## P <br> Pearson Edexcel

## Mark Scheme (Results)

## Summer 2018

Pearson Edexcel International Advanced Level In Mechanics M2 (WME02/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## PEARSON EDEXCEL IAL MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:

## 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.
To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel ' $g$ ' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity - this M mark is often dependent on the two previous $M$ marks having been earned.
'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.
'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. - follow through - marks.

## 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
- $\quad$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

6. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

## (But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by $\cos$ or $\sin )$ are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $\mathrm{g}=9.8$ should be given to 2 or 3 SF .
- Use of $\mathrm{g}=9.81$ should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads - if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.
N2L Newton's Second Law (Equation of Motion)
NEL Newton's Experimental Law (Newton's Law of Impact)
HL Hooke's Law
SHM Simple harmonic motion
PCLM Principle of conservation of linear momentum
RHS, LHS Right hand side, left hand side.

## June 2018 Mechanics 2 - WME02 <br> Mark Scheme

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1a. | Use of $\mathbf{I}=m \mathbf{v}-m \mathbf{u}$ | M1 |
|  | $=0.7((3 \mathbf{i}+4 \mathbf{j})-(\mathbf{i}-2 \mathbf{j}))$ | A1 |
|  | $=0.7(2 \mathbf{i}+6 \mathbf{j})=1.4 \mathbf{i}+4.2 \mathbf{j}(\mathrm{~N} \mathrm{~s})$ | A1 |
|  |  | (3) |
|  |  |  |
| 1b. |  |  |
|  | Required angle $=\tan ^{-1} 3+\tan ^{-1} 2 \quad$ Follow their impulse | $\begin{aligned} & \text { M1 } \\ & \text { A1ft } \end{aligned}$ |
|  | $=135^{\circ}$ (Allow $225^{\circ}$ ) | A1 |
|  |  | (3) |
|  |  |  |
|  |  |  |
|  | Alternative for last 3 marks using scalar product: |  |
|  | Correct use of scalar product with relevant vectors M1 |  |
|  | $\cos \theta=\frac{(\mathbf{i}-2 \mathbf{j}) \cdot(\mathbf{i}+3 \mathbf{j})}{\sqrt{1+4} \sqrt{1+9}}=\frac{-5}{5 \sqrt{2}}$ |  |
|  | $\theta=135^{\circ}$ A1 |  |
|  |  |  |
|  | Alternative for last 3 marks using cosine rule: |  |
|  | Correct use of cosine rule with relevant sides M1 |  |
|  | $\begin{equation*} \cos \theta=\frac{\left(1.4^{2}+4.2^{2}\right)+\left(1^{2}+(-2)^{2}\right)-\left(0.4^{2}+6.2^{2}\right)}{2 \sqrt{\left(1.4^{2}+4.2^{2}\right)} \sqrt{\left(1^{2}+(-2)^{2}\right)}} \tag{A1ft} \end{equation*}$ |  |
|  | $\theta=135^{\circ}$ A1 |  |
|  | 1(a) <br> M1 for Use of $\mathbf{I}=m \mathbf{v}-m \mathbf{u}$ (M0 if $m$ omitted or $g$ included) with $\mathbf{v}$ and u substituted but allow $m$ and allow terms reversed <br> First A1 for a correct equation <br> Second A1 for a correct answer, i's and $\mathbf{j}$ 's need to be collected Isw if they go on and give a magnitude. |  |
|  | 1(b) <br> M1 for a complete correct method to find required angle for their I (M0 if they are finding the wrong angle) <br> First A1 ft for a correct expression from their $\mathbf{I}$ and $\mathbf{u}$ Second A1 for $135^{\circ}$ (Accept 134.9.. or 225.1...) |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  |  |  |
