

3H

Pearson Edexcel
International GCSE

EDEXCEL

IGCSE

MATHEMATICS A

SOLUTIONS

JANUARY 2016

4MA0/3H

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or ‘best’ method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

Here is a list of the ingredients needed to make lentil soup for 6 people.

Lentil Soup (for 6 people)
120 g lentils
300 g carrots
800 ml vegetable stock
3 onions

Jenny wants to make lentil soup for 24 people.

(a) Work out the amount of vegetable stock she needs.

$$\frac{800}{6} \times 24 \quad \text{(ml)}$$

$$\begin{array}{r} \text{(A)} \\ 3200 \text{ ml} \\ \hline (2) \end{array}$$

Ravi is going to make lentil soup.
He uses 450 g of carrots.

(b) How many people is Ravi making the lentil soup for?

$$\frac{450}{300} = 1.5 \quad \text{(ml)}$$

$$6 \times 1.5 = \underline{9}$$

$$\begin{array}{r} \text{(A)} \\ 9 \\ \hline (2) \end{array}$$

Lizzy drove by car to visit her aunt.
She left home at 9:30 am.

Lizzy arrived at her aunt's house at 11:15 am.
She drove a distance of 140 km.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

Work out, in km/h, Lizzy's average speed for the journey.

$$\begin{aligned}\text{TIME} &= 11:15 - 09:30 \\ &= 1 \text{ HOUR } 45 \text{ MIN} \\ &= \underline{\underline{1.75}} \text{ HOURS} \quad \text{(B1)}\end{aligned}$$

$$\begin{aligned}\therefore \text{SPEED} &= \frac{140}{1.75} \quad \text{(M1) [DIVISION]} \\ &= \underline{\underline{80}}\end{aligned}$$

$$\underline{\underline{80}} \quad \text{(A1)} \text{ km/h}$$

Show that $\frac{3}{8} \div \frac{7}{12} = \frac{9}{14}$

$$\frac{3}{8} \div \frac{7}{12} = \frac{3}{8} \times \frac{12}{7} \quad (B1)$$

$$= \frac{36}{56} \quad (B1)$$

$$= \frac{9}{14}$$

(a) Factorise $15r + 10$

$$\frac{5(3r+2)}{(1)} \quad \text{(BI)}$$

(b) Simplify $y^7 \times y^2$

$$\frac{y^9}{(1)} \quad \text{(BI)}$$

(c) Expand and simplify $(x+5)(x-1)$

$$\begin{array}{cccc} \text{F} & \text{O} & \text{I} & \text{L} \\ x^2 & -x & +5x & -5 \end{array} \quad \text{(MI)}$$

$$\frac{x^2 + 4x - 5}{(2)} \quad \text{(AI)}$$

(d) Simplify fully $\frac{36k^3m^4}{30k^5m}$

(MI) [ANY CORRECT OF m^3
OR k^{-2} !]

$$\frac{6m^3}{5k^2} \quad \text{(AI)}$$

(2)

Kim asked 40 people how many text messages they each sent on Monday.
The table shows her results.

MID-VALUE	Number of text messages sent	Frequency	$f \times \text{Mid-val}$
2	0 to 4	6	12
7	5 to 9	3	21
12	10 to 14	5	60
17	15 to 19	12	204
22	20 to 24	14	308
		TOTAL	605

(a) Write down the modal class.

(M) [USING MIDPOINTS]

20 to 24

(1)

(b) Calculate an estimate for the mean number of text messages sent.

$$\text{MEAN} = \frac{\text{TOTAL TEXTS}}{\text{NO. PEOPLE}}$$

$$= \frac{605}{40} \quad \text{(M) [CORRECT DIVISION]}$$

15.125

(4)

