3H

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

JANUARY 2013

4MA0/3H

Disclaimer

These solutions have been produced by Maths4Everyone Limited. While We have used reasonable endeavours to verify the accuracy of these solutions, these solutions are provided on an "as is" basis and We make no warranties of any kind, whether express or implied, in relation to these solutions.

We make no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

We request that You use the 'contact' link on Our web site to inform Us of any errors or omissions that You find. We will update these solutions and correct errors that We become aware of. We recommend that You check Our web site for the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

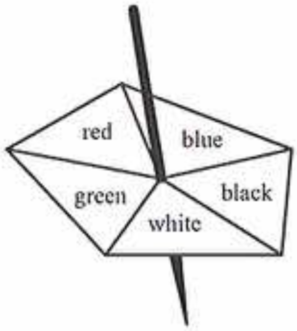
Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

Here is a biased 5-sided spinner.



When the spinner is spun, it can land on red, blue, black, white or green.
The probability that it lands on red, blue, black or white is given in the table.

Colour	red	blue	black	white	green
Probability	0.18	0.20	0.23	0.22	

George spins the spinner once.

(a) Work out the probability that the spinner lands on green.

B1

0.18
0.20
0.23
$+\ 0.22$
0.83

$1 - 0.83 = 0.17$

0.17 A1
(2)

Heena spins the spinner 40 times.

(b) Work out an estimate for the number of times the spinner lands on blue.

0.20×40

M1

8 A1
(2)

Rectangle **A** has a width of x metres and a height of $(x + 2)$ metres.
Rectangle **B** has a width of $2x$ metres and a height of $4x$ metres.

Diagram **NOT** accurately drawn

The perimeter of rectangle **A** is equal to the perimeter of rectangle **B**.

(i) Use this information to write down an equation in x.

$2x + 2(x+2) = 12x$ A1

....

(ii) Find the value of x.

$2x + 2x + 4 = 12x$ M1

$4x + 4 = 12x$

$4 = 8x$ M1

$x = \frac{4}{8}$

$= \frac{1}{2}$ A1

Joseph travels to work each day by train.
The weekly cost of his train journey is £45
Joseph's weekly pay is £625

(a) Work out 45 as a percentage of 625

$\frac{45}{625} \times 100$ M1

7.2 A1 %
(2)

(b) The weekly cost of his train journey increases by 8%.

Increase £45 by 8%.

45×1.08

M1 B1

£ 48.60 A1
(3)

(c) Joseph's weekly pay increases to £640

Calculate the percentage increase from 625 to 640

$640 - 625 = 15$ M1

$\frac{15}{625} \times 100 = 2.4$ M1

2.4 A1 %
(3)

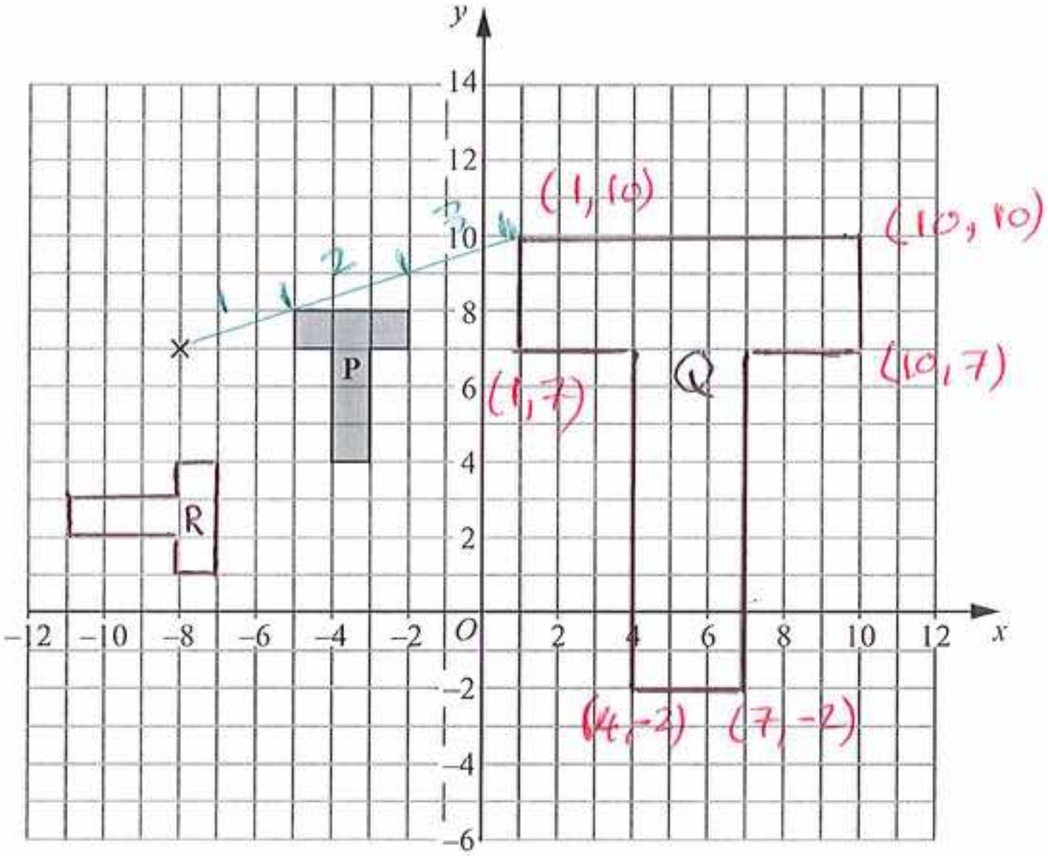
(d) Joseph decides to cycle to work.
He cycles 18 km to work.
His journey to work takes 1 hour 20 minutes. → $1.\dot{3}$ HOURS B1