| 2 a |  |  |
|  | Moments about $A$ : $T \times 1.6 \sin 70^{\circ}=6 \mathrm{~g} \times 0.8 \cos 30^{\circ}$ | M1A2 |
|  | $T=27.1 \quad$ Given Answer | A1 |
|  |  | (4) |
|  |  |  |
| 2b | Resolve $\leftrightarrow$ : $F=T \cos 40$ | B1 |
|  | Resolve $\uparrow: R+T \cos 50=6 \mathrm{~g}$ | M1A1 |
|  | Use of $F \leq \mu R$ and solve for $\mu: \mu \geq \frac{20.76}{41.38}=0.50 \quad(0.502)$ | DM1A1 |
|  |  | (5) |
|  |  | [9] |
|  | Notes for Qu2 |  |
|  | 2(a) <br> M1 for a complete method to obtain an equation in $T$ only, with usual rules (applied to all equations if more than one is used) <br> N.B. Treat wrong angle(s) as A error(s) <br> A2 for a correct equation (or equations) A1A0 if one error (Allow use of $a$ and $2 a$ for lengths) <br> A1 for 27.1 correctly obtained (and no incorrect work seen) <br> N.B. GIVEN ANSWER <br> Other equations: $\begin{aligned} & \nearrow: R \cos 60+F \cos 30=6 g \cos 60+T \cos 70 \\ & \nwarrow: R \sin 60+T \sin 70=F \sin 30+6 g \sin 60 \\ & M(B): 6 g l \cos 30+F 2 l \sin 30=R 2 l \cos 30 \\ & M(G): F l \sin 30+T l \sin 70=R l \cos 30 \end{aligned}$ |  |
|  | 2(b) <br> B1 for $F=T \cos 40$ seen <br> First M1 for a complete method, with usual rules applied to all equations used, to find $R$ <br> N.B. Treat wrong angle(s) as A error(s) <br> First A1 for a correct equation <br> Second DM1, dependent on first M1, for use of $F \leq \mu R$, and solve for $\mu$ <br> (Allow this M if they use $F=\mu R$ or $F<\mu R$ but final A1 not then available but M0 if they use $F \geq \mu R$ or $F>\mu R$ ) <br> Second A1 for either $\mu \geq 0.5(0)$ or $\mu \geq 0.502$ <br> A0 if they also give an upper bound for $\mu$ |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 3a | Use of $P=F v: F=\frac{180}{4}$ | B1 |
|  | Equation of motion: $\quad F-R=75 \times 0.2$ | M1 |
|  | Equation in $R: \quad \frac{180}{4}-R=75 \times 0.2(45-R=15)$ | DM1 |
|  | $R=30$ | A1 |
|  |  | (4) |
| 3b | Equation of motion: $D-75 g \sin \theta-R=0$ | M1 |
|  | $\frac{180}{v}-75 \times g \times \frac{1}{21}-\text { their } R=0$ | A2 ft |
|  | $v=2.77$ or 2.8 | A1 |
|  |  | (4) |
|  |  | [8] |
|  | Notes for Qu3 |  |
|  | 3(a) <br> B1 for $F=\frac{180}{4}$ seen <br> First M1 for equation of motion with usual rules, $F$ does not need to be substituted <br> Second M1, dependent on first M1, for an equation in $R$ only with usual rules <br> A 1 for $R=30$ |  |
|  | 3(b) <br> M1 for equation of motion with usual rules but none of $D, \sin \theta$ nor $R$ need to be substituted <br> A2 $\mathbf{f t}$ for a correct equation, in $v$ only, ft on their $R$ <br> A1A0 if one error <br> Third A1 for 2.77 or 2.8 (Only answers) |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 4a | Area ratios: $9 a^{2}: \frac{9}{4} a^{2}: 5 \times \frac{9}{4} a^{2} \quad(4: 1: 5)$ | B1 |
|  | Distances from $A C: a, \frac{a}{2}(, \bar{y})$ | B1 |
|  | Moments about $A C: 4 \times a+1 \times \frac{a}{2}=5 \bar{y}$ | M1A1ft |
|  | $\bar{y}=\frac{9}{10} a \quad$ (Given Answer) | A1 cso |
|  |  | (5) |
| 4b | Distances from vertical through $B: 0, \frac{3}{2} a(, \bar{x})$ OR: | B1 |
|  | Moments about vertical axis through $B: \quad 1 \times \frac{3}{2} a=5 \bar{x}$ | M1 |
