3H(R)

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

JANUARY 2014

4MA0/3HR

Disclaimer

These solutions have been produced by Maths4Everyone Limited. While We have used reasonable endeavours to verify the accuracy of these solutions, these solutions are provided on an "as is" basis and We make no warranties of any kind, whether express or implied, in relation to these solutions.

We make no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

We request that You use the 'contact' link on Our web site to inform Us of any errors or omissions that You find. We will update these solutions and correct errors that We become aware of. We recommend that You check Our web site for the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

The table shows information about the number of goals scored in each of the 25 matches in a hockey tournament.

Number of goals	Number of matches	x × f
1	6	6
2	8	16
3	7	21
4	3	12
5	1	5
	TOTAL =	60

M1

Work out the mean number of goals.

MEAN = $\frac{60}{25}$ M1

A1

2.4

The ratio of Mark's age to Reeta's age is 3 : 5
Mark's age is 24 years.

(a) Work out Reeta's age.

M : R
3 : 5

$\frac{24}{3} = 8$ (M1)

$5 \times 8 = 40$

40 (A1) years

(2)

The ratio of John's age to Zahra's age is 1 : 4
The sum of their ages is 45 years.

(b) Work out Zahra's age.

J : Z
1 : 4

TOTAL
5

$\frac{45}{5} = 9$ (M1)

$4 \times 9 = 36$ (A1)

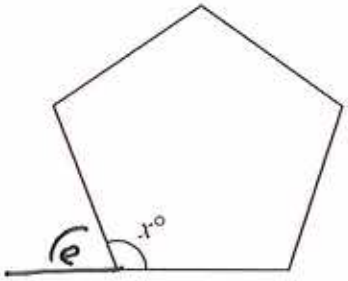


Diagram **NOT** accurately drawn

The diagram shows a regular 5-sided polygon.

(a) Work out the value of x.

EXTERIOR ANGLE $= \frac{360}{5}$

$= 72$ (M1)

INTERIOR ANGLE, $x = 180 - 72$

$= 108$

[LOTS OF OTHER METHODS, TOO]

(A1)

$x = 108$

(2)

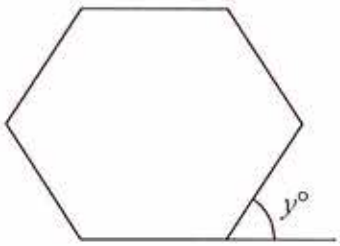


Diagram **NOT** accurately drawn

The diagram shows a regular 6-sided polygon.

(b) Work out the value of y.

y = EXTERIOR ANGLE $= \frac{360}{6}$ (M1)

$= 60^\circ$ (A1)

(a) Factorise $t^2 + 6t$

A1 A1

$t(t+6)$

(2)

(b) Solve $7x - 5 = 5x - 4$

Show clear algebraic working.

$7x - 5x = -4 + 5$ M1

$2x = 1$ M1

$x = \frac{1}{2}$

A1

$x = 0.5$

(3)

(c) Expand and simplify fully $4(2y + 3) + 2(y - 6)$

$8y + 12 \quad + 2y - 12$

M1

$10y$ A1

(2)

$\mathscr{E}$ = {even numbers} {2, 4, 6, 8, ...}
A = {factors of 8} → {1, 2, 4, 8}
B = {factors of 20} → {1, 2, 4, 5, 10, 20}

List the members of $A \cap B$

{2, 4}

A2

(a) Dilip buys a painting for $675
Later, he sells it and makes a percentage profit of 12%.

→ 0.12 (AMOUNT OF PROFIT)
→ 1.12 (FINAL AMOUNT)

Work out the price for which Dilip sells the painting.

675×1.12 (M1)

(B1)

$ 756 (A1)

(3)

(b) Renuka sells her car.
She makes a loss of $2162
Her percentage loss is 23%.

→ 0.23 (AMOUNT OF LOSS)
→ 0.77 (FINAL AMOUNT)

Work out the price for which Renuka sells her car.

$\frac{2162}{0.23}$ (M1) = 9400 (STARTING AMOUNT)

$9400 - 2162 = 7238$ (M1)

ANOTHER METHOD:
$\frac{2162}{23} \times 77$

$ 7238 (A1)

(3)

(c) Lin bought a computer that had a value of $1500
At the end of each year, the value of her computer had depreciated by 40% of its value at the start of that year.