Calculate his average speed in kilometres per hour.

$V = \frac{d}{t} = \frac{18}{1.\dot{3}}$ M1

$= 13.5$

13.5 A1 km/h



(a) On the grid, enlarge shape **P** with scale factor 3 and centre (−8, 7).
Label the new shape **Q**.

(3)

(b) On the grid, rotate shape **P** through 90° clockwise about the point (−8, 7).
Label the new shape **R**.

(2)

Solve the simultaneous equations

$$y - 2x = 6$$
$$y + 2x = 0$$

Show clear algebraic working.

$y - 2x = 6$ — (1)
$y + 2x = 0$ — (2)

ADD $2y = 6$ (M1)
$y = 3$

SUBT INTO $y = 3$ INTO (1)

$\Rightarrow 3 - 2x = 6$
$-2x = 3$
$x = -\frac{3}{2}$ (-1.5)

$x =$ -1.5 (A1)

$y =$ 3 (A1)

A school has 60 teachers.

The table shows information about the distances, in km, the teachers travel to school each day.

Distance (d km)	Frequency	MIDPOINT	$x \times f$
$0 < d \leqslant 5$	12	2.5	30
$5 < d \leqslant 10$	6	7.5	45
$10 < d \leqslant 15$	4	12.5	50
$15 < d \leqslant 20$	6	17.5	105
$20 < d \leqslant 25$	14	22.5	315
$25 < d \leqslant 30$	18	27.5	495

M1

M1

(a) Write down the modal class.

$25 < d \leqslant 30$ A1

(1)

(b) Work out an estimate for the total distance travelled to school by the 60 teachers each day.

$30 + 45 + 50 + \ldots + 495$

1040 A1 km

(3)

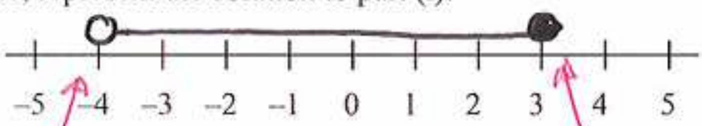
(i) Solve the inequalities $-2 < x + 2 \leqslant 5$

SUBTRACT 2 $-4 < x \leq 3$

A1 A1

$-4 < x \leq 3$

(ii) On the number line, represent the solution to part (i).

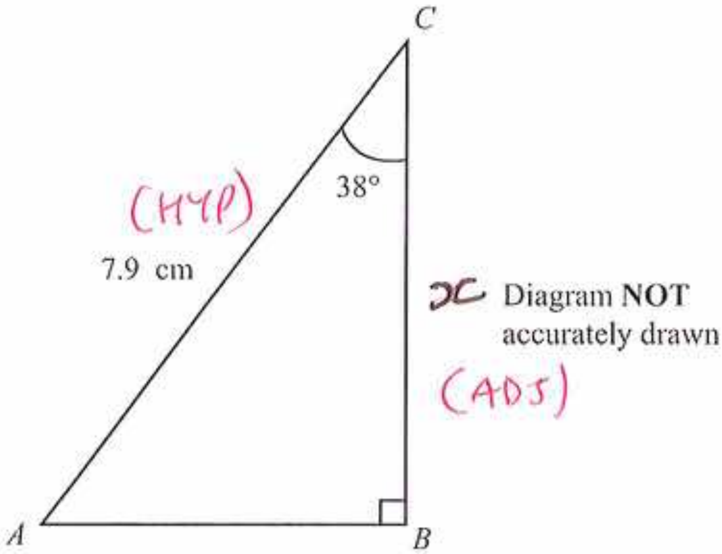


A1

MUST NOT BE SHADED

A1

MUST BE SHADED



ABC is a triangle.
$AC = 7.9$ cm
Angle $B = 90°$
Angle $C = 38°$

(a) Calculate the length of *BC*. (x)
Give your answer correct to 3 significant figures.

