3H

EDEXCEL IGCSE

Pearson Edexcel International GCSE

MATHEMATICS A

SOLUTIONS

JANUARY 2012

4MA0/3H

Disclaimer

These solutions have been produced by Maths4Everyone Limited. While We have used reasonable endeavours to verify the accuracy of these solutions, these solutions are provided on an "as is" basis and We make no warranties of any kind, whether express or implied, in relation to these solutions.

We make no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

We request that You use the 'contact' link on Our web site to inform Us of any errors or omissions that You find. We will update these solutions and correct errors that We become aware of. We recommend that You check Our web site for the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

In January 2007 the population of Canada was 32 million.
7 million of these Canadian people spoke French as their first language.

(a) Express 7 million as a percentage of 32 million.
Give your answer correct to 1 decimal place.

$\frac{7}{32} \times 100 = 21.875$

M1

21.9 B1 %

(2)

Between January 2007 and January 2009 the population of Canada increased by 4%.

(b) Increase 32 million by 4%.
Give your answer correct to the nearest million.

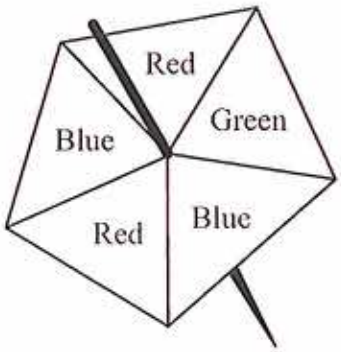
$32 \times 1.04 = 33.28$ MILLION

M1 B1

33 B1 million

(3)

Here is a fair 5-sided spinner.



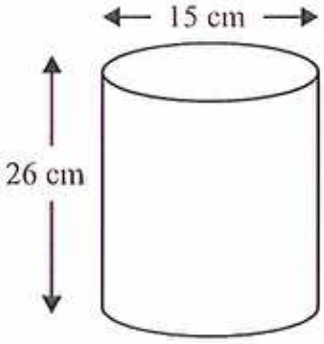
Hans spins the spinner 30 times.

Work out an estimate for the number of times the spinner lands on Red.

$P(\text{RED}) = \frac{2}{5}$ (B1)

$E(\text{RED}) = \frac{2}{5} \times 30$

$= \underline{\underline{12}}$ (A1)



$r = 7.5$

Diagram **NOT** accurately drawn

A cylinder has a diameter of 15 cm and a height of 26 cm.

Work out the volume of the cylinder.
Give your answer correct to 3 significant figures.

