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Mark Scheme (Results)

Summer 2012

GCE Mechanics M2
(6678) Paper 1

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Summer 2012
6678 Mechanics 2
Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso – correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the first 2 A or B marks affected are lost, and the subsequent A marks affected are treated as A ft; but manifestly absurd answers should never be awarded A marks.

General Principles for Mechanics Marking

Usual rules for M marks: correct no. of terms; dim correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
Omission or extra g in a resolution is accuracy error not method error.
Omission of mass from a resolution is method error.
Omission of a length from a moments equation is a method error.
Omission of units or incorrect units is not (usually) counted as an accuracy error.
DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
Use of g = 9.81 should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *ONCE* per complete question.
However, premature approximation should be penalised every time it occurs.
MARKS MUST BE ENTERED IN THE SAME ORDER AS THEY APPEAR ON THE MARK SCHEME.

In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

Accept column vectors in all cases.

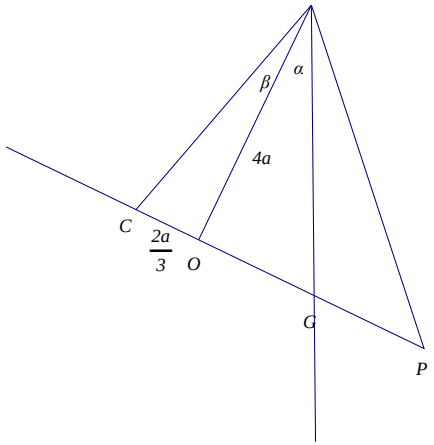
Summer 2012
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks	Notes
1			
(a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 6t\mathbf{i} + (4-2t)\mathbf{j}$	M1	Differentiate **v** to obtain **a**.
		A1	Accept column vector or **i** and **j** components dealt with separately.
	When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	DM1	Substitute $t = 1$ into their **a.** Dependent on 1st M1
	$\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32$ (m s^{-2})	DM1	Use of Pythagoras to find the magnitude of their **a**. Allow with their t. Dependent on 1st M1
		A1	Accept awrt 6.32, 6.3 or exact equivalents.
		(5)	
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\, dt$	M1	Integrate **v** to obtain **r**
	$= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$	A1	Condone C, D missing
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	DM1	Use $t = 0$, **r** = **i** to find C & D
	When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m)	DM1	Substitute $t = 3$ with their C & D to find **r**. Dependent on both previous Ms.
		A1	cao. Must be a vector.
		(5)	
		10	

Question Number	Scheme	Marks	Notes
2			
(a)	$3m.2u - 4mu = 3mv_1 + 4mv_2$	M1 A1	CLM. Need all terms. Condone sign slips. Correct but check their directions for v_1 & v_2.
	$e(2u+u) = -v_1 + v_2$	M1 A1	Impact law. Must be used the right way round, but condone sign slips. Directions of v_1 & v_2 must be consistent between the two equations. (Ignore the diagram if necessary)
	$\frac{u(2+9e)}{7} = v_2$	DM1 A1 (6)	Eliminate v_1 to produce an equation in v_2 only. Dependent on both previous M marks – must be using both equations. DO NOT accept the negative. The question asks for speed.
(b)	$v_1 = \frac{2u(1-6e)}{7}$	M1	Use the work from (a) or restart to find v_1 or λv_1 for a constant λ. If using work from (a) this mark is dependent on the first 2 M marks.
		A1	a.e.f. Correct for their direction. Allow for λv_1
	$v_1 < 0 \Rightarrow e > \frac{1}{6}$	DM1	An appropriate inequality for their v_1 (seen or implied) – requires previous M1 scored. Work on $v_1=0$ scores M0 until the inequality is formed.
		A1	Accept $\frac{2}{12}$. Answer must follow from correct work for v_1
	$1 \geq e > \frac{1}{6}$	B1	For (their value) $< e \leq 1$ SR: from $v_1 \leq 0$ could score M1A0B1
		(5)**11**	