SOH CAH TOA

$\cos 38° = \frac{ADJ}{HYP}$

$\cos 38 = \frac{x}{7.9}$

$x = 7.9 \times \cos 38$

$= 6.22528...$

6.23 cm
(3)

(b) The size of angle *C* is 38°, correct to ~~2 significant figures~~. nearest whole number!

38 ± 0.5

(i) Write down the lower bound of the size of angle *C*.

$38 - 0.5$

37.5 °

(ii) Write down the upper bound of the size of angle *C*.

$38 + 0.5$

°

38.5

The table shows the diameters, in kilometres, of five planets.

Planet	Diameter (km)
Venus	1.2×10^4
Jupiter	1.4×10^5
Neptune	5.0×10^4
Mars	6.8×10^3
Saturn	1.2×10^5

(a) Which of these planets has the smallest diameter?

MARS B1

(1)

(b) Calculate the difference, in kilometres, between the diameter of Saturn and the diameter of Neptune.
Give your answer in standard form.

$$1.2 \times 10^5 - 5.0 \times 10^4$$

$$12 \times 10^4 - 5 \times 10^4 = 7 \times 10^4$$

M1 FOR SUBTRACTING

7×10^4 A1 km

(2)

The diameter of the Moon is 3.5×10^3 km.
The diameter of the Sun is 1.4×10^6 km.

(c) Calculate the ratio of the diameter of the Moon to the diameter of the Sun.
Give your answer in the form $1 : n$

$3.5 \times 10^3 : 1.4 \times 10^6$

$3500 : 1\,400\,000$

$35 : 14\,000$

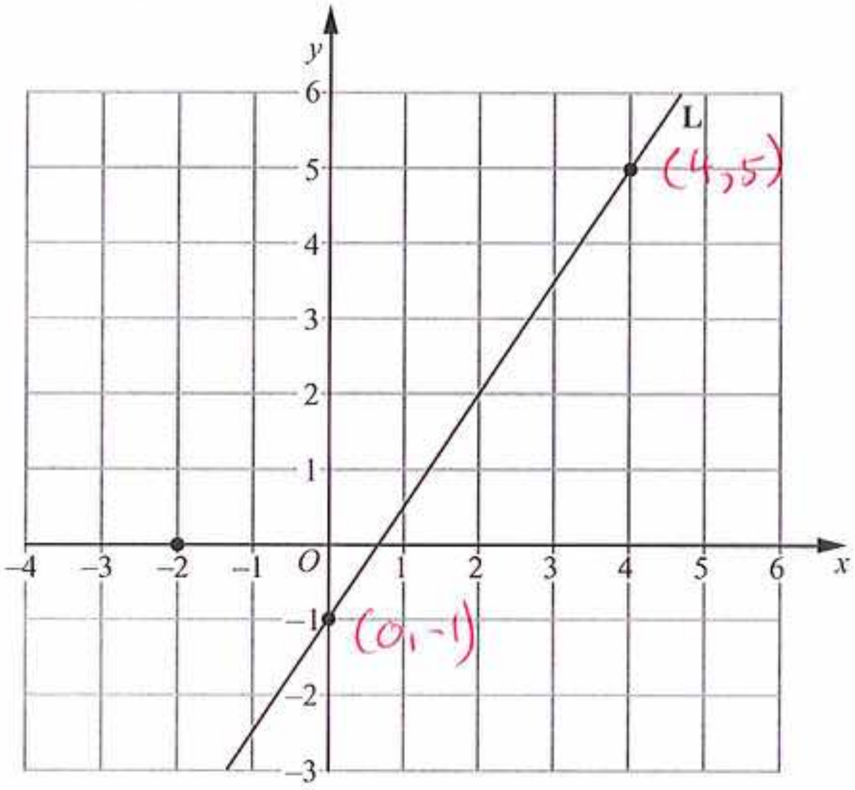
$5 : 2\,000$

$1 : 400$

M1 FOR ANY DIVISION

$1:400$ A1

The points (0, –1) and (4, 5) lie on the straight line **L**.



(a) Work out the gradient of **L**.

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{5 - -1}{4 - 0} \text{ (M1)} = \frac{6}{4} = 1.5$$

1.5 (A1)

(2)

(b) Write down an equation of **L**.

