Mark Scheme (Final)
October 2019

Pearson Edexcel IAL Mathematics
Pure Mathematics (1) (WMA11/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Pearson Mathematics mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.


## 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ or ft will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- d... or dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper or ag- answer given
- $\square$ or $\mathrm{d} \ldots$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A 1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.
8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ' 0 ' or ' 1 ' for each mark, or "trait", as shown:

|  | 0 | 1 |
| :---: | ---: | ---: |
| aM |  | $\bullet$ |
| aA | $\bullet$ |  |
| bM 1 |  | $\bullet$ |
| bA 1 | $\bullet$ |  |
| bB | $\bullet$ |  |
| bM 2 |  | $\bullet$ |
| bA 2 |  | $\bullet$ |

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the ' 0 ' column when it was meant to be ' 1 ' and all correct.

## General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

1. Factorisation

$$
\begin{aligned}
& \left(x^{2}+b x+c\right)=(x+p)(x+q), \text { where }|p q|=|c|, \text { leading to } x=\ldots \\
& \left(a x^{2}+b x+c\right)=(m x+p)(n x+q), \text { where }|p q|=|c| \text { and }|m n|=|a|, \text { leading to } x=\ldots
\end{aligned}
$$

## 2. Formula

Attempt to use correct formula (with values for $a, b$ and $c$ ).

## 3. Completing the square

Solving $x^{2}+b x+c=0: \quad\left(x \pm \frac{b}{2}\right)^{2} \pm q \pm c, \quad q \neq 0, \quad$ leading to $x=\ldots$

## Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^{n} \rightarrow x^{n-1}\right)$

## 2. Integration

Power of at least one term increased by 1. $\left(x^{n} \rightarrow x^{n+1}\right)$

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

## Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1.(a) | $\text { Sets } \begin{aligned} \frac{1}{2} r^{2} \times 1.25=15 & \Rightarrow r^{2}=24 \\ & \Rightarrow r=\sqrt{24} \text { or } 2 \sqrt{6} \text { (only) } \end{aligned}$ | M1 <br> A1 <br> (2) |
| (b) | $\begin{aligned} & \text { Attempts } s=r \theta=2 \sqrt{6} \times 1.25 \\ & \text { Attempts } \begin{aligned} P=2 r+r \theta & =2 \times 2 \sqrt{6}+2 \sqrt{6} \times 1.25 \\ & =\frac{13 \sqrt{6}}{2} \text { oe } \end{aligned} \end{aligned}$ | M1 <br> dM1 <br> A1 |
|  |  | (3) <br> (5 marks) |

(a)

M1 Uses $A=\frac{1}{2} r^{2} \theta$ in an attempt to find $r$
A1 $\quad r=\sqrt{24}$ or $2 \sqrt{6}$ only (oe) (isw after a correct answer is seen) Withhold the A if $r= \pm 2 \sqrt{6}$ is given.
(b)

M1 Uses the formula $s=r \theta$ with their $r$ and $\theta=1.25$ in an attempt to find the arc length.
dM1 For applying $P=2 r+r \theta$ their $r$ and $\theta=1.25$ in an attempt to find the perimeter.
A1 $\frac{13 \sqrt{6}}{2}$ (isw after a correct answer is seen). Accept $6.5 \sqrt{13}$ (oe simplest forms)

| Question <br> Number | Scheme | Marks |
| :---: | :--- | :--- |
| 2. (a) | $1.85=2 a+b$ and $3.45=7 a+b$ <br> Solves simultaneously to get $a=0.32, b=1.21 \quad$ (oe) <br> (b)States 1.21 m or 121 cm (oe) M1 A1 <br> dM1 A1  |  |

(a)

M1 For either $1.85=2 a+b$ or $3.45=7 a+b$
A1 For both $1.85=2 a+b$ and $3.45=7 a+b$
dM1 Solves simultaneously to get a value for $a$ and a value for $b$
A1 $a=0.32, b=1.21$ or equivalent fractions. May be seen in the equation.
(b)

B1 ft States 1.21 m or 121 cm (oe including units). Correct answer or follow through on their positive $b$
Alt part (a)
M1 Attempts $\frac{3.45-1.85}{7-2}$ Allow from attempts at use of arithmetic series, or from incorrect indexing. So
E.g. $1.85=3 a+b, 3.45=8 a+b \Rightarrow a=\ldots$ or $a_{n}=a+(n-1) d, a_{1}=1.85, a_{6}=3.45 \Rightarrow a=\ldots$ gain this mark.