$V = \pi r^2 h$ B1

$= \pi \times 7.5^2 \times 26$ M1

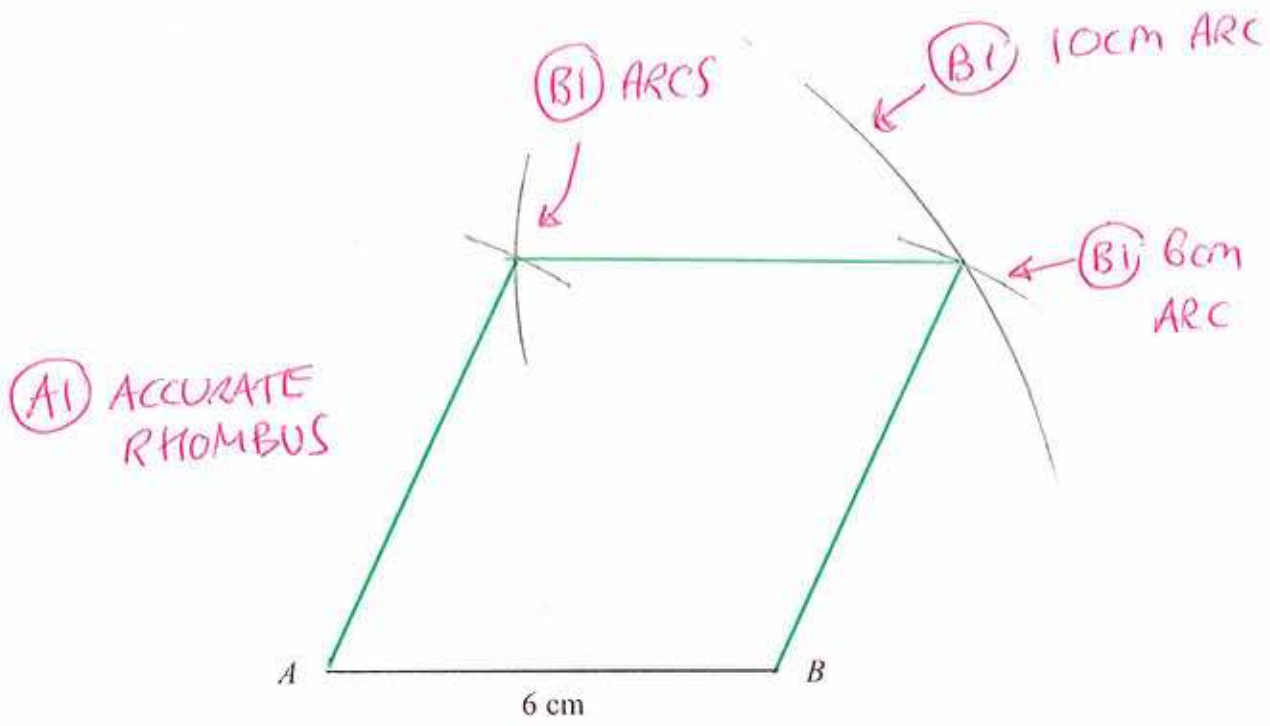
$= 4594.57...$

$= 4590$

4590 A1 cm^3

The lengths of the sides of a rhombus are 6 cm.
The length of the longer diagonal of the rhombus is 10 cm.
AB is a side of the rhombus.

Construct an accurate, full-size drawing of the rhombus.
You must show all construction lines.



(a) Factorise $5a - 3a^2$

A1 A1

$a(5 - 3a)$

(2)

(b) Expand

(i) $2(4 - 3w)$

B1

$8 - 6w$

(ii) $y^2 (y + 10)$

B1 B1

$y^3 + 10y^2$

(3)

(c) $W = \dfrac{5.6a}{b^2}$

$a = 1.28 \quad b = 0.8$

Work out the value of W.

$W = \dfrac{5.6 \times 1.28}{0.8^2}$ M1

$= 11.2$

$W =$ 11.2 B1

(2)

(a) $\mathscr{E}$ = {Students in Year 12}
G = {Students who study German}
F = {Students who study French}
M = {Students who study Maths}

(i) $G \cap M = \emptyset$

Use this information to write a statement about the students who study German in Year 12

NO STUDENTS STUDY BOTH GERMAN AND MATHS IN YEAR 12. (A1)

(ii) Preety is a student in Year 12
Preety $\notin F$.

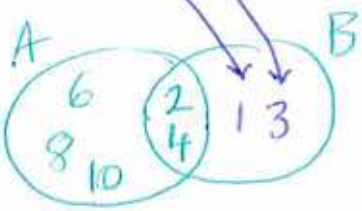
Use this information to write a statement about Preety.

PREETY DOES NOT STUDY FRENCH (A1)

(2)

(b) A = {2, 4, 6, 8, 10}
$A \cap B$ = {2, 4}
$A \cup B$ = {1, 2, 3, 4, 6, 8, 10}

List all the members of set B.



$B = \{1, 2, 3, 4\}$ (A2)

(2)

The table shows information about the numbers of text messages sent by 40 teenagers in one day.

Number of text messages	Number of teenagers (f)	Mid-interval value (x)	f x x
0 to 2	3	1	3
3 to 5	6	4	24
6 to 8	10	7	70
9 to 11	15	10	150
12 to 14	5	13	65
15 to 17	1	16	16
		TOTAL =	328

M1 B1

(a) Write down the modal class.

9 to 11 A1

(1)

(b) (i) Work out an estimate for the mean number of texts sent by the 40 teenagers in one day.

M1 $\frac{328}{40} = 8.2$

8.2 A1

(ii) Explain why your answer to part (b)(i) is an estimate.