$y = 1.5x - 1$ (A1)

(1)

(c) Find an equation of the line which is parallel to **L** and passes through the point (– 2, 0)

$y = 1.5x + c \quad (x = -2, y = 0)$

$\Rightarrow 0 = 1.5 \times (-2) + c$ (M1)

$0 = -3 + c$

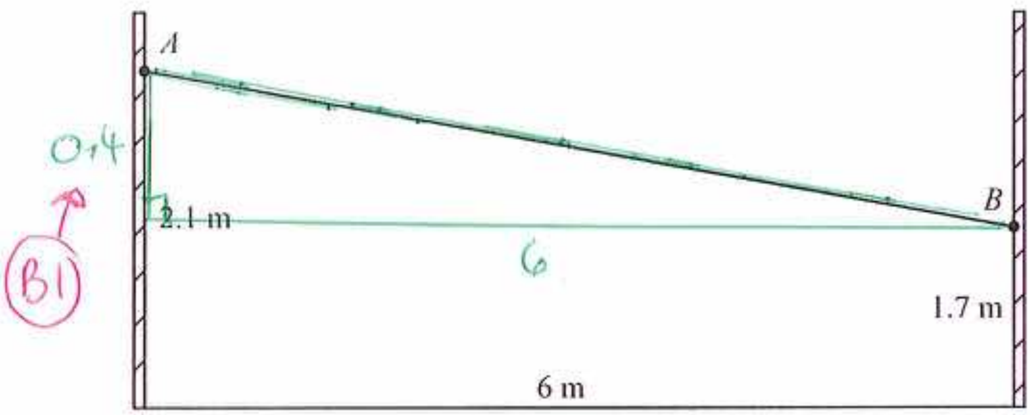
$c = 3$

$y = 1.5x + 3$ (A1)

(2)

A washing line is attached at points A and B on two vertical posts standing on horizontal ground.
Point A is 2.1 metres above the ground on one post.
Point B is 1.7 metres above the ground on the other post.
The horizontal distance between the two posts is 6 metres.

Diagram **NOT** accurately drawn



Calculate the distance AB.
Give your answer correct to 3 significant figures.