|  | $\bar{x}=\frac{3}{10} a$ <br> N.B. You may see $\bar{x}=3.3 a$ (if distances measured from $A$ ) or $\bar{x}=2.7 a$ (if distances measured from $C$ ) | A1 |
|  |  |  |
|  | Required angle $A B G: 45^{\circ}+\theta$ (for their $\bar{x}$; e.g. if $\bar{x}<0$, then $45^{\circ}-\theta$ ) | M1 |
|  | Method for $\theta: \tan \theta=\frac{\bar{x}}{3 a-\frac{9 a}{10}}\left(=\frac{1}{7}\right)$ | M1A1ft |
|  | $45^{\circ}+\tan ^{-1}\left(\frac{1}{7}\right)=53^{\circ}$ nearest degree | A1 |
|  |  | (7) |
|  |  | [12] |
|  | Notes for Qu4 |  |
|  | 4(a) <br> First B1 for correct (unsimplified) area ratios seen Second B1 for correct distances from AC (or from any parallel line) First M1 for a 'moments' equation about $A C$ (or any parallel line) First A1ft for a correct equation, ft on their 'distances' and 'masses'. Second A1 cso for a correct GIVEN ANSWER with no errors seen |  |
|  | N.B. <br> Allow equivalent vector equation for two moments equations. |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | 4(b) <br> B1 for correct distances from vertical through $B$ (or from any parallel line) (May be seen in (a)) Allow missing $a$ 's if they recover <br> First M1 for 'moments' equation about vertical axis through $B$ (or from any parallel line) (May be seen in (a)) <br> First A1 for a correct $\bar{x}$ (allow recovery of missing $a$ 's) <br> Second M1 for use of correct angle (e.g. $45^{\circ}+\theta$ or $45^{\circ}-\theta$ if using tan method) oe <br> Third M1 for appropriate equation in $\theta$ only, using their $\bar{x}$ and $\frac{9 a}{10}$ <br> Second A1ft for a correct equation <br> Third A1 for $53^{\circ}$ (nearest degree) <br> ALTERNATIVE: <br> B1 for correct distances from vertical through $B$ (or from any parallel line) (May be seen in (a)) Allow missing $a$ 's if they recover <br> First M1 for a dim correct moments equation about vertical axis through $B$ (or from any parallel line) (May be seen in (a)) <br> First A1 for a correct $\bar{x}$ (allow recovery of missing $a$ 's) <br> Second M1 for identifying correct angle ( $A B G=\alpha$ ) <br> Third M1 for use of cos rule on triangle $A B G$ to give equation in $\alpha$ only, using their $\bar{x}$ and $\frac{9 a}{10}$ <br> Second A1ft for a correct equation <br> Third A1 for $53^{\circ}$ (nearest degree) <br> OR: <br> B1 for correct distances from vertical through $B$ (or from any parallel line) (May be seen in (a)) Allow missing $a$ 's if they recover <br> First M1 for a dim correct moments equation about vertical axis through $B$ (or from any parallel line) (May be seen in (a)) <br> First A1 for a correct $\bar{x}$ (allow recovery of missing $a$ 's) <br> Second M1 for use of angle $A B G=$ angle $A B C$ - angle $G B C$ <br> Third M1 for use of cos rule on triangle $G B C$ to find angle $G B C$, using their $\bar{x}$ and $\frac{9 a}{10}$ <br> Second Alft for a correct equation <br> Third A1 for $53^{\circ}$ (nearest degree) |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 5a | Differentiate $\mathbf{v}$ : $\quad \mathbf{a}=(6 t-4) \mathbf{i}+(6 t-8) \mathbf{j}$ | M1A1 |
|  | $\mathbf{F}=m \mathbf{a}$ when $t=4: \quad \mathbf{F}=0.3(20 \mathbf{i}+16 \mathbf{j})=6 \mathbf{i}+4.8 \mathbf{j}$ | M1 |
|  |  | (3) |
| 5b | Motion parallel to i: $3 t^{2}-8 t+4=0=(3 t-2)(t-2)$ | M1 |
|  | $t=\frac{2}{3}$ or $t=2$ | A1 |