THE MID-INTERVAL VALUE WAS USED BECAUSE THE EXACT VALUES WERE NOT KNOWN. A1

A bag contains 60 beads.
x of the beads are red and the rest are green.
Altaaf takes at random a bead from the bag.

(a) State, in terms of x, the probability that Altaaf takes a red bead.

$\frac{x}{60}$ B1

(1)

Altaaf puts his bead back in the bag.
Another 20 **red** beads are added to those in the bag.
The probability that Altaaf takes a red bead is now doubled.

(b) (i) Use this information to write down an equation in x
and show that your equation can be expressed as $8x = 3(x + 20)$

$\frac{x+20}{80} = 2 \times \frac{x}{60}$ M1

$\Rightarrow x + 20 = \frac{160x}{60}$ M1

$\Rightarrow x + 20 = \frac{8x}{3}$ M1

$3(x+20) = 8x$

$\Rightarrow 8x = 3(x+20)$

QED

(ii) Solve $8x = 3(x + 20)$
Show your working clearly.

$8x = 3x + 60$
$5x = 60$
$x = \frac{60}{5}$
$= 12$

M1 EITHER

$x =$ 12 A1

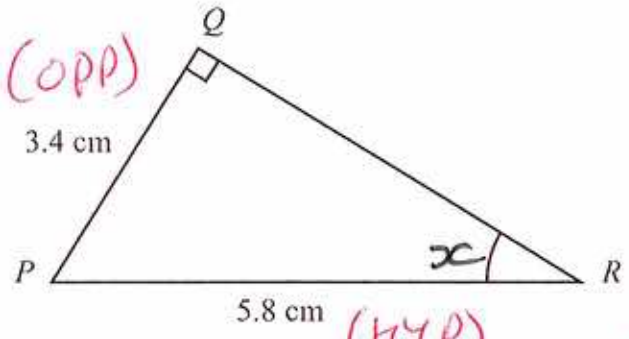


Diagram **NOT** accurately drawn

SOH CAH TOA

$\sin x = \frac{OPP}{HYP}$

Triangle PQR has a right angle at Q.

$PQ = 3.4$ cm and $PR = 5.8$ cm.

(a) Work out the size of angle QRP.
Give your answer correct to 1 decimal place.

$\sin x = \frac{3.4}{5.8}$

$x = \sin^{-1}\left(\frac{3.4}{5.8}\right)$

$= 35.888$

35.9 °

(3)

The length 5.8 cm, of PR, is correct to 2 significant figures. (1 decimal place)

(b) (i) Write down the upper bound of the length of PR.

5.8 ± 0.05

$5.8 + 0.05$

5.85 cm

(ii) Write down the lower bound of the length of PR.

$5.8 - 0.05$

5.75 cm

(2)

A bank pays compound interest of 6% per annum on its savings accounts.
Julia invests $7500 for 3 years.

Calculate the total interest gained after 3 years.

$7500 \times 1.06^3 = 8932.62$

M1 A1

INTEREST

8932.62
−7500.00
1432.62

A1

$ 1432.62

Make y the subject of $3(y + 2x - 1) = x + 5y$

$3y + 6x - 3 = x + 5y$ (M1)

$3y - 5y = x - 6x + 3$

$-2y = -5x + 3$ (M1)

$2y = 5x - 3$

$y = \frac{5x - 3}{2}$

$y =$ $\frac{5x - 3}{2}$ (A1)

OR $2.5x - 1.5$

ABCD and *APQR* are two similar quadrilaterals.

PQ = 9 cm.
BC = 6 cm.
AD = 5 cm.
QR = 12 cm.

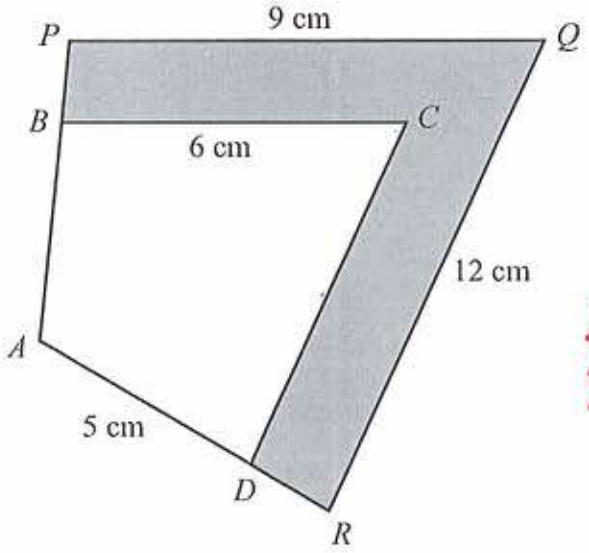


Diagram **NOT** accurately drawn

SF = 9/6 = 1·5

(a) Find the length of *DC*.