A1 $\quad a=0.32$ (may be called $d$ if using AS)
dM1 Full correct method to find $b$ E.g substitutes their $a=0.32$ into either correct equation (with correct indexing), or in " $a_{n}=a+(n-1) d$ " finds " $a$ "(=1.53) and expands to find $b=" a "-" d$ ".
A1 $\quad a=0.32, b=1.21$

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 3. (a) | $\begin{gathered} x^{2}-5 x+13=(x-2.5)^{2}-2.5^{2}+13=(x-2.5)^{2}+6.75 \\ \text { Coordinates } M=(2.5,6.75) \end{gathered}$ | M1 A1 A1 |
| (b) | Attempts the equation of $l$ using their $M \quad y=\frac{6.75}{2.5} x \quad(y=2.7 x)$ | M1 |
|  | Attempts to solve their $y=2.7 x$ with $y=x^{2}-5 x+13$ $\begin{array}{r} \Rightarrow 2.7 x=x^{2}-5 x+13 \Rightarrow x^{2}-7.7 x+13=0 \Rightarrow(x-2.5)(x-5.2)=0 \\ x=5.2 \mathrm{oe} \\ \text { Coordinates } N=(5.2,14.04) \end{array}$ | M1 <br> A1 <br> dM1 A1 |
| (c) | States two of $y<x^{2}-5 x+13, y>2.7 x, 0 \leqslant x<2.5$ <br> States all three of $y<x^{2}-5 x+13, y>2.7 x, 0 \leqslant x<2.5$ | (5) <br> M1 <br> A1ft |
|  |  | (2) <br> (10 marks) |

(a)

M1 For attempting to complete the square. Look for $(x-2.5)^{2}$
A1 $\quad(x-2.5)^{2}+6.75$ or for correctly extracting $x=2.5$ as the $x$ coordinate.
A1 $\quad M=(2.5,6.75)$ or $\left(\frac{5}{2}, \frac{27}{4}\right)$
(b)

M1 Uses their $M$ to find an equation for $l$. Look for a correct attempt at the gradient, so $y=\frac{" 6.75 "}{" 2.5 "} x$ or $y-" 6.75 "=\frac{" 6.75 "}{" 2.5 "}(x-$ "2.5" $)$
M1 Depends on having made an attempt (not necessarily correct) to use $O$ and $M$ to find the equation of $l$. Attempts to solve their equation for $l$ with $y=x^{2}-5 x+13$ Look for a full attempt leading to $x=\ldots$

Correct answers following a correct simplified quadratic is fine for the method.
A1 $\quad x=5.2$ or equivalent such as $x=\frac{26}{5}$
dM1 Depends on second M mark. Substitutes their $x=5.2$ into either equation to find $y=\ldots$ If a $y$ value follows an $x$ value with no clearly incorrect working, then allow as an attempt.
A1 Coordinates $N=(5.2,14.04)$ oe such as $\left(\frac{26}{5}, \frac{351}{25}\right)$. Allow this to be written separately.
(c)

M1 States any two of $y<x^{2}-5 x+13, \quad y>" 2.7 " x, \quad 0 \leqslant x<b$ where " 2.5 " $\leqslant b \leqslant " 5.2$ "
Also allow the first two combined, or with loose inequalities, e.g " 2.7 " $x \leqslant y \leqslant x^{2}-5 x+13$
Also allow either $\leqslant$ or $<$ on the left hand end of $0 \leqslant x<2.5$ due to the $y$-axis.
Use of $\boldsymbol{R}$ instead of $\boldsymbol{y}$ is M0
A1ft States all three inequalities. Same conditions as above, follow through on their $2.7 x$ from (b) and 2.5
Allow part (a) to be attempted via finding minimum point
M1 For setting $\frac{\mathrm{d} y}{\mathrm{~d} x}=a x+b=0$ and finding a value for $x$, or for stating $x_{\text {min }}=-b / 2 a=\ldots$
A1 $x=2.5$
A1 $M=(2.5,6.75) \quad$ Note: Answer with no working score M0A0A0

