then do not give this mark.

Notes for Question 7 Continued	
(b)	**M1:** Sets the denominator of their $\frac{dy}{dx}$ equal to zero (or the numerator of their $\frac{dx}{dy}$ equal to zero) oe. **A1:** Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only). The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$. **M1*:** Substitutes $y = \pm\lambda x$ or or $x = \pm\mu y$ or $y = \pm\lambda x \pm a$ or $x = \pm\mu y \pm b$ $(\lambda \neq 0, \mu \neq 0)$ into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable. **dM1*:** leading to at least either $x^2 = A,\ A > 0$ or $y^2 = B,\ B > 0$ **Note:** This mark is dependent on the previous method mark (M1*) being awarded. **A1:** For $x = -3$ (ignore $x = 3$) or if y was found first, $y = 6$ (ignore $y = -6$) (correct solution only). **ddM1*** Substitutes their value of x into $y = \pm\lambda x$ to give $y =$ value or substitutes their value of x into $x^2 + 4xy + y^2 + 27 = 0$ to give $y =$ value. **Alternatively, s**ubstitutes their value of y into $x = \pm\mu y$ to give $x =$ value or substitutes their value of y into $x^2 + 4xy + y^2 + 27 = 0$ to give $x =$ value **Note:** This mark is dependent on the two previous method marks (M1* and dM1*) being awarded. **A1:** $(-3, 6)$ **cso**. **Note:** If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.** **Note:** $x = -3$ followed later in working by $y = 6$ is fine for A1. **Note:** $y = 6$ followed later in working by $x = -3$ is fine for A1. **Note:** $x = -3, 3$ followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$ **Note:** Candidates who set the numerator of $\frac{dy}{dx}$ equal to 0 (or the denominator of their $\frac{dx}{dy}$ equal to zero) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ **{** or even $(6, -3)$ **}.** **Note:** Candidates who set ***the numerator*** or ***the denominator*** of $\frac{dy}{dx}$ equal to $\pm k$ (usually $k = 1$) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0. **Special Case:** It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{dy}{dx}$) to gain all 7 marks in part (b). Eg: An incorrect part (a) answer of $\frac{dy}{dx} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.