12 / 1·5 (M1)

8 (A1) cm

(2)

(b) Find the length of *AR*.

5 × 1·5 (M1)

7·5 (A1) cm

(2)

The area of the quadrilateral *ABCD* is 32 cm².

(c) Calculate the area of the shaded region.

AREA OF APQR = 32 × 1·5² (M1)

= 72

SHADED REGION = 72 − 32 (M1)

= 40 cm² (A1)

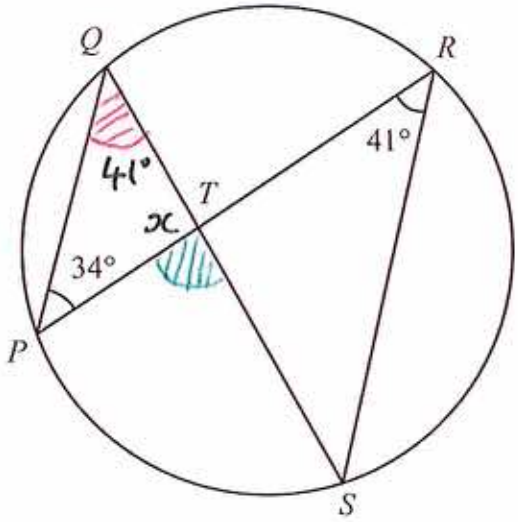


Diagram **NOT** accurately drawn

P, Q, R and S are points on the circumference of a circle.
PR and QS intersect at T.
Angle $QPR = 34°$ and angle $PRS = 41°$

(a) (i) Find the size of angle PQS.

41° .....

(ii) Give a reason for your answer.

ANGLES IN THE SAME SEGMENT ARE EQUAL

(2)

(b) (i) Find the size of angle PTS.

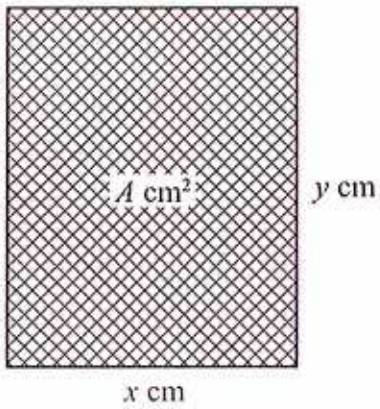
$x = 180 - (41 + 34)$
$= 105$

$P\hat{T}S = 180 - 105$
$= 75$

75 .....

(ii) Explain why T cannot be the centre of the circle.

BECAUSE $P\hat{T}S$ IS NOT TWICE THE ANGLE AT THE CIRCUMFERENCE (41°)



The diagram shows a rectangular photo frame of area A cm^2.
The width of the photo frame is x cm.
The height of the photo frame is y cm.
The perimeter of the photo frame is 72 cm.

$\Rightarrow 2x + 2y = 72$ (B1)

$\hookrightarrow 2y = 72 - 2x$

$y = 36 - x$ (B1)

(a) Show that $A = 36x - x^2$

$A = x \times y$

$= x \times (36 - x)$ (B1)

$= 36x - x^2$

(3)

(b) Find $\frac{dA}{dx}$

(A1) (A1)

$36 - 2x$

(2)

(c) Find the maximum value of A.

AT TURNING POINTS $\frac{dA}{dx} = 0$

$36 - 2x = 0$ (M1)

$\Rightarrow 36 = 2x$

$\Rightarrow x = 18$ (A1)

$\therefore$ MAXIMUM AREA, $A = 36x - x^2 \quad (x = 18)$

$= 36 \times 18 - 18^2$

$= 324 \text{ cm}^2$ (A1)

Two small magnets attract each other with a force, F newtons.
F is inversely proportional to the square of the distance, d cm, between them.