→ 0.4 (DEPRECIATION)
→ 0.6 (FINAL)

Calculate the value of her computer at the end of 3 years.

1500×0.6^3 (M1)

(B1)

$ 324 (A1)

USE PYTHAGORAS

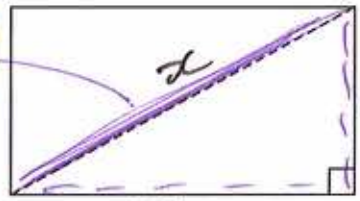


Diagram **NOT** accurately drawn

A TV screen is rectangular.
The width of the rectangle is 64.8 cm and the height is 48.6 cm.
The length of a diagonal of the rectangle gives the 'size' of the TV screen.

(a) Calculate the 'size' of the TV screen.

$x^2 = 64.8^2 + 48.6^2$ (M1)

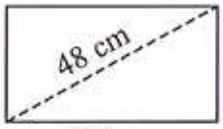
$= 6561$

$\Rightarrow x = \sqrt{6561}$ (M1)

(A1)

81 cm

(3)



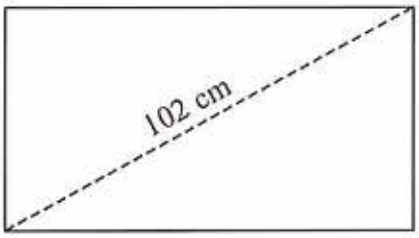


Diagram **NOT** accurately drawn

The diagram shows two rectangular TV screens.
The rectangles are similar.
The 'size' of the smaller screen is 48 cm.
The width of the smaller screen is 38.4 cm.
The 'size' of the larger screen is 102 cm.

FIND SCALE FACTOR!

$SF = \frac{102}{48} = 2.125$

(b) Calculate the width of the larger TV screen.

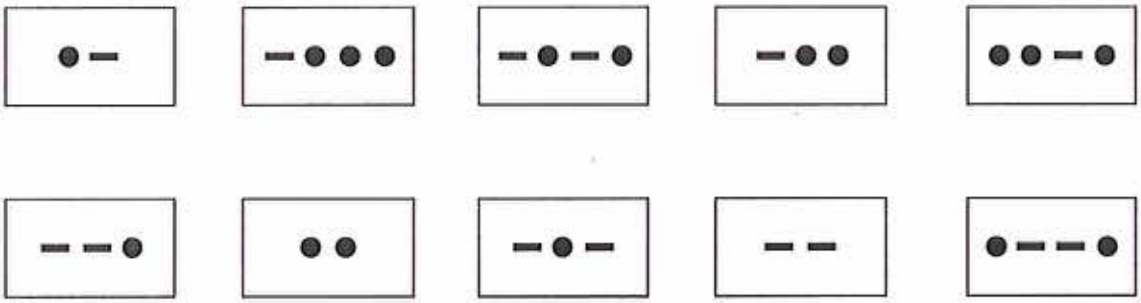
$WIDTH = 38.4 \times 2.125$

(B1)

(A1)

81.6 cm

Morse Code uses dots (●) and dashes (—) to represent each letter of the alphabet.
Here are 10 cards.
Each card has the Morse Code for a letter on it.



(a) Kelly takes at random one of the cards.

Find the probability that she takes a card with 2 dots or a card with 3 dots.

$\frac{6}{10}$ A1

$\frac{3}{5}$ A1

(2)

(b) Hashim has the 10 cards.
He takes at random a card 200 times.
He replaces the card each time.

Work out an estimate for the number of times he will take a card with exactly 2 dots.

$200 \times \frac{4}{10}$ M1

80 A1

(2)

(c) Shani takes at random two of the 10 cards without replacement.

Calculate the probability that

(i) there is exactly 1 dot on each card she takes,

$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$ A1

M1

$\frac{1}{15}$ A1

(3)

(a) Simplify $\frac{y^8}{y^3}$

y^5

(1) A1

(b) Solve the inequality $4(x + 3) > 8$

$4x + 12 > 8$ M1

$4x > 8 - 12$

$4x > -4$

$x > -1$

A1

$x > -1$

(2)

The grouped frequency table gives information about the lengths of time 160 students exercised one day.