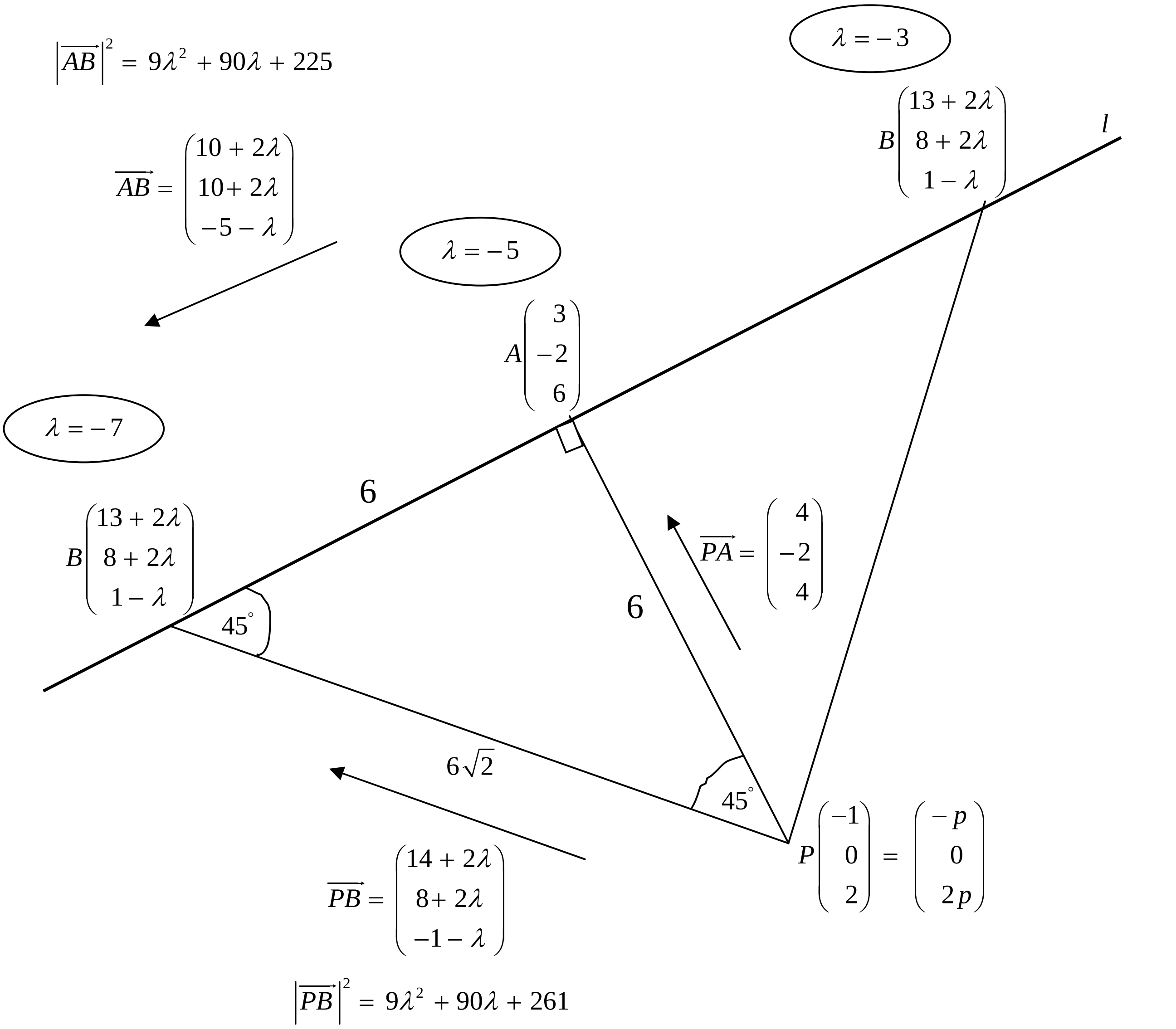
Question Number	Scheme		Marks
8.	$l:\ \mathbf{r}=\begin{pmatrix}13\\8\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\-1\end{pmatrix},\quad A(3,-2,6),\quad \overrightarrow{OP}=\begin{pmatrix}-p\\0\\2p\end{pmatrix}$		
(a)	$\{\overrightarrow{PA}\}=\begin{pmatrix}3\\-2\\6\end{pmatrix}-\begin{pmatrix}-p\\0\\2p\end{pmatrix}$ | $\{\overrightarrow{AP}\}=\begin{pmatrix}-p\\0\\2p\end{pmatrix}-\begin{pmatrix}3\\-2\\6\end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling.	M1
	$=\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ | $=\begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$	Correct difference.	A1
	$\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}=6+2p-4-6+2p=0$	See notes.	M1
	$p=1$		A1 **cso**
			[4]
(b)	$\lvert AP\rvert=\sqrt{4^2+(-2)^2+4^2}$ or $\lvert AP\rvert=\sqrt{(-4)^2+2^2+(-4)^2}$	See notes.	M1
	So, PA or $AP=\sqrt{36}$ or 6 **cao**		A1 **cao**
	It follows that, $AB=\text{"6"}\ \{=PA\}$ or $PB=\text{"}6\sqrt{2}\text{"}\ \{=\sqrt{2}\,PA\}$	See notes.	B1 ft
	{Note that $AB=\text{"6"}=2$(the modulus of the direction vector of l)}		
	$\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}\pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-7\begin{pmatrix}2\\2\\-1\end{pmatrix}$	Uses a correct method in order to find both possible sets of coordinates of *B*.	M1
	$=\begin{pmatrix}7\\2\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\-6\\8\end{pmatrix}$	Both coordinates are correct.	A1 **cao**
			[5]
			9

Notes for Question 8

8. (a)	**M1:** Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. **A1:** Accept any of $\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ or $(3+p)\mathbf{i}-2\mathbf{j}+(6-2p)\mathbf{k}$ or $\begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$ or $(-3-p)\mathbf{i}+2\mathbf{j}+(2p-6)\mathbf{k}$