When $d = 2$, $F = 12$

(a) Express F in terms of d.

$F = \frac{k}{d^2}$ (B1) $(d = 2, F = 12)$

$\Rightarrow 12 = \frac{k}{2^2}$

$\Rightarrow k = 12 \times 2^2$

$= 48$ (B1)

$F = \frac{48}{d^2}$ (A1)

(3)

(b) Calculate the value of F when $d = 5$

$F = \frac{48}{d^2}$ $(d = 5)$

$F = \frac{48}{5^2}$

$= 1.92$

$F =$ 1.92 (A1)

(1)

(c) Calculate the value of d when $F = 3$

$F = \frac{48}{d^2}$ $(F = 3)$

$3 = \frac{48}{d^2}$

$\Rightarrow d^2 = \frac{48}{3}$ (M1)

$d = \sqrt{\frac{48}{3}}$

$= 4$

$d =$ 4 (A1)

(2)

The incomplete table shows information about the times, in minutes, that runners took to complete a race.

Time (t minutes)	$30 \leqslant t < 35$	$35 \leqslant t < 40$	$40 \leqslant t < 50$	$50 \leqslant t < 60$	$60 \leqslant t < 80$
Number of runners	12	20	30	12	16

(a) Use the histogram to calculate the number of runners who took between 40 and 50 minutes to complete the race.

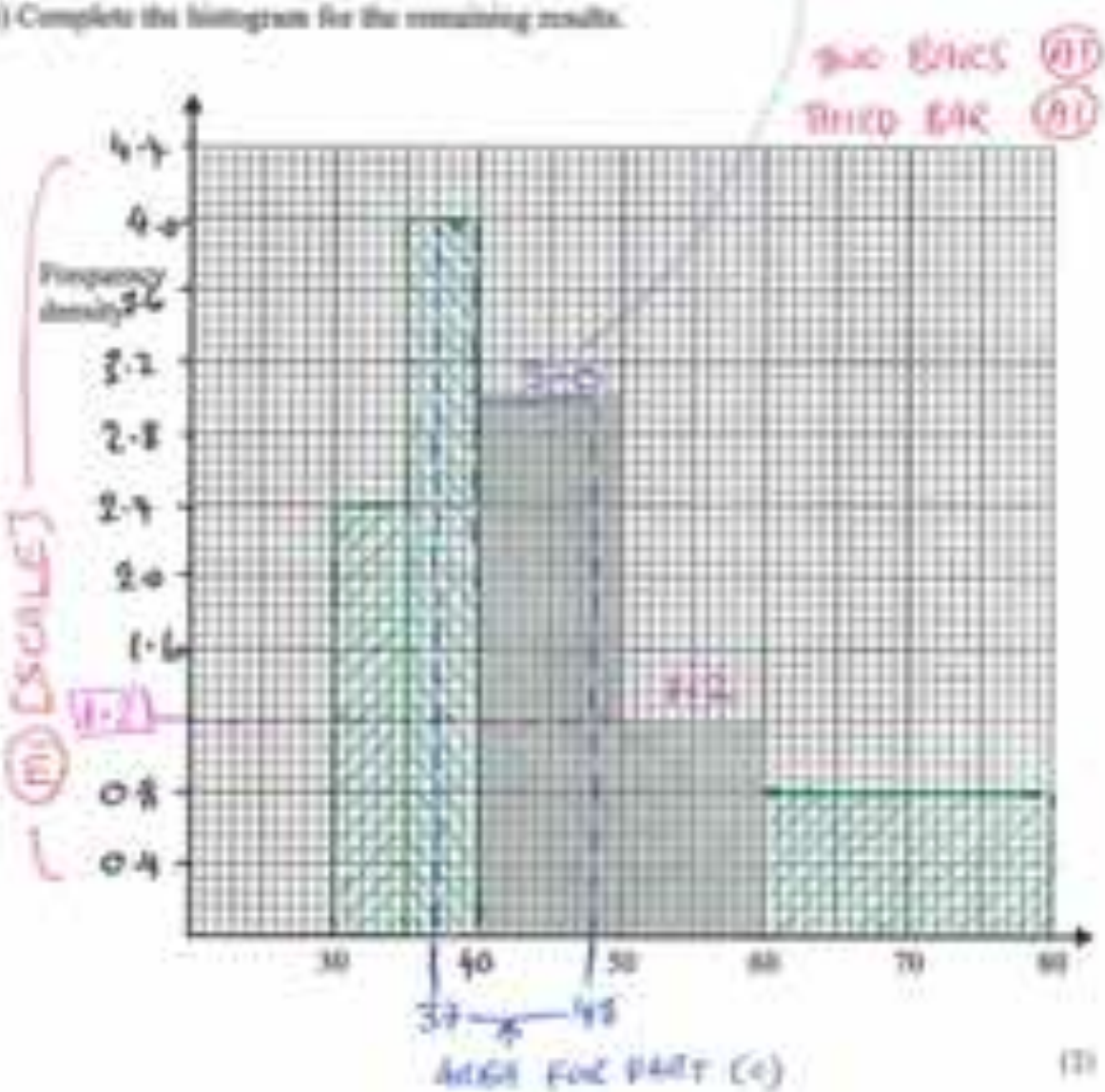
WIDTH	5	5	10	10	20
HEIGHT	$\frac{12}{5} = 2.4$	$\frac{20}{5} = 4$	3	$\frac{12}{10} = 1.2$	$\frac{16}{20} = 0.8$