$AB^2 = 6^2 + 0.4^2 = 36.16$ M1

$AB = \sqrt{36.16}$ M1

$= 6.01331\ldots$

$= \underline{\underline{6.01}}$ m B1

Make h the subject of the formula $A = 2\pi r(r + h)$

$$A = 2\pi r^2 + 2\pi rh$$

$$\Rightarrow 2\pi rh = A - 2\pi r^2 \quad \text{M1}$$

$$\Rightarrow h = \frac{A - 2\pi r^2}{2\pi r} \quad \text{A1}$$

METHOD 2

$$r + h = \frac{A}{2\pi r} \quad \text{M1}$$

$$h = \frac{A}{2\pi r} - r \quad \text{A1}$$

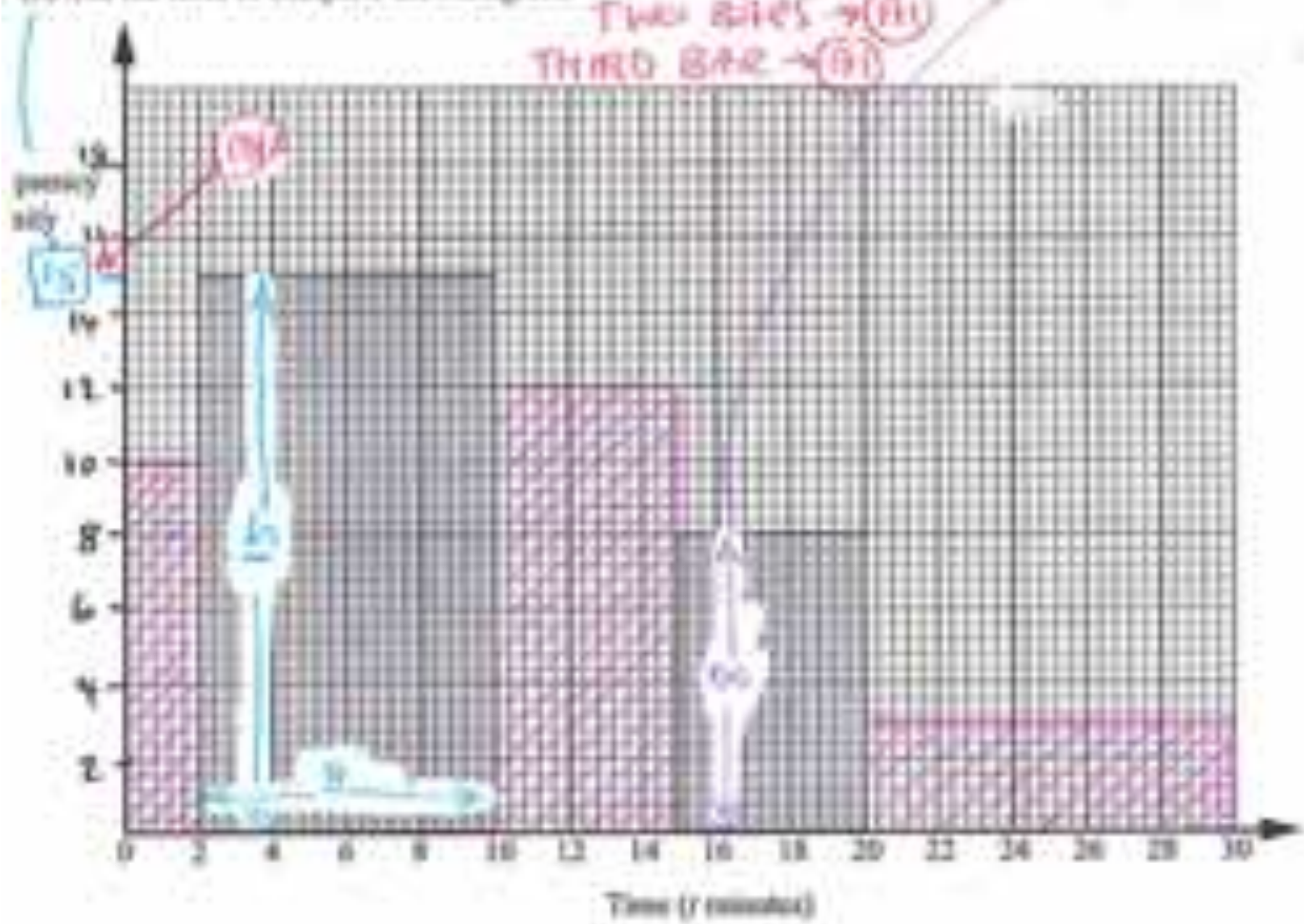
The incomplete table and histogram show information about the lengths of time, t minutes, students spent waiting for their school bus one morning.

Time (t minutes)	Number of students	WIDTH	HEIGHT
$0 < t \leqslant 2$	20	2	10
$2 < t \leqslant 10$	120	8	$\boxed{15}$
$10 < t \leqslant 15$	60	5	12
$15 < t \leqslant 20$	40 (A1)	5	8
$20 < t \leqslant 30$	30	10	3

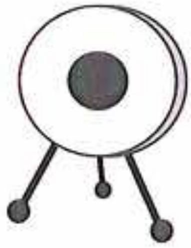
(i) Use the histogram to complete the table.

(ii) Use the table to complete the histogram.

TWO BARS → (A1)
THIRD BAR → (A1)

