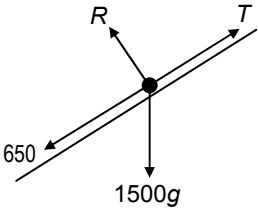
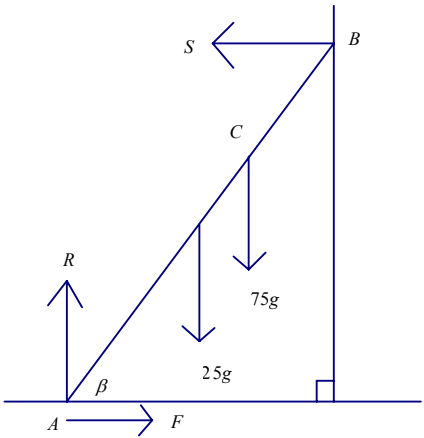


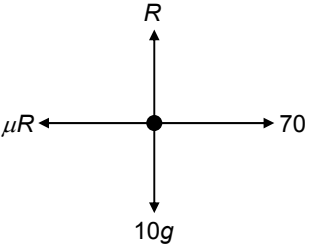
Mark Scheme (Results) January 2009






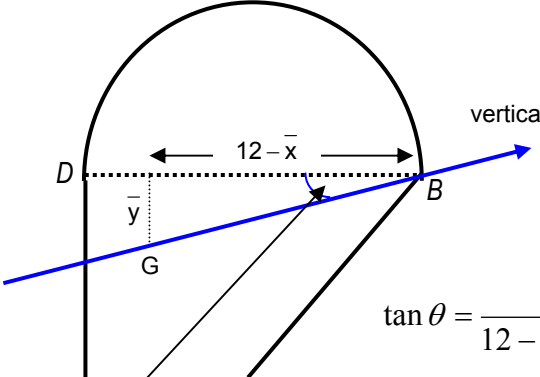
GCE

GCE Mathematics (6678/01)


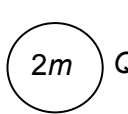
January 2009
6678 Mechanics M2
Mark Scheme

| Question Number | Scheme | Marks |
|-----------------|---|--|
| 1 |  <p> $F = ma$ parallel to the slope, $T - 1500g \sin \theta - 650 = 1500a$ Tractive force, $30000 = T \times 15$ $a = \frac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$ $\underline{0.2} \text{ (m s}^{-2}\text{)}$ </p> | M1* A1 M1* d*M1 A1 (5) [5] |
| 2 | <p>(a)</p>  <p> $R(\uparrow) : R = 25g + 75g (= 100g)$ $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ $= 44g (= 431)$ </p> <p>(b)</p> <p>M(A): $25g \times 2 \cos \beta + 75g \times 2.8 \cos \beta$ $= S \times 4 \sin \beta$ $R(\leftrightarrow) : F = S$ $176g \sin \beta = 260g \cos \beta$ $\beta = 56(^{\circ})$ </p> <p>(c) So that Reece's weight acts directly at the point C.</p> | B1 M1 A1 (3) M1 A2,1,0 M1A1 A1 (6) B1 [10] |