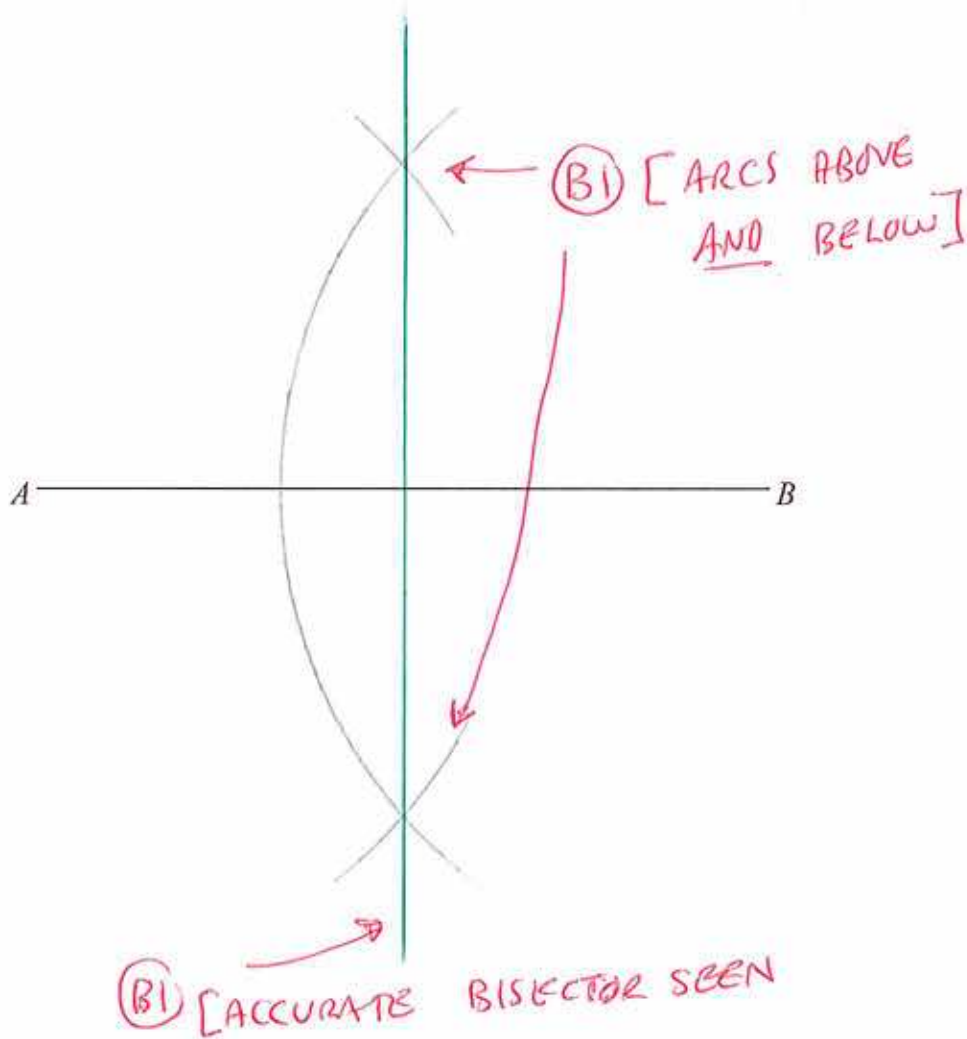
(c) What percentage of these 40 people sent 20 or more text messages?

$$\frac{14}{40} \times 100 \quad \text{(M)}$$

35%

(2)

Use ruler and compasses only to construct the perpendicular bisector of line AB .
You must show all your construction lines.



$$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{\text{odd numbers}\} = \{1, 3, 5, 7, 9\}$$

(a) List the members of $A \cup B$

$$\{1, 2, 3, 4, 5, 6, 7, 9\}$$

(1)

C is a set such that $A \cap C = \{4, 5\}$

The set C has 4 members.

(b) List the members of one possible set C

$$\{4, 5, \text{ANY TWO OTHERS NOT IN } A\}$$

[ALSO $\{4, 5, 7, 9\}$ $\{4, 5, 7, 10\}$ $\{4, 5, 8, 9\}$ ETC.]

$$\{4, 5, 7, 8\}$$

(2)

Solve $3(2x + 5) = 4 - x$

Show clear algebraic working.

$$6x + 15 = 4 - x \quad \text{(M1) [EXPAND BRACKETS]}$$

$$\left. \begin{aligned} 6x + x &= 4 - 15 \\ 7x &= -11 \end{aligned} \right\} \text{(M1) [EITHER]}$$

$$x = -\frac{11}{7} \quad \text{(A1)}$$

$$= -1\frac{4}{7}$$

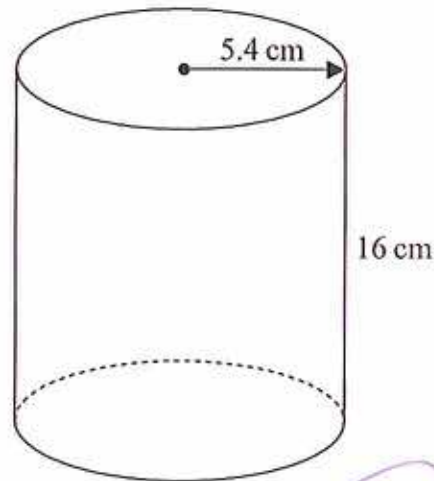


Diagram NOT
accurately drawn

A cylinder has radius 5.4 cm and height 16 cm.

(a) Work out the volume of the cylinder.