|  | Integrate $\mathbf{v}$ : $\quad \mathbf{r}=\left(t^{3}-2 t^{2}(+p)\right) \mathbf{i}+\left(t^{3}-4 t^{2}+4 t(+q)\right) \mathbf{j}$ | M1A1 |
|  | Use limits: $\quad \mathbf{r}_{2}=(8-8(+p)) \mathbf{i}+(8-16+8(+q)) \mathbf{j}$ | M1A1 |
|  | $\mathbf{r}_{\frac{2}{3}}=\left(\frac{8}{27}-\frac{8}{9}(+p)\right) \mathbf{i}+\left(\frac{8}{27}-\frac{16}{9}+\frac{8}{3}(+q)\right) \mathbf{j}$ | A1 |
|  | $\overrightarrow{A B}= \pm\left(\frac{16}{27} \mathbf{i}-\frac{32}{27} \mathbf{j}\right)$ |  |
|  | Pythagoras' theorem: $\|\overrightarrow{A B}\|=\frac{16}{27} \sqrt{5}=1.3$ (or better) (m) | DM1A1 |
|  |  | (9) |
|  |  | [12] |
|  |  |  |
|  | Notes for Qu5 |  |
|  | Accept column vectors throughout |  |
|  | 5(a) <br> First M1 for attempt to differentiate $\mathbf{v}$, at least two powers of $t$ decreasing by one. <br> A1 for a correct expression. (A0 if $\mathbf{i}$ or $\mathbf{j}$ omitted) Second M1 for multiplying their a by 0.3 , substituting $t=4$ and collecting i's and $\mathbf{j}$ 's. Isw if they find the magnitude. |  |
|  | 5(b) <br> First M1 for $3 t^{2}-8 t+4=0$ and attempting to solve. This M mark can be implied by two correct answers but if answer(s) are incorrect, we need to see an explicit attempt at factorising, using the formula or completing the square. <br> First A1 for two correct answers, allow 0.67 or better. <br> Second M1 for attempt to integrate $\mathbf{v}$, to produce a vector, with at least two powers of $t$ increasing by 1 . <br> Second A1 for a correct $\mathbf{r}$ (constant not needed). <br> Third M1for substituting both their values of $t$ (which must have come from using a velocity vector) into their $\mathbf{r}$. <br> Third A1 for correct unsimplified $\mathbf{r}_{2}$ ( constant not needed) Allow a point. <br> Fourth A1 for correct unsimplified $\mathbf{r}_{2}$ ( constant not needed). Allow a <br> point. <br> Fourth DM1, dependent on previous M mark, for subtracting their velocity vectors (or points) either way and using Pythagoras to find the length. <br> Fifth A1 for correct surd answer oe or 1.3 or better. |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 6 a | Energy: $\frac{1}{2} m \times 144=\frac{1}{2} m \times 64+m g h$ | M1A2 |
|  | $h=4.1$ or 4.08 (m) | A1 |
|  | N.B. If they find an $h$, using a non-energy method, they score nothing in (a) BUT it can be used in (b), (c) and (d) without penalty. | (4) |
| 6b | Vertical distance: $h=12 \sin \alpha \times 1.5-\frac{g}{2} \times 1.5^{2} \quad$ (their $h$ ) | M1A1ft |
|  | $(\sin \alpha=0.839) \quad \alpha=57.1^{\circ}\left(57^{\circ}\right)(0.996$ rads $)$ | A1 |
|  |  | (3) |
|  | Alt: Use $12 \cos \alpha=8 \cos \beta$ and $-8 \sin \beta=12 \sin \alpha-1.5 g$ |  |
|  | and eliminate $\beta$ to give equation in ONE trig ratio for $\alpha$ | M1A1ft |
|  | $(\sin \alpha=0.839) \quad \alpha=57.1^{\circ}\left(57^{\circ}\right)(0.996$ rads $)$ | A1 |
|  |  | (3) |
| 6c | Horizontal cpts: $12 \cos \alpha=8 \cos \beta$ (with their $\alpha$ for the A mark) | M1A1ft |
|  | $\beta=35.4^{\circ} \quad\left(35^{\circ}\right) \quad(0.617$ or 0.62 rads$)$ | A1 |
|  | Alt: $\tan \beta=\frac{12 \sin \alpha-1.5 g}{12 \cos \alpha}(=-0.709 \ldots$.$) \quad M1A1ft$ |  |
|  | $\beta=35.4^{\circ} \quad\left(35^{\circ}\right) \quad(0.617$ rads or 0.62 rads$) \quad \mathrm{A} 1$ |  |