| Question Number | Scheme | Marks |
|---|--|---|
| <p>3 (a)</p>  | <p>$R(\uparrow) : R = 10g$</p> <p>$F = \mu R \Rightarrow F = \frac{4}{7}(10g) = 56$</p> <p>$\therefore$ WD against friction = $\frac{4}{7}(10g)(50)$</p> <p>2800(J)</p> <p>(b) $70(50) - "2800" = \frac{1}{2}(10)v^2 - \frac{1}{2}(10)(2)^2$</p> <p>$700 = 5v^2 - 20, 5v^2 = 720 \Rightarrow v^2 = 144$</p> <p>Hence, $v = \underline{12}$ (m s⁻¹)</p> <p>Or (b) N2L(\rightarrow): $70 - \frac{4}{7}R = 10a$</p> <p>$70 - \frac{4}{7} \times 10g = 10a, (a = 1.4)$</p> <p>AB ($\rightarrow$): $v^2 = (2)^2 + 2(1.4)(50)$</p> <p>Hence, $v = \underline{12}$ (m s⁻¹)</p> | <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>(4)</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>(4)</p> <p>[8]</p> |
| <p>4 (a)</p> | <p>$v = 10t - 2t^2, s = \int v dt$</p> <p>$= 5t^2 - \frac{2t^3}{3} (+C)$</p> <p>$t = 6 \Rightarrow s = 180 - 144 = \underline{36}$ (m)</p> <p>(b) $\underline{s} = \int v dt = \frac{-432t^{-1}}{-1} (+K) = \frac{432}{t} (+K)$</p> <p>$t = 6, s = "36" \Rightarrow 36 = \frac{432}{6} + K$</p> <p>$\Rightarrow K = -36$</p> <p>At $t = 10, s = \frac{432}{10} - 36 = \underline{7.2}$ (m)</p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>d*M1</p> <p>A1</p> <p>(5)</p> <p>[8]</p> |

| Question Number | Scheme | | | Marks | |
|--|--|--|---|---|--------------------------|
| 5 (a) | MR |  108 |  18π |  $108 + 18\pi$ | B1 |
| | x_i (\rightarrow) from AD | 4 | 6 | \bar{x} | B1 |
| | y_i (\downarrow) from BD | 6 | $-\frac{8}{\pi}$ | \bar{y} | |
| $AD(\rightarrow): 108(4) + 18\pi(6) = (108 + 18\pi)\bar{x}$ | | | | | M1 |
| $\bar{x} = \frac{432 + 108\pi}{108 + 18\pi} = 4.68731\dots = \underline{4.69}$ (cm) (3 sf) AG | | | | | A1 (4) |
| (b) | y_i (\downarrow) from BD |  6 |  $-\frac{8}{\pi}$ | \bar{y} | B1 oe |
| $BD(\downarrow): 108(6) + 18\pi(-\frac{8}{\pi}) = (108 + 18\pi)\bar{y}$ | | | | | M1 |
| $\bar{y} = \frac{504}{108 + 18\pi} = 3.06292\dots = \underline{3.06}$ (cm) (3 sf) | | | | | A1ft A1 (4) |
| (c) |  <p style="text-align: center;">$\theta =$ required angle</p> $\tan \theta = \frac{\bar{y}}{12 - 4.68731\dots}$ $= \frac{3.06392\dots}{12 - 4.68731\dots}$ | | | | M1 dM1 A1 |
| $\theta = 22.72641\dots = \underline{23}$ (nearest degree) | | | | | A1 (4) [12] |

| Question Number | Scheme | Marks |
|-----------------|--|---|
| 6 | (a) Horizontal distance: $57.6 = p \times 3$ $p = 19.2$ | M1 A1 (2) |
| | (b) Use $s = ut + \frac{1}{2}at^2$ for vertical displacement. $-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$ $-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$ $q = \frac{43.2}{3} = 14.4$ *AG* | M1 A1 A1 cso (3) |
| | (c) initial speed $\sqrt{p^2 + 14.4^2}$ (with their p) $= \sqrt{576} = 24$ (m s ⁻¹) | M1 A1 cao (2) |
| | (d) $\tan \alpha = \frac{14.4}{p}$ (= $\frac{3}{4}$) (with their p) | B1 (1) |
| | (e) When the ball is 4 m above ground: $3.1 = ut + \frac{1}{2}at^2$ used $3.1 = 14.4t - \frac{1}{2}gt^2$ o.e. ($4.9t^2 - 14.4t + 3.1 = 0$) $\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied $t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389\dots$ or $2.70488\dots$ awrt 0.23 and 2.7 duration = $2.70488\dots - 0.023389\dots$ $= 2.47$ or 2.5 (seconds) | M1 A1 M1 A1 M1 A1 (6) |
| | or 6 (e) M1A1M1 as above $t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e. $= 2.47$ or 2.5 (seconds) | A1 M1 A1 (6) |
| | (f) Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle. | B1 (1) [15] |