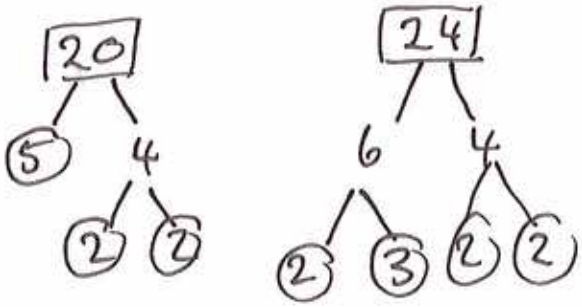
Time (t minutes)	Frequency
$0 < t \leqslant 40$	20
$40 < t \leqslant 80$	35
$80 < t \leqslant 120$	60
$120 < t \leqslant 160$	33
$160 < t \leqslant 200$	7
$200 < t \leqslant 240$	5

(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 40$	20
$0 < t \leqslant 80$	55
$0 < t \leqslant 120$	115
$0 < t \leqslant 160$	148
$0 < t \leqslant 200$	155
$0 < t \leqslant 240$	160

B1

Find the Lowest Common Multiple (LCM) of 20 and 24



$20 = 2^2 \times 5$

$24 = 2^3 \times 3$ } M1

$= 2^3 \times 3 \times 5$

$= 120$ A1

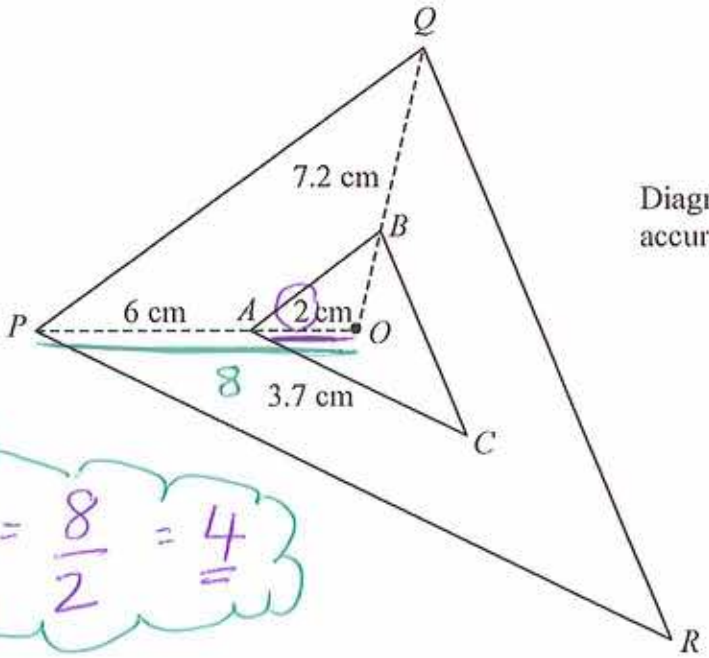


Diagram **NOT** accurately drawn

SCALE FACTOR = $\frac{8}{2} = \frac{4}{1}$

Triangle PQR is an enlargement, centre O, of triangle ABC.
OAP and OBQ are straight lines.
$OA = 2$ cm.
$AP = 6$ cm.
$BQ = 7.2$ cm.
$AC = 3.7$ cm.

SIMILAR SHAPES!

(a) Work out the length of OB.

BECAUSE CENTRE OF ENLARGEMENT IS KNOWN

$\frac{BQ}{AP} = \frac{7.2}{6} = 1.2$ (M1) $\therefore \frac{OB}{OA} = 1.2$

$\Rightarrow OB = 1.2 \times OA$
$= 1.2 \times 2$
$= 2.4$ (A1)

(b) Work out the length of PR.

(B1)

$PR = 3.7 \times 4$ (M1)
$= 14.8$ cm (A1)

(a) Solve the simultaneous equations $3x + 5y = 14$ ——— (1) ×3
$4x + 3y = 4$ ——— (2) ×5

Show clear algebraic working.