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 4. (a) | $\begin{aligned} & \text { Area } A B C D \text { is } 40 \mathrm{~cm}^{2} \Rightarrow 40=6 \times 10 \times \sin \theta \quad \text { oe } \\ & \qquad \begin{aligned} \sin \theta & =\frac{2}{3} \Rightarrow \theta=180^{\circ}-41.8^{\circ} \\ \angle D A B & =\operatorname{awrt} 138.19^{\circ} \end{aligned} \end{aligned}$ | M1 <br> M1 <br> A1 |
| (b) | Attempts $D B^{2}=10^{2}+6^{2}-2 \times 10 \times 6 \cos " 138.19^{\circ} "$ $D B=$ awrt $15.0(\mathrm{~cm})$ | M1 <br> A1 |
|  |  | $\begin{array}{r} (2) \\ \text { (5 marks) } \end{array}$ |

(a)

M1 Scored for a correct attempt at using the area of $A B C D$ is $40 \mathrm{~cm}^{2}$
Score for $40=6 \times 10 \times \sin \theta$ or $20=\frac{1}{2} \times 6 \times 10 \times \sin \theta$ where $\theta$ is one of the corner angles.
M1 Score for $\sin \theta=k \Rightarrow \theta=180^{\circ}-\arcsin k$
A1 $\angle D A B=$ awrt $138.19^{\circ}$
(b)

M1 Attempts $D B^{2}=10^{2}+6^{2}-2 \times 10 \times 6 \cos " 138.19^{\circ}$ " - allow if the angle used is acute as long as it is clearly their attempt at angle $D A B$. So allow use of $41.8^{\circ}$ unless they have correctly found angle $D A B$ and chosen the wrong one here.
A1 $D B=$ awrt $15.0(\mathrm{~cm})$ Accept 15 in place of 15.0. Allow from attempts using awrt $138^{\circ}$

Alt for (a)
M1 Area $A B C D$ is $40 \mathrm{~cm}^{2} \Rightarrow h=\frac{40}{10}=\ldots \Rightarrow \sin \angle A B C=\frac{4 "}{6}$ OR $\cos \angle A B C=\frac{" 4 "}{6}$ oe
Essentially this mark is for using the area together with an appropriate trig identity to form an equation in the sine or cosine of one of the angles of the parallelogram. Attempts finding " $D X$ " where $X$ is where the perpendicular to $D C$ through $A$ meets $D C$ are possible.

M1 $\angle D A B=180^{\circ}-\arcsin \left(\frac{" 4 "}{6}\right)=\ldots \quad$ or may see $\angle D A B=90^{\circ}+\arccos \left(\frac{" 4 "}{6}\right)=\ldots$
This is for a complete correct method to find the angle $D A B$
A1 $\quad \theta=\operatorname{awrt} 138.19^{\circ}$

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :--- |
| 5.(a) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1}{2} x^{2}+2 x^{-\frac{1}{2}}$ | M1A1 A1 |
| (b) $\|$$\left.\frac{\mathrm{d} y}{\mathrm{~d} x}\right\|_{x=4}=\frac{1}{2} \times 4^{2}+2 \times \frac{1}{\sqrt{4}}=(9)$ <br> Gradient of normal is $-\frac{1}{9}$ <br> $y-\frac{11}{3}=-\frac{1}{9}(x-4) \Rightarrow x+9 y-37=0$ | M1 |  |
|  |  | MM1 A1 |
| (3) marks) |  |  |

(a)

M1 For reducing the power by one on any $x$ term
A1 Two correct terms which may be unsimplified (but ignore spurious extra terms like -15 for this mark)
A1 Fully correct and simplified. Accept exact simplified equivalents. $\operatorname{Eg} \frac{x^{2}}{2}+\frac{2}{\sqrt{x}}$
Withhold the final A mark if " +c " is included.
(b)

M1 For substituting $x=4$ into their $\frac{\mathrm{d} y}{\mathrm{~d} x}$
dM1 For the correct method of using the negative reciprocal to find the gradient of the normal.
M1 For an attempt at finding the equation of the normal. It is for using a changed gradient and the point $\left(4, \frac{11}{3}\right)$ If the form $y=m x+c$ is used they must proceed to $c=\ldots$
A1 $x+9 y-37=0$ or any (positive or negative) integer multiple thereof.
Accept with terms in a different order, but must include " $=0$ ". ISW after a correct answer.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 6. (a) | $y^{1}$ |  |
|  | Shape | B1 |
|  |  <br> States asymptote as $y=k$ | B1 |
|  | States intercept as $-\frac{4}{k}$ | B1 |
|  |  | (3) |
| (b) | $10-2 x=\frac{4}{x}+k \Rightarrow 10 x-2 x^{2}=4+k x$ | M1 |
|  | $\Rightarrow 2 x^{2}+(k-10) x+4=0$ | A1 |
|  | Attempts " $b^{2}-4 a c$ " $=0 \Rightarrow(k-10)^{2}-4 \times 2 \times 4=0$ | M1 |
|  | $k=10 \pm 4 \sqrt{2}$ oe | M1 A1 |
|  |  | $\begin{array}{r} (5) \\ \text { (8 marks) } \end{array}$ |