Give your answer correct to the nearest whole number.

$$V = \pi r^2 h$$

$$V = \pi \times 5.4^2 \times 16 \quad (\text{ml})$$

$$\approx 1465.74\dots$$

$$\begin{array}{r} 1466 \text{ cm}^3 \\ \hline (2) \end{array} \quad (\text{A1})$$

The radius 5.4 cm is correct to 2 significant figures.

(b) (i) Write down the upper bound of the radius.

$$5.4 + 0.05$$

$$\begin{array}{r} 5.45 \text{ cm} \\ \hline (\text{B1}) \end{array}$$

(ii) Write down the lower bound of the radius.

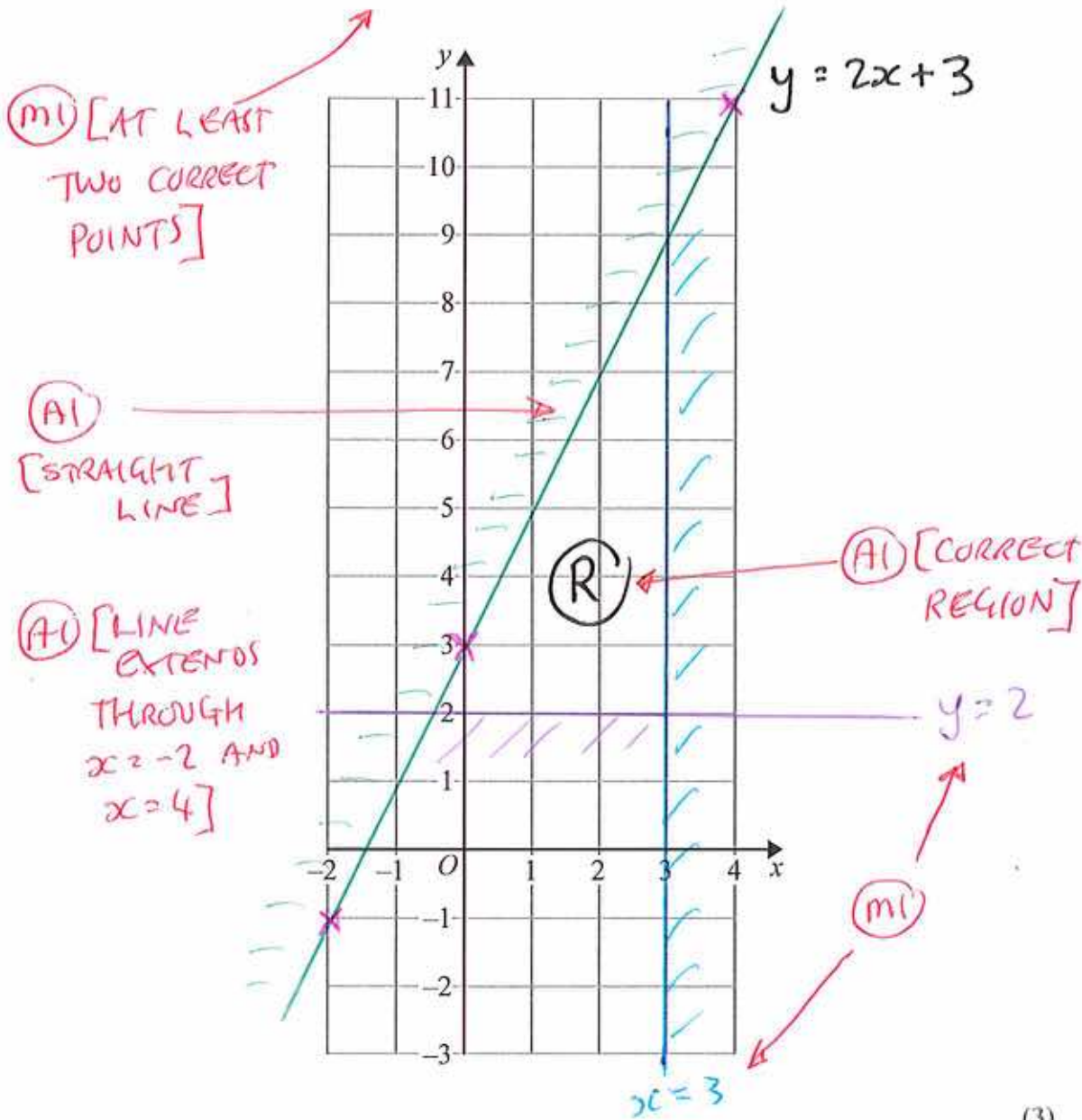
$$5.4 - 0.05$$

$$\begin{array}{r} 5.35 \text{ cm} \\ \hline (\text{B1}) \\ (2) \end{array}$$

(a) On the grid, draw the graph of $y = 2x + 3$ for values of x from -2 to 4

x	-2	0	4
y	-1	3	11

$y = 2(-2) + 3$
ETC.