Question Number	Scheme	Marks	Notes
3 **(a)**	$M(A), F.4\sin 40^o = 5g.2\cos 25^o$	M1	A complete method to find F , e.g. take moments about A. Condone sin/cos confusion. Requires correct ratio of lengths.
		A1	Correct terms with at most one slip
		A1	All correct
	$F = 35$	A1	35 or 34.5 (>3sf not acceptable due to use of 9.8, but only penalise once in a question)
		(4)	
(b)	$F\cos 75^o \pm Y = 5g$	M1	Resolve vertically. Need all three terms but condone sign errors. Must be attempting to work with their 75° or 15°.
		A1	Correct equation (their F)
	$Y = 40$; UP	A1	40 or 40.1 Apply ISW if the candidate goes on to find R.
		A1	cso (the Q does specifically ask for the direction, so this must be clearly stated)
		(4)	
		8	
(b)	OR1: $4m\cos 25\times Y = 5g\times 2m\cos 25 + F\cos 15\times 4m\sin 25$ etc.	M1 A1	Taking moments about the point vertically below B and on the same horizontal level as A.(Their F)
	OR2: $R\cos\alpha = F\cos 40 + 5g\cos 65$ $R\sin\alpha + F\sin 40 = 5g\cos 25$		Resolve parallel & perpendicular to the rod
	R=52.1, α=25.3°		Solve for R, α
	$Y = R\sin(25+\alpha)$ Etc.	M1A1	Need a complete strategy to find Y for M1.

Question Number	Scheme	Marks	Notes
4 **(a)**	$\pi(4a)^2 \quad \pi(2a)^2 \quad (\pi(4a)^2 - \pi(2a)^2)$ $4 \quad 1 \quad 3$ $4a \quad 2a \quad \bar{x}$	B1	Correct mass ratios
		B1	Distance of c of m from P (or from a point on QP).
	$(4 \text{ x } 4a) - (1 \text{ x } 2a) = 3\bar{x}$	M1	Moments about axis through P, or about a parallel axis then convert the answer to distance from P. Condone a sign slip.
	$\frac{14a}{3} = \bar{x}$ *	A1	Answer given – check working carefully. Must reach positive answer legitimately.
		(4)	
(b)	$OG = 4a\tan\alpha = \frac{10a}{3}\left(\Rightarrow PG = \frac{2a}{3}\right)$	M1	Vertical through S cuts OP at G. Use trig to find the position of G on OP.
		A1	$OG = \frac{10a}{3}$, $QG = \frac{22a}{3}$ or $PG = \frac{2a}{3}$ seen or implied
	$M(P), (m+km)g.\frac{2a}{3}\cos\alpha = mg.\frac{14a}{3}\cos\alpha$ $M(G): km \times \frac{2}{3}a = m \times \left(\frac{10}{3}a + \frac{2}{3}a\right) = 4ma$ $M(O): m(1+k) \times \frac{10}{3}a + m \times \frac{2}{3}a = km \times 4a$	M1	Take moments about a point on QP – terms should be dimensionally consistent. Masses must be associated with the appropriate distances, which might be incorrectly evaluated or not yet found – e.g. accept with QG. Must have the right terms but condone trig confusion. Also condone absence of trig.
	$M(C): \frac{12}{3}a \times (1+k)m = \frac{14}{3}a \times km$	A1	cso (C is the position of the original centre of mass.)
	$M(Q): \frac{22}{3}a \times m(1+k) = \frac{10}{3}a \times m + 8a \times km$ $k = 6$	A1	cso
		(5)	See next page for more alternatives…..
		9	