|  |  | (3) |
|  | Alt: $h=\frac{(12 \sin \alpha-8 \sin \beta)}{2} \times 1.5 \quad$ M1A1ft |  |
|  | $\beta=35.4^{\circ} \quad\left(35^{\circ}\right) \quad(0.617$ rads or 0.62 rads $) \quad \mathrm{A} 1$ |  |
|  | Alt: $-8 \sin \beta=12 \sin \alpha-(9.8 \times 1.5) \quad$ M1A1ft |  |
|  | $\beta=35.4^{\circ} \quad\left(35^{\circ}\right) \quad(0.617$ rads or 0.62 rads $) \quad \mathrm{A} 1$ |  |
| 6d | Correct strategy e.g. $2(1.5-T)$ where $T=$ time to max height | M1 |
|  | Correct equations e.g.Time to max height: $T=\frac{12 \sin \alpha}{g}=1.028$ $t=2\left(1.5-\frac{12 \sin \alpha}{g}\right)$ | DM1A1 |
|  | Correct answer: $t=0.94$ or 0.945 (s) | A1 |
|  | Alt 1: Time between the two points when $v=8 \quad$ M1 |  |
|  | $0=8 \sin \beta t-\frac{1}{2} g t^{2}\left(=t\left(8 \sin \beta-\frac{1}{2} g t\right)\right) \quad$ DM1A1 |  |
|  | $t=0.94,0.945$ A1 |  |
|  | Alt 2: Find the difference between the two times when $h=4.08$ M1 |  |
|  | $h=12 \sin \alpha t-\frac{1}{2} g t^{2}$ for their $h, \alpha \quad$ DM1A1 |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | $t=1.5-0.555=0.94$ or 0.945 (s) A1 |  |
|  |  | (4) |
|  | Alt 3: Use $v=u+a t$ (vert) between the two points M1 |  |
|  | $8 \sin \beta=-8 \sin \beta+g t \quad$ DM1A1 |  |
|  | $t=\frac{16 \sin \beta}{g}=0.94,0.945$ | [14] |
|  |  |  |
|  | Notes for Qu6 |  |
|  | 6(a) <br> M1 for an energy equation with correct no. of terms etc. (M0 if no energy equation used) <br> First A1 and Second A1 for a correct equation. <br> Third A1 for 4.1 or 4.08 (m) |  |
|  | 6(b) <br> M1 for a complete method to find an equation in $\sin \alpha$ only, using their $h$. <br> (N.B. This equation could be obtained by getting two equations in $\alpha$ and $\beta$, which don't require $h$, and eliminating $\beta$, see above) <br> First A1ft for a correct equation, ft on their $h$ (if used) <br> Second A1 for $57^{\circ}, 57.1^{\circ}, 1.0,1.00 \mathrm{rad}$ |  |
|  | 6(c) <br> M1 for a complete method to find equation in $\alpha$ and $\beta$ only <br> Either by equating horizontal velocity components: $12 \cos \alpha=8 \cos \beta$ <br> Or by finding the vertical velocity component at $B, v_{\mathrm{v}}$, and then using $\tan \beta=\frac{\left\|v_{\mathrm{v}}\right\|}{12 \cos \alpha}$ or $\sin \beta=\frac{\left\|v_{\mathrm{v}}\right\|}{8}$. <br> First A1ft for a correct equation. <br> N.B. <br> $v_{\mathrm{V}}$ can be found in a number of ways: $\begin{aligned} & v=u+a t: v_{\mathrm{v}}=12 \sin \alpha-1.5 g \\ & v^{2}=u^{2}+2 a s: v_{\mathrm{V}}=\sqrt{(12 \sin \alpha)^{2}-2 g \times \text { their } h} \\ & s=v t-\frac{1}{2} a t^{2}: h=1.5 v_{\mathrm{V}}+\frac{1}{2} g \times 1.5^{2} \Rightarrow v_{\mathrm{V}}=\frac{h-\frac{1}{2} g \times 1.5^{2}}{1.5}=\frac{2 h}{3}-\frac{3 g}{4} \\ & s=\left(\frac{u+v}{2}\right) t: h=\left(\frac{12 \sin \alpha+v_{\mathrm{V}}}{2}\right) \times 1.5 \Rightarrow v_{\mathrm{V}}=\frac{4 h}{3}-12 \sin \alpha \end{aligned}$ <br> Second A 1 for $\beta=35.4^{\circ} \quad\left(35^{\circ}\right) \quad$ ( 0.617 rads or 0.62 rads) |  |
|  | 6(d) <br> First M1 for a strategy to solve the problem <br> Second DM1, dependent on first M, for attempt to set up an equation based on the strategy <br> First A1 for a correct equation in $t$. <br> Second A1 for 0.94 or 0.945 |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 7a | (A) $2 m$ <br> (B) $3 m$ <br> (C) $4 m$ $v \longrightarrow$ $\begin{aligned} & w \longrightarrow \\ & v \longrightarrow \end{aligned}$ <br> $W \longrightarrow$ |  |