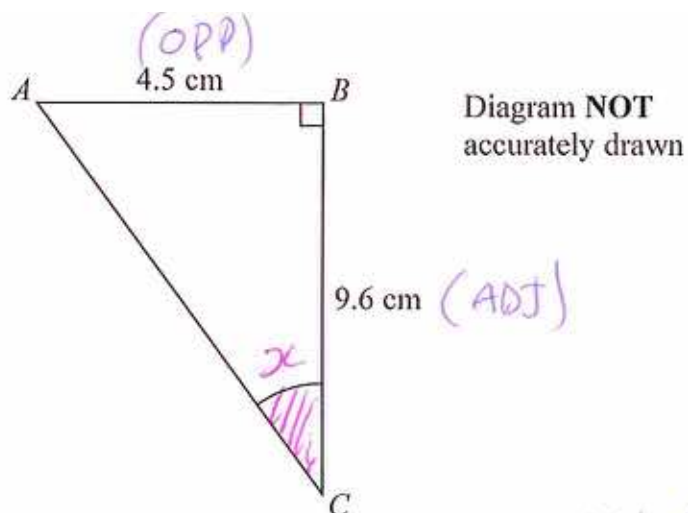
(3)

(b) Show, by shading on the grid, the region that satisfies all three of the inequalities

$x \leq 3$ and $y \geq 2$ and $y \leq 2x + 3$

Label your region R.

(2)



Work out the size of angle ACB .
Give your answer correct to 1 decimal place.

$$\tan \alpha = \frac{\text{OPP}}{\text{ADJ}}$$

$$\tan \alpha = \frac{4.5}{9.6} \quad (\text{mi})$$

SOH CAH TOA (mi)

$$\alpha = \tan^{-1} \left(\frac{4.5}{9.6} \right) \quad (\text{mi})$$

$$= 25.1148\dots$$

$$\underline{25.1} \quad (\text{A1}) \quad ^\circ$$

Here are the marks that James scored in eleven maths tests.

16 12 19 18 17 13 13 20 11 19 17

(a) Find the interquartile range of these marks.

(M) [ORDERED LIST]

11 12 13 13 16 17 18 19 19 20
 1 2 3 4 5 6 7 8 9 10 11

QUARTILES EVERY $\frac{11+1}{4}$
 = 3 VALUES

$IQR = Q_3 - Q_1$
 = 19 - 13 (M)
 = 6 (A)

Sunil did the same eleven maths tests.
 The median mark Sunil scored in his tests is 17
 The interquartile range is 8

(b) Which one of Sunil or James has the more consistent marks?
 Give a reason for your answer.

JAMES - HIS INTERQUARTILE RANGE
 WAS LOWER (A)

(1)

Sunil did four more maths tests.
 His scores in these four tests were 16, 20, 18 and 10

(c) How does his new median mark for the fifteen tests compare with his median mark of 17 for the eleven tests?

Tick (✓) one box.

new median is lower

new median is 17

new median is higher

Explain your answer.

TWO OF THE NEW TESTS WERE HIGHER THAN ORIGINAL MEDIAN AND TWO WERE LOWER, SO MEDIAN (MIDDLE VALUES) IS THE SAME (A)

(1)

Liam invests £8000 in a savings account for 4 years.
The savings account pays compound interest at a rate of

4.5 % for the first year

2.75 % for all subsequent years.

→ USE 1.045
→ USE 1.0275

(a) Work out the value of Liam's investment at the end of 4 years.

$$8000 \times 1.045 = 8360 \text{ (M)}$$

$$8360 \times 1.0275^3 = 9068.8406... \text{ (M)}$$

$$\begin{array}{r} \text{£ } 9068.84 \\ \hline (3) \end{array} \text{ (A)}$$

Max invests some money in a savings bond.