| Question Number | Scheme | Marks |
|-----------------|--|--|
| <p>7 (a)</p> | <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>Before $\xrightarrow{2u}$</p>  <p>After \xrightarrow{x} \xrightarrow{y}</p> </div> <div style="text-align: center;"> <p>\xleftarrow{u}</p>  </div> </div> <p style="text-align: right;">Correct use of NEL</p> <p style="text-align: right;">$y - x = e(2u + u)$ o.e.</p> <p>CLM (\rightarrow): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$) Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$) Hence, speed of $Q = \frac{1}{5}(9e + 4)u$ AG</p> | <p>M1*</p> <p>A1</p> <p>B1</p> <p>d*M1</p> <p>A1 cso</p> <p style="text-align: right;">(5)</p> |
| <p>(b)</p> | <p>$x = y - 3eu = \frac{1}{5}(9e + 4)u - 3eu$</p> <p>Hence, speed P $= \frac{1}{5}(4 - 6e)u = \frac{2u}{5}(2 - 3e)$ o.e.</p> <p>$x = \frac{1}{2}u = \frac{2u}{5}(2 - 3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e gives, $e = \frac{3}{12} \Rightarrow e = \frac{1}{4}$ AG</p> | <p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p style="text-align: right;">(4)</p> |
| <p>Or (b)</p> | <p>Using NEL correctly with given speeds of P and Q</p> <p>$3eu = \frac{1}{5}(9e + 4)u - \frac{1}{2}u$</p> <p>$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e $\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.</p> | <p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p style="text-align: right;">(4)</p> |
| <p>or (c)</p> | <p>Time taken by Q from A to the wall $= \frac{d}{y} = \left\{ \frac{4d}{5u} \right\}$</p> <p>Distance moved by P in this time $= \frac{u}{2} \times \frac{d}{y} (= \frac{u}{2} \left(\frac{4d}{5u} \right) = \frac{2}{5}d)$</p> <p>Distance of P from wall $= d - x \left(\frac{d}{y} \right); = d - \frac{2}{5}d = \frac{3}{5}d$ AG</p> | <p>M1†</p> <p>A1</p> <p>d†M1; A1 cso</p> <p style="text-align: right;">(4)</p> |
| <p>or (c)</p> | <p>Ratio speed P: speed Q $= x:y = \frac{1}{2}u : \frac{1}{5} \left(\frac{9}{4} + 4 \right)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$</p> <p>So if Q moves a distance d, P will move a distance $\frac{2}{5}d$</p> <p>Distance of P from wall $= d - \frac{2}{5}d; = \frac{3}{5}d$ AG</p> | <p>M1†</p> <p>A1</p> <p>d†M1; A1</p> <p style="text-align: right;">(4)</p> |

| Question Number | Scheme | Marks |
|------------------------|---|---------------------------------------|
| (d) | After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y Time for P , $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q , $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$ gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x$, $3x = \frac{3d}{5} \Rightarrow x = \frac{1}{5}d$ | B1ft B1ft M1 A1 cao (4) |
| or (d) | After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y speed $P = x = \frac{1}{2}u$, speed P : new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y Distance of B from wall = $\frac{1}{3} \times \frac{3d}{5} = \frac{d}{5}$ their $\frac{1}{2+1}$ | B1ft B1ft M1; A1 (4) |
| 2 nd or (d) | After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$ Time from wall to 2 nd collision = $\frac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y Distance of B from wall = (their speed)x(their time) = $\frac{u}{4} \times \frac{4d}{5u} = \frac{1}{5}d$ | B1ft B1ft M1; A1 (4) [17] |