$9x + 15y = 42$ ——— (3) M1
$20x + 15y = 20$ ——— (4)

SUBTRACT: $-11x = 22$ M1

$x = \frac{22}{-11}$

$x = -2$

SUBSTITUTE INTO (1)

$3 \times (-2) + 5y = 14$

$5y = 20$

$y = 4$

$x =$ -2 A1

$y =$ 4 A1

(4)

(b) Write down the coordinates of the point of intersection of the two lines whose equations are $3x + 5y = 14$ and $4x + 3y = 4$

(-2 , 4) A1

(1)

$A = 2\pi rh$

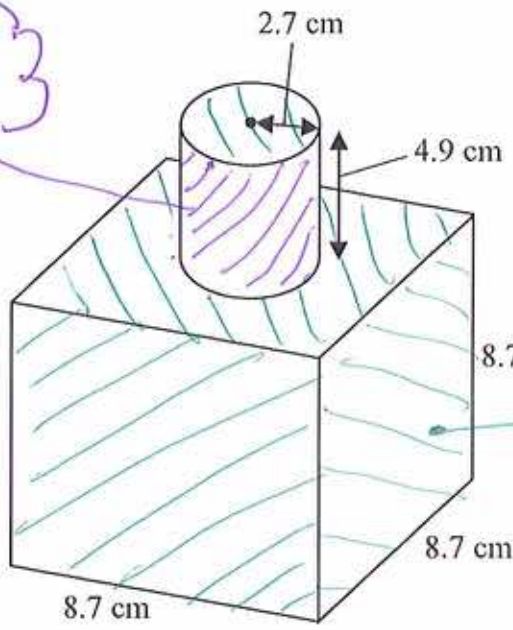


Diagram **NOT** accurately drawn

6×8.7^2

The diagram shows a shape made from a solid cube and a solid cylinder.
The cube has sides of length 8.7 cm.
The cylinder has a radius of 2.7 cm and a height of 4.9 cm.

Calculate the total surface area of the solid shape.
Give your answer correct to 3 significant figures.

NOTE
AREA OF CIRCLE AT THE TOP IS THE SAME AS AREA 'MISSING' FROM THE CUBE!

CUBE:

$6 \times 8.7^2 = 454.14$ (M1)

CYLINDER:

$2 \times \pi \times 2.7 \times 4.9 = 83.126...$ (M1)

TOTAL $= 454.14 + 83.126...$

$= 537.266...$

$= 537 \text{ cm}^3$ (A1)

A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds is s metres, where

DIFFERENTIATE $s = t^3 - 6t + 3$

(a) Find an expression for the velocity, v m/s, of the particle at time t seconds.

$$v = \frac{ds}{dt}$$

A1 A1

$v = 3t^2 - 6$

(2)

DIFFERENTIATE AGAIN!

(b) Find the acceleration of the particle at time 5 seconds.

$$a = \frac{dv}{dt} = 6t \quad \text{B1}$$

$$= 6 \times 5$$

A1

30 m/s²

(2)

Make r the subject of the formula $A = 4r^2 - \pi r^2$ where r is positive.

$4r^2 - \pi r^2 = A$

$r^2(4 - \pi) = A$ (m1) FACTORISING

$r^2 = \frac{A}{4 - \pi}$ (m1) DIVIDING

$r = \sqrt{\frac{A}{4 - \pi}}$ (A1)

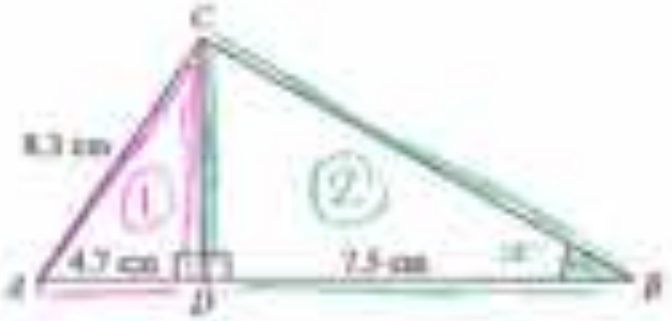


Diagram **NOT** accurately drawn

The diagram shows triangle ABC.
D is the point on AB, such that CD is perpendicular to AB.
$AC = 8.3$ cm
$AD = 4.7$ cm
$BD = 7.5$ cm