The savings bond pays interest at a rate of 2% per year. → USE 1.02

At the end of the first year, his savings bond is worth £5763

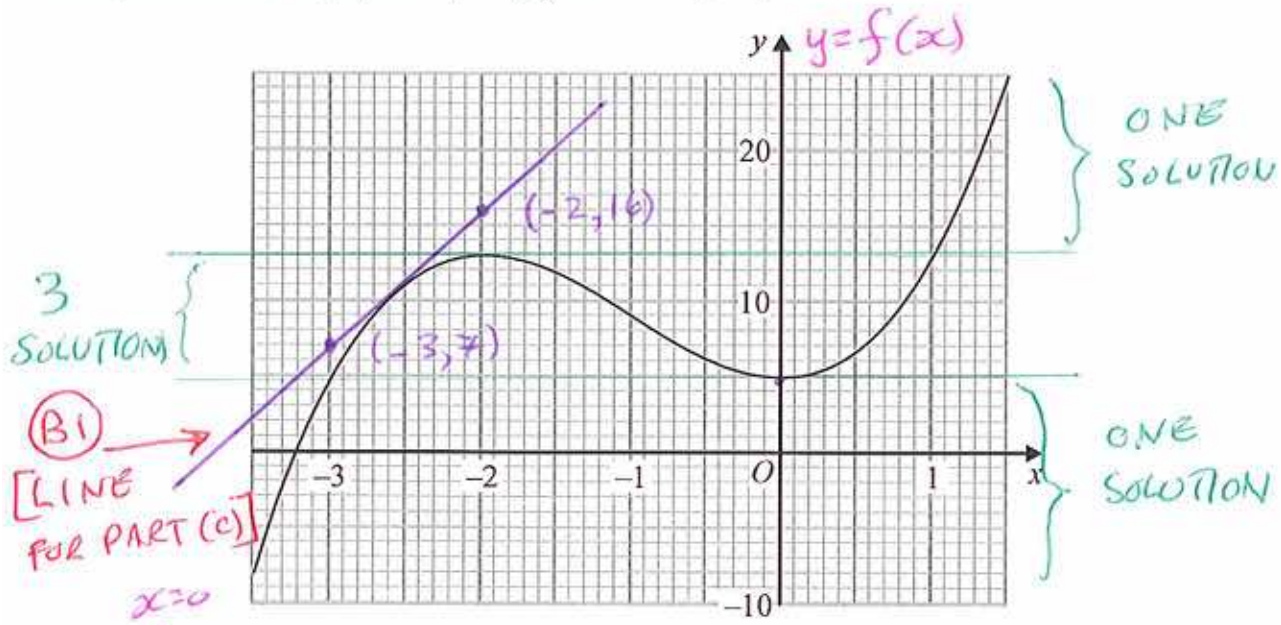
(b) How much money did Max invest in the savings bond?

$$\text{ORIGINAL} \times 1.02 = 5763 \text{ (B)}$$

$$\therefore \text{ORIGINAL} = \frac{5763}{1.02} \text{ (M)}$$

$$\begin{array}{r} \text{£ } 5650 \\ \hline (3) \end{array} \text{ (A)}$$

The diagram shows the graph of $y = f(x)$ for $-3.5 \leq x \leq 1.5$



(a) Find $f(0)$

[LOOK UP Y VALUE WHEN $x=0$]

5 (B1)

(1)

(b) For which values of k does the equation $f(x) = k$ have only one solution?

[HORIZONTAL LINES, WHICH CROSS THE GRAPH ONLY ONCE]

$k < 5$ AND $k > 13$ (B1) (B1)

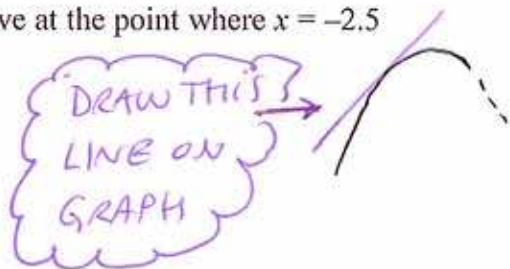
(2)

(c) Find an estimate for the gradient of the curve at the point where $x = -2.5$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{16 - 7}{-2 - -3} \quad | \quad \text{(M1)}$$

$$= 9 \quad [\text{APPROX.}]$$



9 (A1)

(3)

$$g(x) = \frac{1}{2+x}$$

DENOMINATORS CAN NEVER BE ZERO

(d) State which value of x must be excluded from any domain of g

$$2+x \neq 0 \quad \therefore x \neq -2$$

$$\frac{-2 \text{ (B1)}}{(1)}$$

(e) Find $fg(-3)$

$$g(-3) = \frac{1}{2+(-3)} \text{ (M1)} \rightarrow f(-1) = 9 \text{ [FROM GRAPH]}$$

$$= -1$$

$$\frac{9 \text{ (A1)}}{(2)}$$

Solve the inequality $5x^2 - 13 < 32$

Show clear algebraic working.

$$5x^2 < 45$$

$$x^2 < 9 \quad (M1)$$

$$x < 3$$

[1ST ANSWER]

$$x > -3$$

(A1) [EITHER ANSWER]

[SECOND ANSWER]