(a)

B1 Correct shape for graph. Look for a $y=\frac{1}{x}$ curve translated in any direction. Be tolerant with slips of pen how close the approach to asymptotes are, but the curve must not bend back on itself.
B1 Curve has horizontal asymptote above the $x$-axis with asymptote stated as $y=k$ on diagram or in text. (Do not accept just $k$ marked on the axis for this mark.)
B1 Curve crosses (not just touches) the negative $x$-axis, with intercept marked or stated as $-\frac{4}{k}$
(b)

M1 Equates and attempts to multiply by $x$ obtaining terms in $x^{2}, x$ and constant(s).
A1 $2 x^{2}+(k-10) x+4=0$ This may be implied by a correct $a, b$ and $c$
M1 Attempts " $b^{2}-4 a c$ " $=0 \Rightarrow(k-10)^{2}-4 \times 2 \times 4=0$ Withhold this mark if an inequality is applied.
M1 For a correct method of finding at least one value for $k$ from this attempt at the discriminant which is quadratic in $k$. Allow this mark for attempts using inequalities rather than equality, e.g. $b^{2}-4 a c<0$
A1 $k=10 \pm 4 \sqrt{2}$ oe (Do not accept decimal approximations and do not isw if an inequality is later stated.)

## Alt for 6(b)

M1 Attempts to set $\frac{\mathrm{d} y}{\mathrm{~d} x}=-2$ (usual rules for differentiation)
A1 Finds at least one value on the curve at which $\frac{\mathrm{d} y}{\mathrm{~d} x}=-2 \Rightarrow-\frac{4}{x^{2}}=-2 \Rightarrow x= \pm \sqrt{2} \quad($ accept awrt $\pm 1.41)$
dM1 Substitutes one of their $x$ values into $y=10-2 x$ to find at least one $y$ value. $\operatorname{Eg} x=\sqrt{2}, y=10-2 \sqrt{2}$ ddM1 Substitutes one of their $(x, y)$ values into $y=\frac{4}{x}+k$ and proceeds to find one value for $k$.
A1 $k=10 \pm 4 \sqrt{2}$

Examples for part (a)


B1B1B1 Pen slip rather than turning back


B1B1B0 Generous with a bit of gap between curve and asymptote


B0B1B1 Turning back on itself


B0B0B1 All four approaches bend away from the asymptote

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :--- |
| 7.(a) | Attempts $\frac{\mathrm{d} y}{\mathrm{~d} x}=4 x$ at $x=2$ <br> At $x=2$ gradient of tangent $=8$ | M1 |
| (b) | $\left(y_{Q}=\right) 2(2+h)^{2}+5$ <br> Gradient $P Q=\frac{\text { their } y_{Q}-13}{2+h-2}$ <br> $\left(=\frac{8 h+2 h^{2}}{h}\right)=8+2 h$ | B1 |
| (c) | States as $h \rightarrow 0$ Gradient $P Q \rightarrow 8=$ Gradient of tangent | A1 |

(a)

M1 Attempts to find the value of $\frac{\mathrm{d} y}{\mathrm{~d} x}=a x, \quad a>0$ at $x=2$
A1 For 8. No need to state this is the gradient.
(b)

B1 $\left(y_{Q}=\right) 2(2+h)^{2}+5$
M1 Attempts $\pm \frac{y_{Q}-y_{P}}{x_{Q}-x_{P}}$ condoning slips, but must be a genuine attempt at $y_{\mathrm{Q}}$
A1 Gradient is $8+2 h$ (with no errors seen)
(c)

B1 States as $h \rightarrow 0$ Gradient $P Q \rightarrow 8=$ Gradient of tangent (oe)
There should be reference to "limit" or "as $h$ tends to 0 " (words or symbols) and linked to part (a) (so same gradient, or showing the answers agree). But be generous with the explanation beyond these constraints.