|  | CLM: $\quad 2 m u=2 m v+3 m w$ | M1A1 |
|  | Impact law: $\quad e u=w-v$ | M1A1 |
|  | Eliminate $w$ : $5 v=u(2-3 e) \mathrm{oe}$ | M1 |
|  | KE: $\quad v=\frac{u}{3}$ | M1 |
|  | $e=\frac{1}{9}, 0.11$ or better | A1 |
|  | OR: $v=\frac{u}{3}$ is used in CLM and NIL equations (See Alternative below) |  |
|  | ALTERNATIVE: $\begin{array}{lll}  & v=\frac{u}{3} & \text { M1 } \quad \text { (KE) } \\ 2 m u=2 m v+3 m w & \left(2 m u=2 m \frac{u}{3}+3 m w\right) & \text { M1 A1 } \quad \text { (CLM) } \\ \Rightarrow w=\frac{4 u}{9} & & \text { M1 } \quad \text { (eliminate } w) \\ e=\frac{w-v}{u} & \left(e=\frac{\frac{4 u}{9}-\frac{u}{3}}{u}\right) & \text { M1 A1 (Impact Law) } \\ e=\frac{1}{9} & & \text { A1 } \end{array}$ |  |
| 7b(i) | Substitute $e\left(=\frac{1}{9}\right): v=\frac{u}{3}, \quad w=\frac{4}{9} u$ | M1 |
|  | $\begin{gathered} \text { CLM \& Impact equations: } 3 m \times w=3 m V+4 m W \quad\left(\frac{4}{3} u=3 V+4 W\right) \\ w f=W-V \quad\left(\frac{16 u}{9} f=4 W-4 V\right) \end{gathered}$ <br> Follow their $w$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
|  | Solve for $V$ : $V=\frac{4 u}{21}-\frac{16 u f}{63}$ <br> N.B. answer must be positive | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |
| 7b(ii) | since $f \geq 0, V \leq \frac{4 u}{21}<\frac{u}{3}$ and hence $A$ collides with $B$ | M1A1 |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | N.B. If $f=0$ is considered, to give $V=\frac{4 u}{21}<\frac{u}{3} \quad$ M1A0 | (7) |
|  |  | [14] |
|  | Notes for Qu7 |  |
|  | 7(a) <br> First M1 for CLM, usual rules <br> First A1 for a correct equation Second M1 for NIL, with $e$ on the correct side <br> Second A1 for a correct equation <br> Third M1 for eliminating $w$ <br> Fourth M1 (this is really a B mark) for use of $v=\frac{u}{3}$ oe <br> (M0 for use of $v=\frac{u}{9}$ oe) <br> Third A1 for $1 / 9,0.11$ or better |  |
|  | 7(b) <br> (i) <br> First M1 for using their $e$ to find $v$ and $w$ (can be done in (a) but must be used in (b)) <br> Second M1 for CLM and NIL equations for $2^{\text {nd }}$ collision <br> First A1 for two correct equations for their $w, V$ and $W$. <br> Third M1 for finding $V$ in terms of $f$ and $u$ <br> Second A1 for any correct expression (must be $>0$ ) <br> (ii) <br> Fourth M1 (only available if they have a correct expression for $V$ ) for a complete strategy to show that there is a $2^{\text {nd }}$ collision between $A$ and $B$, using $f \geq 0$ (Allow M1A0 if they use $f=0$, see above) <br> Third A1 for a fully correct justification. N.B. GIVEN ANSWER <br> (ii) Alternative: <br> Fourth M1 (only available if they have a correct expression for $V$ ) for a complete strategy to show that there is a $2^{\text {nd }}$ collision between $A$ and $B$ : $\begin{array}{cr} 2^{\text {nd }} \text { collision } \quad \text { if } V<\frac{u}{3} \\ \text { i.e. if } & \frac{4 u}{21}-\frac{16 u f}{63}<\frac{u}{3} \\ \text { i.e. if } & \frac{-9}{16}<f . \end{array}$ <br> Since $f \geq 0$ (given), there will be a $2^{\text {nd }}$ collision. <br> Third A1 for a fully correct justification. N.B. GIVEN ANSWER. |  |