OR	$(k+1)m\times PG = m\times\frac{14}{3}a$	M1	Moments about P
	$PG = \frac{14a}{3(k+1)}$	A1	Correct expression for PG
	$\tan\alpha = \frac{OG}{4a} = \frac{4a-\frac{14a}{3(k+1)}}{4a}\left(=1-\frac{7}{6(k+1)}\right)$	M1	Use of $\tan\alpha$ in the correct triangle.
	$\frac{5}{6} = 1-\frac{7}{6(k+1)} \qquad k=6$	A1, A1	Correct equation in *k*, correct solution
OR			
	$\tan(\angle CSO) = \tan\beta = \frac{\frac{2a}{3}}{4a} = \frac{1}{6}$	M1	
	$km.\sqrt{32}a.\sin(45-\alpha) = m.\sqrt{16\frac{4}{9}}a.\sin(\alpha+\beta)$	M1A1	Moments about S
	$k.\sqrt{32}.\left(\frac{6-5}{\sqrt{2}.\sqrt{61}}\right) = \frac{\sqrt{148}}{3}.\frac{6\times5+1\times6}{\sqrt{37}.\sqrt{61}}$	A1	Do not expect accurate working
	$4k = \frac{2}{3}\times36, \quad k=6$	A1	Final answer 6.0

Question Number	Scheme	Marks	Notes
5			NB In a Q with parts labelled (i) & (ii) marks are awarded when seen – they do not belong to a particular part of the Q.
	$12.5\sin\alpha = \frac{1}{4}(v_1 - -30)$	M1	Impulse = change in momentum parallel to the initial direction. .
	or $-12.5\sin\alpha = \frac{1}{4}(v_1 - 30) \quad (v_1 = 0)$	A1	Correct equation
	$12.5\cos\alpha = \frac{1}{4}(v_2 - 0) \qquad (v_2 = 40)$	M1 A1	Impulse = change in momentum perpendicular to the initial direction. Condone sin/cos confusion Correct equation NB could be in the form: $\begin{pmatrix} -12.5\sin\alpha \\ 12.5\cos\alpha \end{pmatrix} = 0.25v - 0.25\begin{pmatrix} 30 \\ 0 \end{pmatrix}$
	speed is 40 m s^{-1};	A1	cwo. Correct magnitude of speed after impulse. NB Must be speed, not velocity.
	perpendicular to original direction	A1	cwo. Correct direction (relative to the line given on the diagram – e.g. accept "vertically", "North", **j** direction, "up").
		6	
OR	Using a vector triangle:	M1	Use cosine rule to find $\frac{1}{4}v$. Terms must be of correct form, but
	$(\frac{1}{4}v)^2 = 7.5^2 + 12.5^2 - 2\text{x}7.5\text{x}12.5\cos(90^\circ - \alpha)$		accept unsimplified or slips e.g. their $\frac{1}{4}\times 30$
		A1	Correct equation
	$v = 40\text{m s}^{-1}$	A1	cao (penultimate mark on epen)
	$\frac{12.5}{\sin\theta} = \frac{7.5}{\sin\alpha}$	M1	Use sine rule to find angle between initial and final directions.
		A1	Correct equation in α and θ
	$\theta = 90^\circ$	A1	cao. (final mark on epen)
		6	

Question Number	Scheme	Marks	Notes
6			
(a)	$F = \frac{60000}{10} = 6000$	B1	Correct application of $P = Fv$ seen or implied.
	$F - 1200g\sin\alpha - 400g\sin\alpha - 1000 - 200 = 1600a$	M1	Use of $F = ma$ parallel to the slope for the car and trailer. Must have all the terms, but condone sign errors.
		A1	At most one error (with F or their F)
		A1	Correct equation (with F or their F)
	$a = 2.3$ (m s^{-2})	A1	only
		(5)	
(b)		M1	Use of $F = ma$ parallel to the slope for the trailer
	$T - 400g\sin\alpha - 200 = 400$ x 2.3	A1 ft	At most one error (their a)
		A1 ft	All correct (their a)
	$T = 1400$	A1	only
		(4)	
OR		M1	Use of $F = ma$ parallel to the slope for the car
	$6000 - 1200g\sin\alpha - 1000 - T = 1200$ x 2.3	A1 ft	At most one error (their a)
		A1 ft	All correct (their a)
	$T = 1400$	A1	only
		(4)	
OR (a)	$F = 6000$	B1	
	$T - 400g\sin\alpha - 200 = 400$ x a		Simultaneous equations in T and a
	$6000 - 1200g\sin\alpha - 1000 - T = 1200$ x a		
	$6000 - 1600g\sin\alpha - 1200 = 1600a$	M1A1A1	Add to eliminate T
	$a = 2.3$ (m s^{-2})	A1	
(b)	$-800a = 2T + 800g\sin\alpha + 800 - 6000$	M1A1A1	Subtract and / or substitute to eliminate a
	$2T = 5200 - 800g\sin\alpha - 800\times 2.3$		
	$T = 1400$	A1	