THIS HAS GIVEN THE SCALE

30 (3)

(2)

(b) Complete the histogram for the remaining results.



(3)

Runners who achieved a time between 37 and 48 minutes to complete the race were each awarded a silver medal.

(ii) Calculate an estimate of the number of runners awarded silver medals.

$3 \times 4{\cdot}0 + 8 \times 3{\cdot}0$ (M1)

AREA BETWEEN 37 & 40

AREA BETWEEN 40 & 48

36 (M1)

(2)

Show that the recurring decimal $0.1\dot{7} = \frac{8}{45}$

ONE RECURRING DIGIT
∴ MULTIPLY BY 10

B1 $10x = 1.7777\ldots$

$x = 0.1777\ldots$

SUBTRACT $9x = 1.6$ MATCHING DECIMAL PART!

$\Rightarrow x = \frac{1.6}{9}$ M1 EITHER

$= \frac{16}{96}$ $= \frac{8}{45}$ QED!

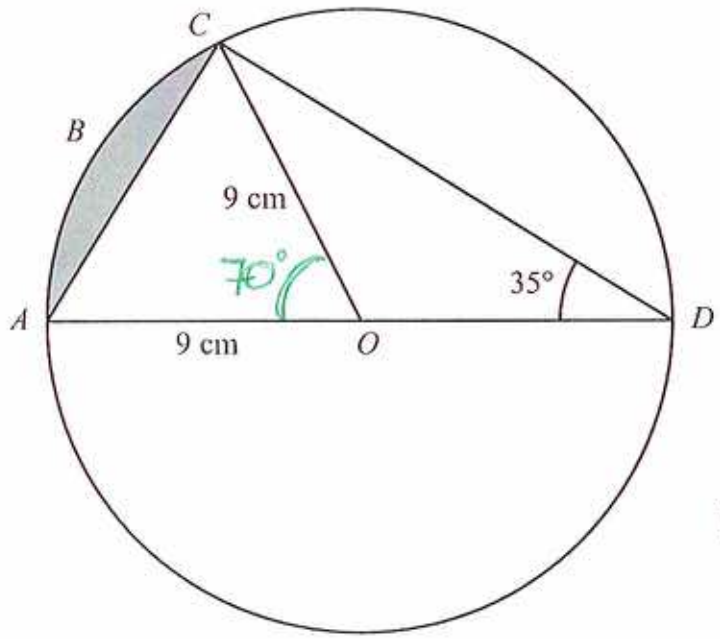


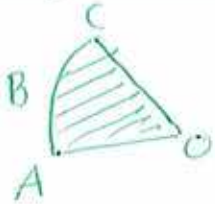
Diagram **NOT** accurately drawn

$A\hat{O}C = 70°$ (B1)

[ANGLE AT CENTRE = 2 × ANGLE AT CIRCUM.]