Notes for Question 8 Continued	
8. (a)	**M1:** ***Applies*** the formula $\overrightarrow{PA}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{AP}\bullet\begin{pmatrix}2\\2\\-1\end{pmatrix}$ correctly to give a linear equation in p which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix}2\\2\\-1\end{pmatrix}$. Eg: Some candidates may find $\begin{pmatrix}13\\8\\1\end{pmatrix}-\begin{pmatrix}3\\-2\\6\end{pmatrix}=\begin{pmatrix}10\\10\\-5\end{pmatrix}$, for instance, and use this in their dot product which is fine for M1. **A1:** Finds $p=1$ from ***a correct solution only.*** **Note:** The direction of subtraction is not important in part (a).
(b)	**M1:** Uses their value of p and Pythagoras to obtain a numerical expression for either AP or PA or AP^2 or PA^2. Eg: PA or AP $=\sqrt{4^2+(-2)^2+4^2}$ or $\sqrt{(-4)^2+2^2+(-4)^2}$ or $\sqrt{4^2+2^2+4^2}$ or PA^2 or AP^2 $=4^2+(-2)^2+4^2$ or $(-4)^2+2^2+(-4)^2$ or $4^2+2^2+4^2$ **A1:** AP or PA $=\sqrt{36}$ or 6 **cao** or $AP^2=36$ **cao** **B1ft:** States or it is clear from their working that $AB=\text{"6"}$ $\{=$ their evaluated $PA\}$ or $PB=\text{"6"}\sqrt{2}$ $\{=\sqrt{2}$ (their evaluated PA)$\}$. **Note:** So a correct follow length is required here for either AB or PB using their evaluated PA. **Note:** This mark may be found on a diagram. **Note:** If a candidate states that $\lvert\overrightarrow{AP}\rvert=\lvert\overrightarrow{AB}\rvert$ and then goes on to find $\lvert\overrightarrow{AP}\rvert=6$ then the B1 mark can be implied. **IMPORTANT:** This mark may be implied as part of expressions such as: $\{AB=\}\sqrt{(10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2}=\mathbf{6}$ or $\{AB^2=\}(10+2\lambda)^2+(10+2\lambda)^2+(-5-\lambda)^2=\mathbf{36}$ or $\{PB=\}\sqrt{(14+2\lambda)^2+(8+2\lambda)^2+(-1-\lambda)^2}=\mathbf{6\sqrt{2}}$ or $\{PB^2=\}(14+2\lambda)^2+(8+2\lambda)^2+(-1-\lambda)^2=\mathbf{72}$ **M1:** Uses a full method in order to find **both** possible sets of coordinates of B: Eg 1: $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}\pm2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ Eg 2: $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}13\\8\\1\end{pmatrix}-7\begin{pmatrix}2\\2\\-1\end{pmatrix}$ **Note:** If a candidate achieves at least one of the correct $(7, 2, 4)$ or $(-1, -6, 8)$ then award SC M1 here. **Note:** $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}-3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}-7\begin{pmatrix}2\\2\\-1\end{pmatrix}$ is M0. **A1:** For both $(7, 2, 4)$ and $(-1, -6, 8)$. Accept vector notation or $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ notation. **Note:** All the marks are accessible in part (b) if $p=1$ is found from incorrect working in part (a). **Note: Imply M1A1B1 and award M1** for candidates who write: $\overrightarrow{OB}=\begin{pmatrix}3\\-2\\6\end{pmatrix}\pm2\begin{pmatrix}2\\2\\-1\end{pmatrix}$, with little or no earlier working.

Notes for Question 8 Continued	
8.	***Helpful Diagram!***



8. (b)	**Way 2:** Setting $AB = "6"$ or $AB^2 = "36"$ **Note:** It is possible for you to apply the main scheme for Way 2. $\{AB = "6" \Rightarrow AB^2 = "36" \Rightarrow\}$ $(10+2\lambda)^2 + (10+2\lambda)^2 + (-5-\lambda)^2 = "36"$ **B1ft** could be implied here. $9\lambda^2 + 90\lambda + 225 = 36 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$ $\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$ $\lambda = -3, -7$ Then apply final M1 A1 as in the original scheme. | ... M1 A1
8. (b)	**Way 3:** Setting $PB = "6\sqrt{2}"$ or $PB^2 = "72"$ **Note:** It is possible for you to apply the main scheme for Way 3. $\{PB = "6"\sqrt{2} \Rightarrow PB^2 = "72" \Rightarrow\}$ $(14+2\lambda)^2 + (8+2\lambda)^2 + (-1-\lambda)^2 = "72"$ **B1ft** could be implied here. $9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$ $\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$ $\lambda = -3, -7$ Then apply final M1 A1 as in the original scheme. | ... M1 A1