| Question <br> Number | Scheme | Marks |
| :---: | :--- | :--- |
| $\mathbf{8}$ | $x-6 x^{\frac{1}{2}}+4=0$ |  |
|  | $x^{\frac{1}{2}}=3 \pm \sqrt{5}$ oe |  |
| $x=(3 \pm \sqrt{5})^{2} \Rightarrow x=14 \pm 6 \sqrt{5}$ |  | M1 A1 |
|  |  | M1 A1 A1 |
| (5 marks) |  |  |

M1 For attempting to solve an equation of the form $y^{2}-6 y+4=0$ by completing the square or quadratic formula to reach at least one solution. There must be some working shown for this mark to be awarded, accept as a minimum identifying $y=x^{\frac{1}{2}}$ and writing the quadratic in $y$ before solutions.
A1 $\quad\left(x^{\frac{1}{2}}\right)=3 \pm \sqrt{5}$ Both required (though one may be later rejected) but need not be simplified, so accept $\frac{6 \pm 2 \sqrt{5}}{2}$
M1 For attempting to square a solution of the form $p \pm q \sqrt{r}$ with 2 (out of 4) correct terms (may be implied by correct answers for their terms, but must have seen at least one solution for $x^{\frac{1}{2}}$ )
A1 $x=14+6 \sqrt{5}$ or $x=14-6 \sqrt{5}$ as an answer Accept equivalents for this mark.
A1 $x=14+6 \sqrt{5}$ and $x=14-6 \sqrt{5}$ as answers, must be simplified.
Special Case: For candidates who show no initial working and write $x^{\frac{1}{2}}=3 \pm \sqrt{5}$ as their first step, M0A0M1A1A1 is possible if they go on to achieve correct answers

| Question <br> Number | Scheme | Marks |
| :---: | :--- | :--- |
| 8 Alt | $x+4=6 x^{\frac{1}{2}}$ <br> $(x+4)^{2}=36 x$ <br> $x^{2}-28 x+16=0 \Rightarrow(x-14)^{2}=180 \Rightarrow x=14 \pm \sqrt{180} \Rightarrow x=14 \pm 6 \sqrt{5}$ | M1 A1 |
|  |  |  |

M1 Isolates the square root term and squares both sides.
A1 Correct squared expression, $(x+4)^{2}$ need not be expanded (as in scheme).
M1 Expands and solves the quadratic in $x$ Note that candidates who square term by term will score no marks.
A1A1 As main scheme. Note for the final A both solutions must be fully simplified.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 9. (a) | $24 \pi$ | B1 <br> (1) |
| (b) | $(18 \pi,-1)$ | B1ft |
| $(\mathbf{c})(\mathbf{i})$ | $-12 \pi-\alpha$ | (1) $\mathrm{B} 1 \mathrm{ft}$ |
|  | $6 \pi-\alpha$ | B1 ft |
|  |  | (2) <br> (4 marks) |

(a) Do not allow the mark if an inequality or coordinates are given, but you may assume if $0<x<p$ or $(p, 0)$ is given that their period is $p$ for the purposes of the remaining marks.

## See above

Note that (b) and (c) are for (correct or) follow through and hence
(b) is scored for $\left(\frac{3 p}{4},-1\right)$-- this may be seen on the graph but take clearly labelled part (b) as precedent.

Follow through on $p \neq 2 \pi$
(c) (i) is scored for $-\frac{1}{2} p-\alpha$
(ii) is scored for $\frac{1}{4} p-\alpha$
where $p$ is their period stated in (a) (which may be $2 \pi$ )
For answers in degrees penalise the first mark due only, but allow if degree symbol is missing.
For reference (a) $4320^{\circ}$ (b) $\left(3240^{\circ},-1\right)$ (c)(i) $-2160^{\circ}-\alpha$ (ii) $1080^{\circ}-\alpha$
Note for example that (a) $0<x<4320^{\circ}$ (b) $\left(3240^{\circ},-1\right)$ (c)(i) $-2160^{\circ}-\alpha$ (ii) $1080^{\circ}-\alpha$ would score B0 B0 B1ft B1ft

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| $\mathbf{1 0}$ (a) | $\mathrm{f}(x) \leqslant 0 \Rightarrow x \leqslant-\frac{5}{2}, \quad x=3$ | M1 A1 |


(a)