COMBINE

$$\underline{\underline{(A1) \quad -3 < x < 3}}$$

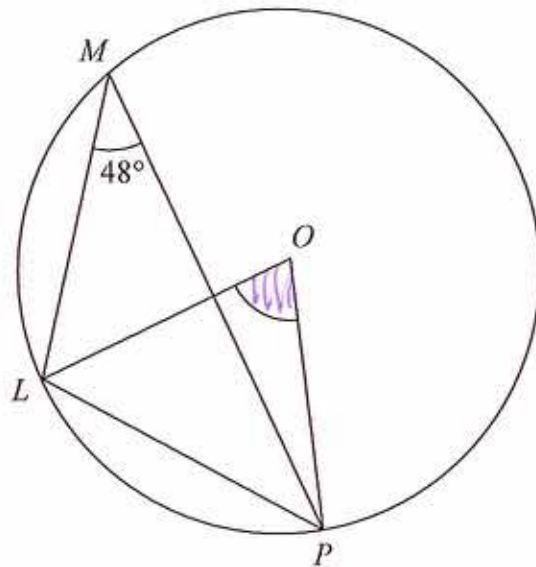


Diagram NOT
accurately drawn

L , M and P are points on a circle, centre O
Angle $LMP = 48^\circ$

(a) (i) Write down the size of angle LOP



2×48

96°

(ii) Give a reason for your answer.

ANGLE AT THE CENTRE IS TWICE THE
ANGLE AT THE CIRCUMFERENCE

(2)

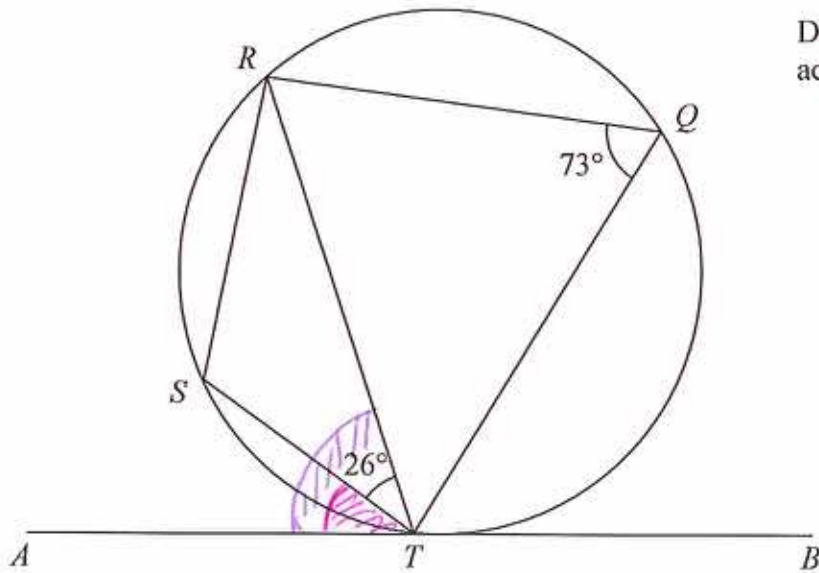



Diagram NOT accurately drawn

Q, R, S and T are points on a circle.
 ATB is the tangent to the circle at T

Angle $STR = 26^\circ$
 Angle $RQT = 73^\circ$

(b) Work out the size of angle STA 
 Give a reason for each stage in your working.

$$RTA = 73^\circ \quad [\text{ALTERNATE SEGMENT THEOREM}]$$

(B1) [MUST SEE REASON]

$$\begin{aligned} \therefore STA &= 73 - 26 \quad (\text{B1}) \quad [\text{MUST SEE CALCULATION}] \\ &= \underline{47^\circ} \quad (\text{A1}) \end{aligned}$$

A is the point with coordinates $(1, 3)$

B is the point with coordinates $(-2, -1)$

The line L has equation $3y = 4 - 2x$

Is line L parallel to AB ?
Show your working clearly.

PARALLEL LINES HAVE
EQUAL GRADIENTS

LINE L

$$3y = 4 - 2x$$

$$\Rightarrow y = \frac{4}{3} - \frac{2}{3}x \quad \text{GRADIENT} = -\frac{2}{3} \quad \text{(BI)}$$