Notes for Question 8 Continued		
8. (b)	**(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).** **Way 4:** Using the dot product formula between $\overrightarrow{PA}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{PA} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{PA}\rvert.\lvert\overrightarrow{PB}\rvert}$. $\overrightarrow{PA} \bullet \overrightarrow{PB} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 14+2\lambda \\ 8+2\lambda \\ -1-\lambda \end{pmatrix} = 56 + 8\lambda - 16 - 4\lambda - 4 - 4\lambda = 36$ $\{\cos 45^\circ =\}\ \dfrac{1}{\sqrt{2}} = \dfrac{36}{6\sqrt{9\lambda^2 + 90\lambda + 261}}$ For finding $\lvert\overrightarrow{PA}\rvert$ as before. $\sqrt{36}$ or 6 $\lvert\overrightarrow{PB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 261}$ $\dfrac{1}{2} = \dfrac{36}{9\lambda^2 + 90\lambda + 261}$ $9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$ $\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$ $\lambda = -3, -7$ Then apply final M1 A1 as in the original scheme.	M1 A1 **cao** B1 oe ... M1 A1
8. (b)	**(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 5).** **Way 5:** Using the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{PB}\rvert}$ $\cos 45^\circ = \dfrac{1}{\sqrt{2}} = \dfrac{\begin{pmatrix} 10+2\lambda \\ 10+2\lambda \\ -5-\lambda \end{pmatrix} \bullet \begin{pmatrix} 14+2\lambda \\ 8+2\lambda \\ -1-\lambda \end{pmatrix}}{\sqrt{9\lambda^2 + 90\lambda + 225}\ \sqrt{9\lambda^2 + 90\lambda + 261}}$ Attempts the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$. Correct statement with $\lvert\overrightarrow{AB}\rvert$ and $\lvert\overrightarrow{PB}\rvert$ simplified as shown. Either $\lvert\overrightarrow{AB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 225}$ or $\lvert\overrightarrow{PB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 261}$ $\{\cos 45^\circ =\}\ \dfrac{1}{\sqrt{2}} = \dfrac{140 + 20\lambda + 28\lambda + 4\lambda^2 + 80 + 20\lambda + 16\lambda + 4\lambda^2 + 5 + 5\lambda + \lambda + \lambda^2}{\sqrt{9\lambda^2 + 90\lambda + 225}\ \sqrt{9\lambda^2 + 90\lambda + 261}}$ $\{\cos 45^\circ =\}\ \dfrac{1}{\sqrt{2}} = \dfrac{9\lambda^2 + 90\lambda + 225}{\sqrt{9\lambda^2 + 90\lambda + 225}\ \sqrt{9\lambda^2 + 90\lambda + 261}}$ $\dfrac{1}{2} = \dfrac{(9\lambda^2 + 90\lambda + 225)^2}{(9\lambda^2 + 90\lambda + 225)(9\lambda^2 + 90\lambda + 261)}$ $\dfrac{1}{2} = \dfrac{(9\lambda^2 + 90\lambda + 225)}{(9\lambda^2 + 90\lambda + 261)}$ $9\lambda^2 + 90\lambda + 261 = 2(9\lambda^2 + 90\lambda + 225) \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$ $\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$ $\lambda = -3, -7$ Then apply final M1 A1 as in the original scheme.	M1 A1 B1 ... M1 A1

Notes for Question 8 Continued	
8. (b)	**Way 6:** $\overrightarrow{PA} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and direction vector of l is $\mathbf{d} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ So, $\left\lvert\overrightarrow{PA}\right\rvert = 2\,\lvert\mathbf{d}\rvert$ or $PA = 2\,\lvert\mathbf{d}\rvert$ — A correct statement relating these distances (and not vectors) — M1 A1 B1 Apply final M1 A1 as in the original scheme. — ... M1 A1 **Note:** $\overrightarrow{PA} = 2\mathbf{d}$ with no other creditable working is M0A0B0... **Note:** $\overrightarrow{PA} = 2\mathbf{d}$, followed by $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ is M1A1B1M1 and the final A1 mark is for both sets of correct coordinates.

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