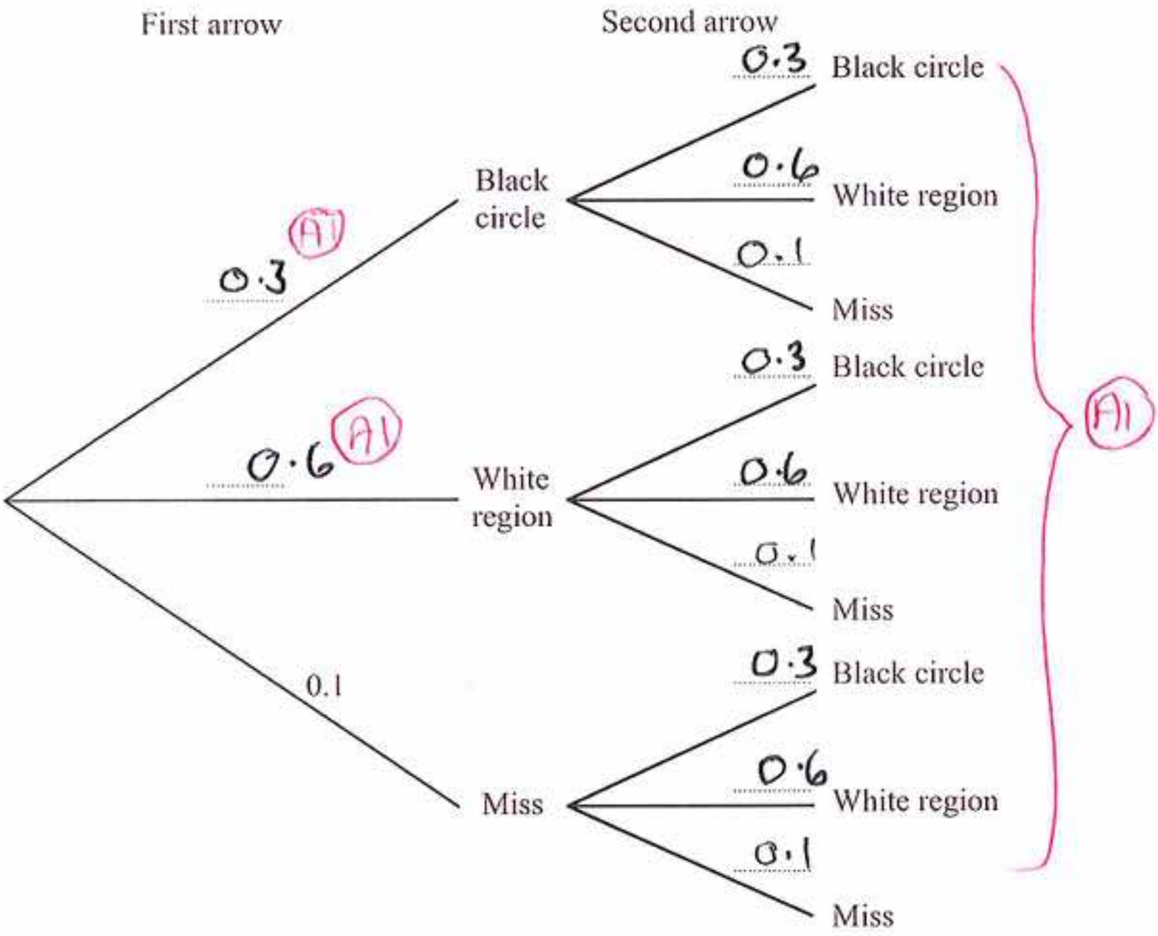


A target has a black circle and a white region.
Arrows can hit the black circle, the white region or miss the target.



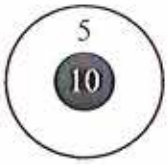
Peter shoots two arrows at the target.
On each shot, the probability that Peter's arrow misses the target is 0.1
On each shot, the probability that Peter's arrow hits the white region is twice the probability that it hits the black circle.

(a) Complete the probability tree diagram for Peter's two arrows.



(3)

(b) An arrow which hits the black circle scores 10 points.
An arrow which hits the white region scores 5 points.
An arrow which misses the target scores 0 points.



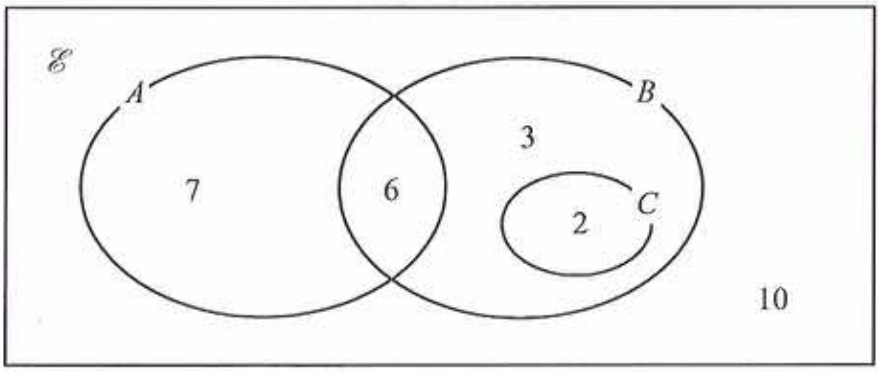
Calculate the probability that Peter scores exactly 10 points with his 2 arrows.

P(BM) = 0.3 × 0.1 = 0.03
P(MB) = 0.1 × 0.3 = 0.03
P(WW) = 0.6 × 0.6 = 0.36 A2

TOTAL = 0.42 B1

(2)

The Venn diagram shows a universal set $\mathscr{E}$ and three sets A, B and C.



7, 6, 3, 2 and 10 represent the **numbers** of elements.

Find

(i) $n(A \cup B)$

$7 + 6 + 3 + 2$

18 A1

(ii) $n(A')$

$3 + 2 + 10$

15 A1

(iii) $n(B \cap C')$

$6 + 3$

9 A1

(iv) $n(A' \cup B')$

$3 + 2 + 10 + 7$

22 A1

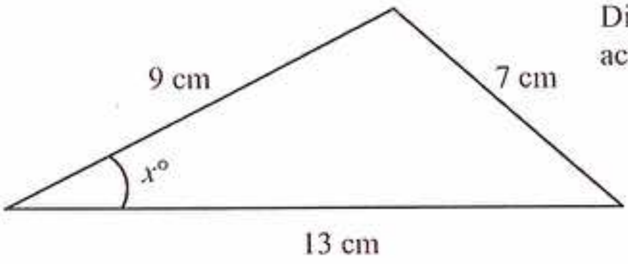


Diagram **NOT** accurately drawn

$$\cos x = \frac{b^2+c^2-a^2}{2bc}$$

Calculate the value of x.
Give your answer correct to 1 decimal place.