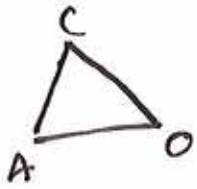
AOD is a diameter of a circle, with centre *O* and radius 9 cm.
ABC is an arc of the circle.
AC is a chord.
Angle *ADC* = 35°

Calculate the area of the shaded segment.
Give your answer correct to 3 significant figures.



AREA OF SECTOR $= \frac{70}{360} \times \pi \times 9^2$ (M1)

$= 49.480...$



AREA OF $\triangle AOC = \frac{1}{2} \times 9 \times 9 \times \sin 70$ (M1)

$= 38.057...$ (A1)

EITHER SECTOR OR TRIANGLE

∴ AREA OF SHADED SEGMENT

$49.480... - 38.057...$ (M1)

$= 11.423$

$= 11.4\ cm^2$ (A1)

Show that $\frac{\sqrt{3}+\sqrt{27}}{\sqrt{2}}$ can be expressed in the form $\sqrt{k}$ where k is an integer.

State the value of k.

$$\frac{\sqrt{3}+\sqrt{27}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{2}+\sqrt{27}\sqrt{2}}{\sqrt{2}\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{54}}{2}$$

$$= \frac{\sqrt{6}+3\sqrt{6}}{2}$$

$$= \frac{4\sqrt{6}}{2} = 2\sqrt{6} = \sqrt{4\times 6}$$

$$= \sqrt{24}$$

$k = 24$

Simplify fully $\frac{4}{x}+\frac{3}{2-x}$

$= \frac{4(2-x)+3x}{x(2-x)}$ M1

$= \frac{8-4x+3x}{x(2-x)}$ M1

$= \frac{8-x}{x(2-x)}$

$\frac{8-x}{x(2-x)}$ A1

21

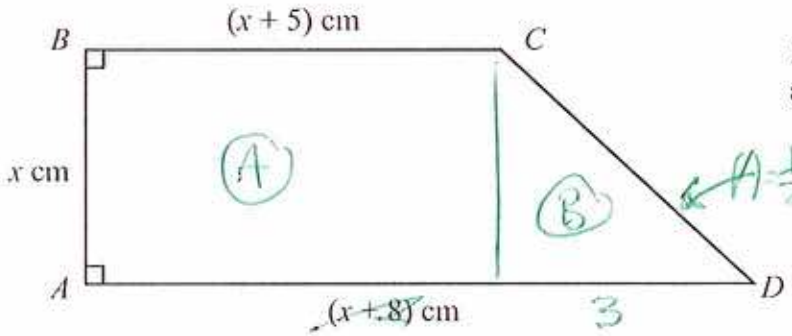


Diagram **NOT** accurately drawn

A = $\frac{1}{2} \times 3 \times x$

The diagram shows a trapezium $ABCD$ with AD parallel to BC.
$AB = x$ cm, $BC = (x + 5)$ cm and $AD = (x + 8)$ cm.
The area of the trapezium is 42 cm^2.

(a) Show that $2x^2 + 13x - 84 = 0$

A + B = 42

$x(x+5) + 1.5x = 42$ M1

$\Rightarrow x^2 + 5x + 1.5x = 42$ M1

$\Rightarrow x^2 + 6.5x = 42$

$\Rightarrow x^2 + 6.5x - 42 = 0$

$\Rightarrow 2x^2 + 13x - 84 = 0$

(2)

(b) Calculate the perimeter of the trapezium.

$2x^2 + 13x - 84 = 0$

$(2x + 21)(x - 4) = 0$ B1

$x = -\frac{21}{2}$ NOT POSSIBLE -VE

$x = 4$ A1

$CD = \sqrt{3^2 + x^2}$

$= \sqrt{9 + 16} = 5$ A1

PERIMETER $= (x + 8) + x + (x + 5) + CD$ M1

$= (4 + 8) + 4 + (4 + 5) + 5$

$= 30$ A1

30 cm

(5)