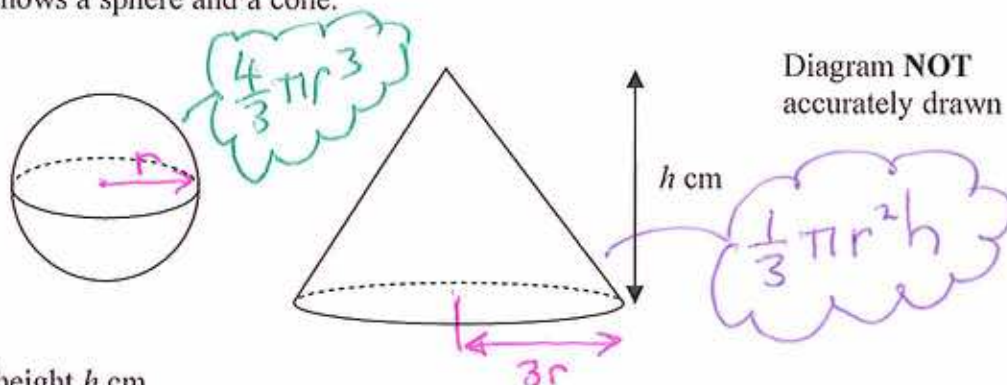
LINE AB

$$\begin{aligned} m &= \frac{y_1 - y_2}{x_1 - x_2} \\ &= \frac{3 - (-1)}{1 - (-2)} \\ &= \frac{4}{3} \quad \text{(BI)} \end{aligned}$$

DIFFERENT GRADIENTS so

(AI) LINE L IS NOT PARALLEL TO AB

The diagram shows a sphere and a cone.



The cone has height h cm.

The radius of the base of the cone is 3 times the radius of the sphere.

Given that the volume of the sphere is equal to the volume of the cone, find an expression for the radius of the sphere in terms of h .

Give your expression in its simplest form.

$$\frac{4}{3} \pi r^3 = \frac{1}{3} \pi (3r)^2 h \quad (M1)$$

$$\Rightarrow \frac{4}{3} \pi r^3 = \frac{1}{3} \pi \times 9r^2 \times h$$

$$\Rightarrow \frac{4}{3} \pi r^3 = 3\pi r^2 h$$

$$\Rightarrow \pi r^3 = \frac{9}{4} \pi r^2 h$$

$$\Rightarrow r^3 = \frac{9}{4} r^2 h$$

$$\Rightarrow r = \frac{9h}{4} \quad (A1)$$

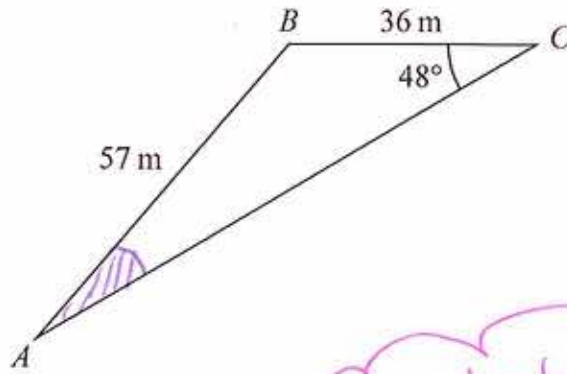
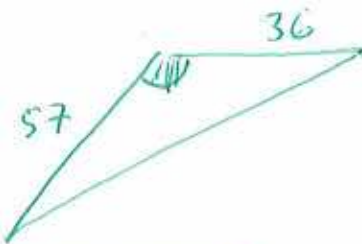


Diagram NOT
accurately drawn

Work out the area of triangle ABC.
Give your answer correct to 3 significant figures.

$$A = \frac{1}{2} ab \sin c$$

I CHOOSE TO USE...



STEP ONE

[FIND CAB]

$$\frac{\sin CAB}{36} = \frac{\sin 48}{57} \text{ (m)}$$

$$\sin CAB = \frac{\sin 48}{57} \times 36$$

$$= 0.46935\dots$$

$$\Rightarrow CAB = \underline{\underline{28}} \text{ (m)}$$

STEP TWO

[FIND ABC]

$$\begin{aligned} ABC &= 180 - (48 + 28) \\ &= \underline{\underline{104}} \end{aligned}$$

STEP 3

$$A = \frac{1}{2} \times 57 \times 36 \times \sin 104 \text{ (m)}$$

$$= 995.523\dots$$

$$\underline{\underline{996}} \text{ (m)} \text{ m}^2$$

Peter travels to work either by bus or by bike.

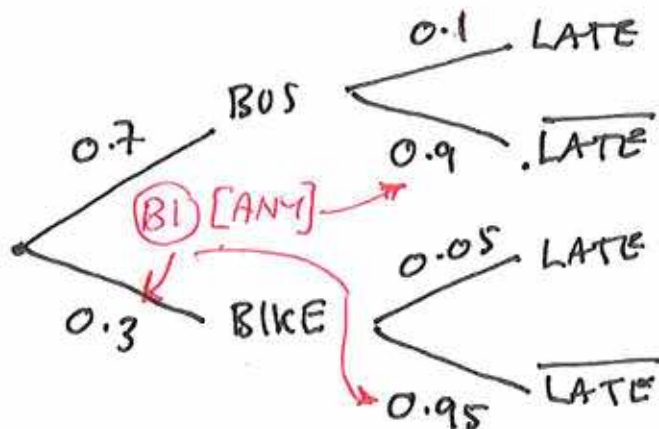
The probability that Peter will travel to work by bus on any one day is 0.7

Whenever Peter travels to work by bus, the probability that he will be late is 0.1

Whenever Peter travels to work by bike, the probability that he will be late is 0.05

Peter is going to go to work on Monday and on Tuesday.