$$\cos x = \frac{13^2+9^2-7^2}{2\times13\times9}$$ (M1)

$$= 0.85897...$$

$$x = \cos^{-1}(0.85897...)$$ (M1)

$$= 30.798...$$

$x =$ 30.8 (B1)

Simplify fully $\frac{4x^2 - 25}{6x^2 + 13x - 5}$

$$= \frac{\cancel{(2x+5)}(2x-5)}{\cancel{(2x+5)}(3x-1)}$$ ← M1 FACTORISING ← M1 FACTORISING.

$$= \frac{2x-5}{3x-1}$$

$\frac{2x-5}{3x-1}$ A1

(a) Differentiate with respect to x

(i) $8x^2$

$16x$ (B1)

(ii) $\frac{2}{x}$ $= 2x^{-1}$ (M1)

$-2x^{-2}$ (B1)

(3)

(b) The curve with equation $y = 8x^2 + \frac{2}{x}$ has one turning point.

Find the coordinates of this turning point.
Show your working clearly.

$\frac{dy}{dx} = 16x - \frac{2}{x^2}$ (M1)

$16x - \frac{2}{x^2} = 0$

$16x = \frac{2}{x^2}$

$16x^3 = 2$

$x^3 = \frac{1}{8}$ (M1)

$x = \sqrt[3]{\frac{1}{8}}$

$= \frac{1}{2}$

$y = 8 \times \left(\frac{1}{2}\right)^2 + \frac{2}{\left(\frac{1}{2}\right)}$

$= 2 + 4$

$= 6$

(A1) (A1)

($\frac{1}{2}$, 6)

(4)

The diagram shows a rectangular playground of width x metres and length $3x$ metres.

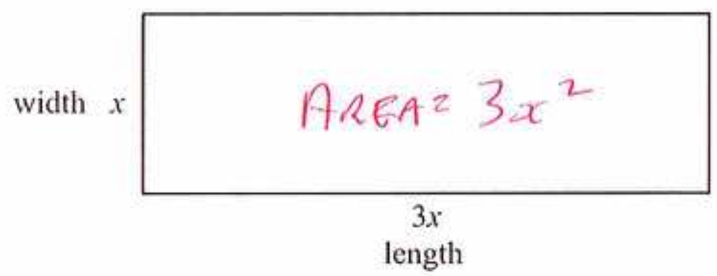
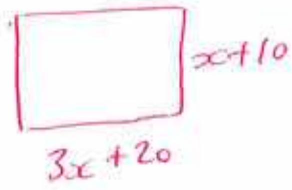


Diagram **NOT** accurately drawn

The playground is extended, by adding 10 metres to its width and 20 metres to its length, to form a larger rectangular playground.



The area of the larger rectangular playground is double the area of the original playground.

AREA = $6x^2$

(a) Show that $3x^2 - 50x - 200 = 0$

B1 $(3x+20)(x+10) = 6x^2$ B1

$3x^2 + 30x + 20x + 200 = 6x^2$

$6x^2 = 3x^2 + 50x + 200$ B1

$\Rightarrow$

$\Rightarrow 3x^2 - 50x - 200 = 0$

(3)

(b) Calculate the area of the original playground.

$3x^2 - 50x - 200 = 0$

$(3x+10)(x-20) = 0$

M1

$3x+10=0$

$3x = -10$

$x = -\frac{10}{3}$

$x - 20 = 0$ A1

$x = \underline{20}$ $\Rightarrow$ AREA = $3x^2$

$= 3 \times 20^2$ M1

$= \underline{\underline{1200\ m^2}}$ A1

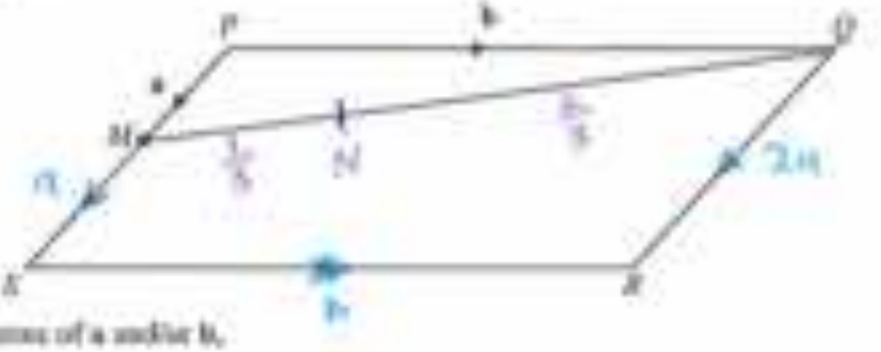
-VE VALUE IS NOT POSSIBLE

The diagram shows a parallelogram $PQRS$.