(c)	$200d = \frac{1}{2}400.12^2 - 400gd\sin\alpha$	M1	Use of work-energy. Must have all three terms. Do not accept duplication of terms, but condone sign errors.
			Equation in only one unknown, but could be vertical distance.
		A1	At most one error in the equation
		A1	All correct in one unknown
	$d = 60$ (m)	DM1	Solve for d – dependent on M for work-energy equation.
		A1	only
		(5)	For vertical distance $\left(=\frac{60}{14}=4.29\right)$ allow 3/5
		14	

Question Number	Scheme	Marks	
7 (a)	$0^2 = u_V{}^2 - 2 \times 9.8 \times 10$	M1	Complete method using *suvat* to form an equation in u_v.
		A1	Correct equation e.g. $0 = u^2 - 20g$
	$u_V = 14$ *	A1	*Answer given* requires equation and working, including 196, seen.
		(3)	
OR	conservation of energy:	M1	Initial KE = gain in GPE + final KE
	$\frac{1}{2}m\left(u_h{}^2 + u_v{}^2\right) = mg \times 10 + \frac{1}{2}mu_h{}^2$, $\frac{1}{2}u_v{}^2 = 98$	A1	Correct equation
	$u_V = 14$ *	A1	*Answer given*
		(3)	
(b)		M1	Use the vertical distance travelled to find the total time taken.
	$(\uparrow),\ -52.5 = 14t - \frac{1}{2}gt^2$	A1	At most one error
		A1	Correct equation
	$49t^2 - 140t - 525 = 0$	DM1	Solve for t. Dependent on the preceding M mark
	$(t-5)(49t+105) = 0 \quad t = 5$	A1	only
	$(\rightarrow),\quad 50 = 5u_H$	M1	Use their time of flight to form an equation in u_H
	$u_H = 10$	A1	only
	$u = \sqrt{10^2 + 14^2}$	M1	Use of Pythagoras with two non-zero components, or solution of a pair of simultaneous equations in u and α.
	$= \sqrt{296}$; 17.2 m s^{-1}	A1	17.2 or 17 (method involves use of g = 9.8 so an exact surd answer is not acceptable)
		(9)	
			See next page for an alternative route to u, and (c).

OR			First 3 marks for the quadratic as above.
	$50 = u\cos\alpha t$ or $50 = u_H t$	M1	Used in their quadratic
	$49\left(\frac{50}{u_H}\right)^2 - 140\left(\frac{50}{u_H}\right) - 525 = 0$	A1	Correct quadratic in u_H
	$525(u_H)^2 + 140(u_H) - 122500 = 0$		
	Solve for u_H	DM1	Dependent on the M mark for setting up the initial quadratic equation in t.
	u_H = 10	A1	only
	etc.		Complete as above.
(c)	$\tan OBA = \frac{52.5}{50} = 1.05$	B1	Correct direction o.e. (accept reciprocal)
	$v_V = 1.05 \text{ x } 10 = 10.5$	M1	Use trig. with their u_H and correct interpretation of direction to find the vertical component of speed. Working with distances is M0. (condone $10 \div 1.05$)
	$(\uparrow),\ -10.5 = 14 - gt$	DM1	Use suvat to form an equation in t. Dependent on the preceding M.
		A1	Correct equation for their $u_{H.}$ For incorrect direction give A0 here.
	$t = 2.5$	A1	only
		(5)	
		17	

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