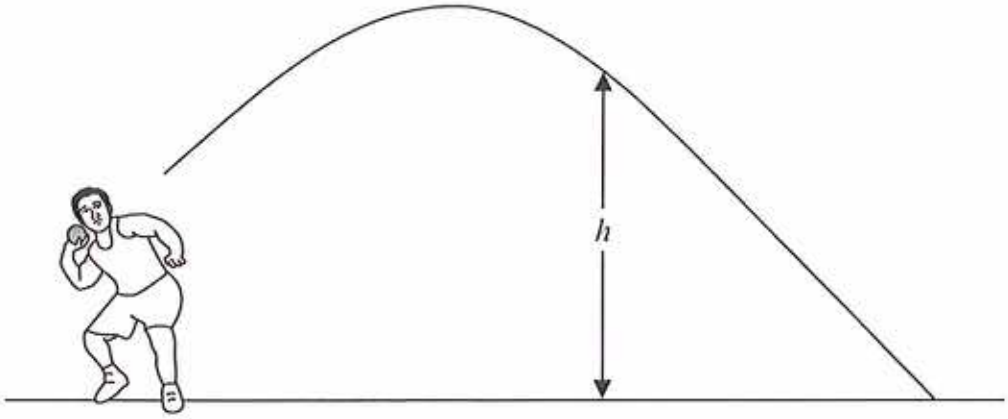
Calculate the size of angle ABC.
Give your answer correct to 1 decimal place.

1ST

$$CD^2 = 8.3^2 - 4.7^2 \quad (m1)$$
$$= 46.8$$
$$\Rightarrow CD = \sqrt{46.8}$$
$$= 6.84105... \quad (A1)$$

2ND

$$\tan x = \frac{OPP}{ADJ}$$
$$\Rightarrow \tan x = \frac{6.84105}{7.5} \quad (m1)$$
$$x = \tan^{-1}\left(\frac{6.84105}{7.5}\right)$$
$$= 42.369...$$
$$= 42.4^\circ \quad (A1)$$



Ivan is a shot putter.
The formula $h = 2 + 6t - 5t^2$ gives the height, h metres, of the shot above the ground t seconds after he has released the shot.

(i) Solve $2 + 6t - 5t^2 = 0$
Give your solutions correct to 3 significant figures.
Show your working clearly.

USE $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = -5, \quad b = 6, \quad c = 2$

$$x = \frac{-(6) \pm \sqrt{36 - 4\times(-5)\times(2)}}{(2)\times(-5)} \quad \text{(M1)}$$

$$= \frac{-6 \pm \sqrt{36+40}}{-10} \quad \text{(M1)}$$

$$\frac{-6+\sqrt{76}}{-10} = -0.272 \qquad \frac{-6-\sqrt{76}}{-10} = 1.047$$

(A1) [BOTH]

The shot hits the ground after T seconds.

(ii) Write down the value of T.
Give your answer correct to 3 significant figures.

IN PART (i) WE SOLVED FOR $h = 0$ (WHEN SHOT IS AT GROUND LEVEL) NEGATIVE TIMES ARE NOT POSSIBLE!

1.047 (A1)

Given that x and y are positive integers such that $(1 + \sqrt{x})(3 + \sqrt{x}) = y + 4\sqrt{5}$
find the value of x and the value of y.

(1) FOCUS ON LHS:

F O I L

$(1+\sqrt{x})(3+\sqrt{x}) = 3 + \sqrt{x} + 3\sqrt{x} + \sqrt{x}\sqrt{x}$

$= 3 + 4\sqrt{x} + x$

$= (3+x) + 4\sqrt{x}$ (M1)

INTEGER PART — SURD PART

(2) NOW COMPARE WITH THE RHS:

$(3+x) + 4\sqrt{x} = y + 4\sqrt{5}$

(3) SURD PARTS MUST EQUAL EACH OTHER

$4\sqrt{x} = 4\sqrt{5}$

$\Rightarrow x = 5$ (A1)