M1 For either $x \leqslant-\frac{5}{2}$ or $x=3 \quad$ Condone for this mark $x<-\frac{5}{2}$
A1 For both $x \leqslant-\frac{5}{2}$ and $x=3$ Accept answers given in set notation. Accept with "and" or "or" between - it is for both correct inequalities.
Note: Mark the final answer. Answers such as $-\frac{5}{2} \leqslant x \leqslant 3$ or $3 \leqslant x \leqslant-\frac{5}{2}$ are M0A0.
(b)

M1 Attempts to multiply two of the brackets together, achieving $x^{2}, x$ and constant terms.
M1 Multiplies the result by the third bracket to reach a four term cubic expression (not necessarily simplified).
A1 $2 x^{3}-7 x^{2}-12 x+45$ Ignore any reference to " $=0$ " or " $<0$ " etc for this mark.
(c)(i)

B1ft $\quad P(0,45)$ following through on their ' $d$ ' . Do not accept e.g. $(45,0)$ or just 45.
(c)(ii)

B1 ft Gradient $=-12$ following through on their ' $c$ '
(d)(i)

M1 Attempts to replace $x$ with $(x-2)$ in $(2 x+5)(x-3)^{2}$ or in their expansion of this.
Allow M1 for one correct bracket if no incorrect working is seen (bod). Condone invisible brackets
A1 $\mathrm{g}(x)=(2 x+1)(x-5)^{2}$ If substituting into the expand form, must factorise correctly to achieve A1. (d)(ii)

B1 25 Accept $(0,25)$.

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
|  |  |  |

11. (a) Attempts $y-\frac{32}{3}=5(x-4) \Rightarrow y=5 x-\frac{28}{3}$
(b)

$$
\begin{aligned}
& \mathrm{f}^{\prime \prime}(x)=\frac{4}{\sqrt{x}}-3 \Rightarrow \mathrm{f}^{\prime}(x)=8 x^{\frac{1}{2}}-3 x+k \\
& \text { Substitutes } x=4, \mathrm{f}^{\prime}(x)=5 \Rightarrow k=1 \\
& \mathrm{f}^{\prime}(x)=8 x^{\frac{1}{2}}-3 x+1 \Rightarrow \mathrm{f}(x)=\frac{16}{3} x^{\frac{3}{2}}-\frac{3}{2} x^{2}+x+d \\
& \text { Substitutes } x=4, \mathrm{f}(x)=\frac{32}{3} \Rightarrow d=-12 \\
& \qquad \mathrm{f}(x)=\frac{16}{3} x^{\frac{3}{2}}-\frac{3}{2} x^{2}+x-12
\end{aligned}
$$


(2)
(a)

M1 Uses gradient of 5 at point $P\left(4, \frac{32}{3}\right)$ to form tangent. For example, $y-\frac{32}{3}=5(x-4)$
A1 $y=5 x-\frac{28}{3}$ Accept recurring decimal, but $y=5 x-9.33$ is A0.
(b)

M1 Attempts to integrate $\frac{4}{\sqrt{x}}-3$ with one index correct
A1 $\quad\left(\mathrm{f}^{\prime}(x)\right)=\frac{4}{\sqrt{x}}-3 \rightarrow 8 x^{\frac{1}{2}}-3 x+k$ with or without the $+k$
dM1 Substitutes $x=4, \mathrm{f}^{\prime}(x)=5$ into an integrated form (with $+k$ ) and proceeds to find the value of $k$
A1 $\quad \mathrm{f}^{\prime}(x)=8 x^{\frac{1}{2}}-3 x+1$ which may be implied (allow if $k=1$ is found following a correct integral with $k$ ) dM1 Dependent upon the first M. It is for integrating 'again' with one term correct
A1 $\mathrm{f}(x)=\frac{16}{3} x^{\frac{3}{2}}-\frac{3}{2} x^{2}+k x+d$ following through on their $k \neq 0$ (which may be letter or number) and with or without $d$ (Both constants may be called $c$ for this mark.)
ddM1 Dependent upon $1^{\text {st }}$ and third M's (ie having attempted to integrate twice), and both " $k$ " and " $d$ " must have been added and processed correctly. Note " $c x+c$ " will score M0 as this is not a correct process for both constant
This mark is scored for using $x=4, \mathrm{f}(x)=\frac{32}{3}$ in an attempt to find ' $d$ '
A1 $\mathrm{f}(x)=\frac{16}{3} x^{\frac{3}{2}}-\frac{3}{2} x^{2}+x-12$ oe $\left(x \sqrt{x}\right.$ instead of $x^{\frac{3}{2}}$ is fine $)$