M is the midpoint of PS.

$\overrightarrow{PM} = \mathbf{a}$ $\quad \overrightarrow{PQ} = \mathbf{b}$

Diagram NOT accurately drawn



(a) Find, in terms of **a** and/or **b**,

(i) $\overrightarrow{PS}$

$2a$ (B1)

(ii) $\overrightarrow{PR}$ $= \overrightarrow{PQ} + \overrightarrow{QR}$

$= b + 2a$

$2a + b$ (B1)

(iii) $\overrightarrow{MQ}$ $= \overrightarrow{MP} + \overrightarrow{PQ}$

$= -a + b$

$-a + b$ (B1)

(3)

N is the point on MQ such that $MN = \frac{1}{3}MQ$

(b) Use a vector method to prove that PNR is a straight line.

$\overrightarrow{PN} = \overrightarrow{PM} + \overrightarrow{MN}$

$= \overrightarrow{PM} + \frac{1}{3}\overrightarrow{MQ}$

$= a + \frac{1}{3}(-a + b)$ (M1)

$= a - \frac{1}{3}a + \frac{1}{3}b$

$= \frac{2}{3}a + \frac{1}{3}b$

$= \frac{1}{3}(2a + b)$ (B1)

$= \frac{1}{3}\overrightarrow{PR}$

SINCE $\overrightarrow{PN} = k\overrightarrow{PR}$

PN AND PR ARE PARALLEL.

THEY ALSO SHARE A COMMON POINT, AND SO

PNR IS A STRAIGHT LINE

The diagram shows a pyramid with a horizontal rectangular base *PQRS*.
PQ = 16 cm.
QR = 10 cm.
M is the midpoint of the line *PR*.
The vertex, *T*, is vertically above *M*.
MT = 15cm.

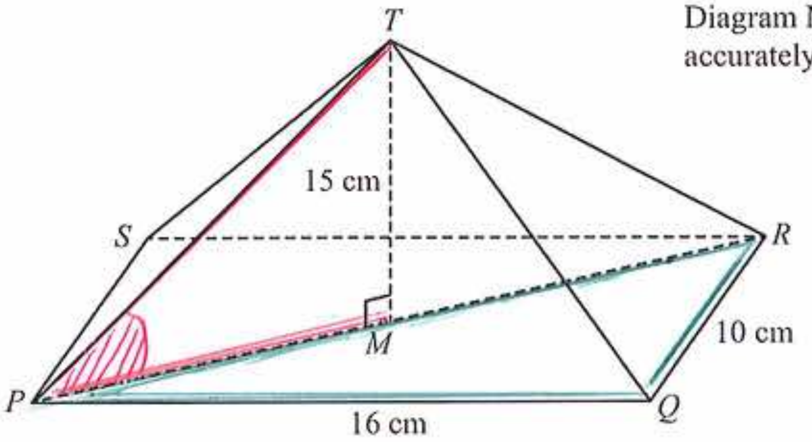
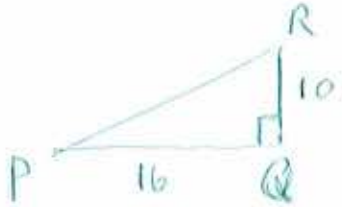


Diagram **NOT** accurately drawn

Calculate the size of the angle between *TP* and the base *PQRS*.
Give your answer correct to 1 decimal place.

(NEED TO KNOW PM)



$PR^2 = 10^2 + 16^2$

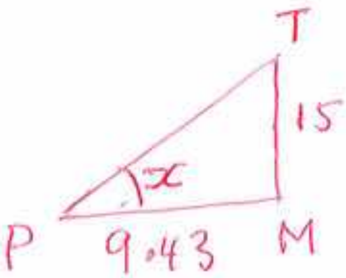
$= 356$ (M1)

$PR = \sqrt{356}$

$= 18.867\ldots$

$PM = \dfrac{18.867\ldots}{2}$

$= 9.4339\ldots$ (B1)



$\tan x = \dfrac{15}{9.43\ldots}$ (M1)

$x = \tan^{-1}\left(\dfrac{15}{9.43\ldots}\right)$

$= 57.832\ldots$

57.8 (A1) °