(4) INTEGER PARTS MUST EQUAL EACH OTHER

$y = 3 + x$

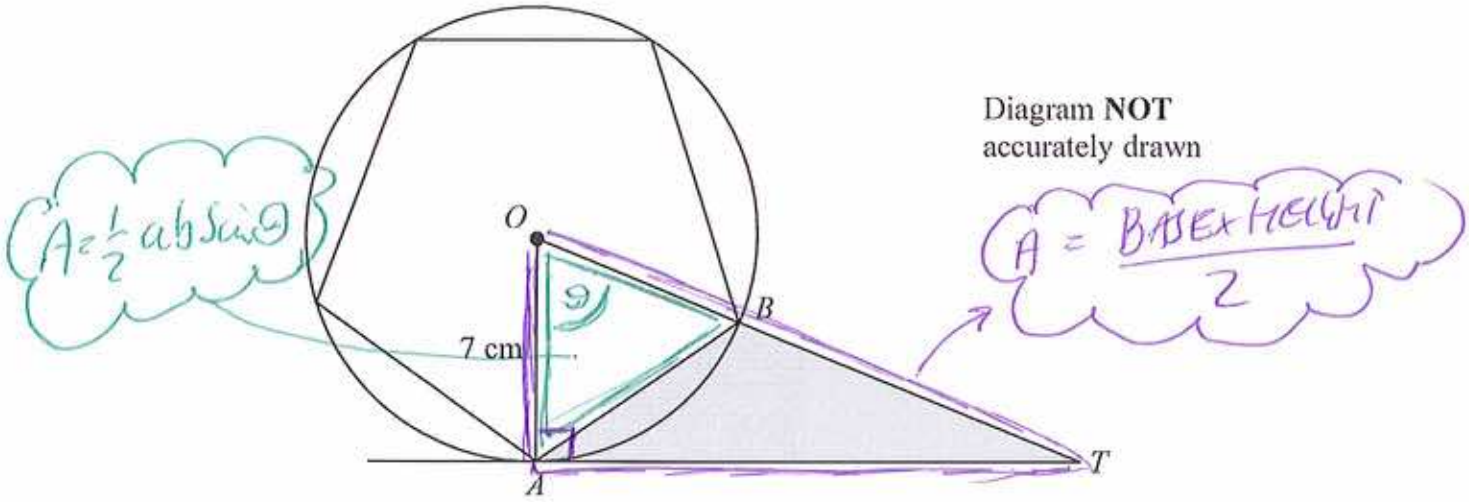
$= 3 + 5$

$= 8$ (A1)

Simplify fully $\dfrac{x^2 - 16}{x^2 - 6x + 8}$

$= \dfrac{(x-4)(x+4)}{(x-4)(x-2)}$ M1 M1

$= \dfrac{x+4}{x-2}$ A1

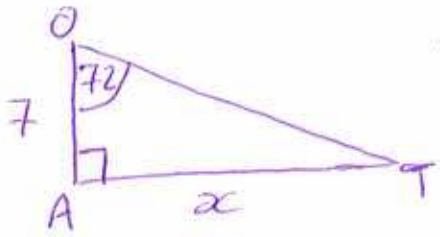


The diagram shows a regular pentagon inside a circle, centre O.
The points A and B lie on the circle such that AB is a side of the pentagon.
$OA = 7$ cm.
TA is a tangent to the circle and OBT is a straight line.

Calculate the area of triangle ABT.
Give your answer correct to 3 significant figures.

WORK OUT PURPLE − GREEN

$\theta = \frac{360}{5} = 72°$ (B1)



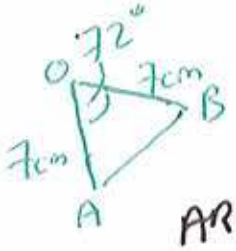
$\tan 72 = \frac{x}{7}$

$\Rightarrow x = 7 \tan 72$

$= 21.543...$ (M1)

$\therefore$ AREA OF AOT $= \frac{7 \times 21.543...}{2}$

$= 75.4032...$ (M1)



AREA OF OAB $= \frac{1}{2} \times 7 \times 7 \times \sin 72$

$= 23.300...$ (M1)

(A1)

SHADED AREA $= 75.4032 - 23.30... = 52.1 \text{ cm}^2$

The functions f and g are such that $f(x) = x + 3$ and $g(x) = \frac{1}{x-2}$

(a) Find $fg(x)$

Give your answer as a single algebraic fraction expressed as simply as possible.

$f(x) = x + 3$

$fg(x) = \left(\frac{1}{x-2}\right) + 3$ (M1)

$= \frac{1 + 3(x-2)}{x-2}$ (M1)

$= \frac{1 + 3x - 6}{x-2}$

$= \frac{3x - 5}{x-2}$ (A1)

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \ldots$

$x = \frac{1}{y-2}$ (M1)

$\Rightarrow y - 2 = \frac{1}{x}$ (M1)

$\Rightarrow y = \frac{1}{x} + 2$

$g^{-1}(x) = \frac{1}{x} + 2$ (A1)

(3)