Work out the probability that he will be late for work on at least one of these days.



MONDAY (OR TUESDAY)

$$P(\text{BUS, LATE}) = 0.7 \times 0.1 = 0.07 \quad \text{(M1)}$$

$$P(\text{BIKE, LATE}) = 0.3 \times 0.05 = 0.015$$

$$\therefore P(\text{LATE}) = \underline{\underline{0.085}} \quad \text{(A1)}$$

$$P(\overline{\text{LATE}}) = \underline{\underline{0.915}}$$

$$P(\overline{\text{LATE}}) = 0.915 \times 0.915 \quad \text{(M1) [OR ANY]}$$

$$= 0.837225$$

$$\therefore P(\text{LATE AT LEAST ONCE}) = 1 - 0.837225$$

$$\underline{\underline{0.1628}} \quad \text{(A1)}$$

The diagram shows a rectangle.

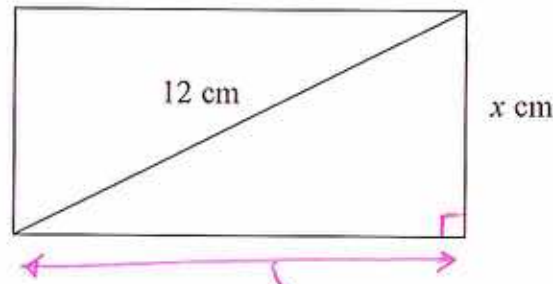


Diagram NOT accurately drawn

The width of the rectangle is x cm.
The length of a diagonal of the rectangle is 12 cm.

The perimeter of the rectangle is 28 cm.

Find the possible values of x .
Give your values correct to 3 significant figures.
Show your working clearly.

$$\sqrt{12^2 - x^2}$$

(B) [EXPRESSION FOR LENGTH]

$$\sqrt{144 - x^2} + x = 14 \quad (M1) \text{ [EQUATION]}$$

$$\sqrt{144 - x^2} = 14 - x$$

$$144 - x^2 = (14 - x)^2 \quad (M2) \text{ [NO SQUARE ROOT]}$$

$$144 - x^2 = 196 - 28x + x^2$$

$$2x^2 - 28x + 52 = 0$$

$$x^2 - 14x + 26 = 0$$

$$a=1 \quad b=-14 \quad c=26$$

(M3) [ANS]

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times (1) \times (26)}}{2(1)} \quad (M1)$$

$$= \frac{14 \pm \sqrt{196 - 104}}{2} \quad (M2) \text{ [SIMPLIFYING]}$$

$$= \underline{\underline{11.8 \text{ cm}}} \quad \underline{\underline{2.20 \text{ cm}}}$$

(A1) [BOTH]

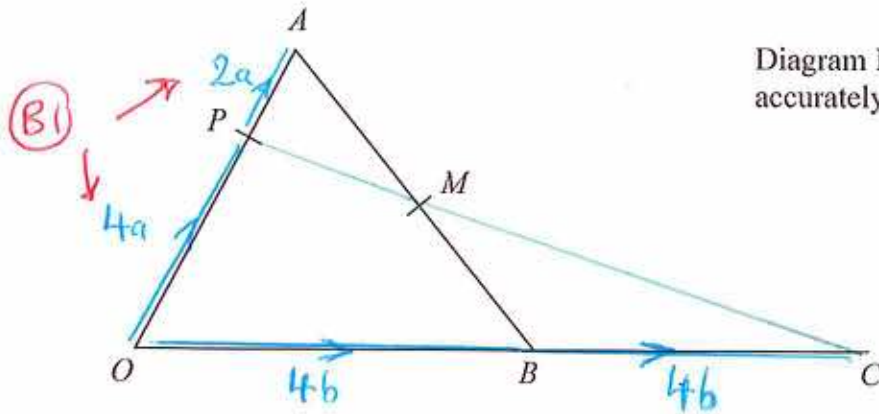


Diagram NOT
accurately drawn

OAB is a triangle.

P is the point on OA such that $OP:PA = 2:1 = 4a:2a$

C is the point such that B is the midpoint of OC .

M is the midpoint of AB .

$$\vec{OA} = 6a$$

$$\vec{OB} = 4b$$

Show that PMC is a straight line.

MUST SHOW THAT

$$\vec{PM} = k \times \vec{MC}$$

$$\text{OR } \vec{PM} = k \times \vec{PC}$$

$$\text{OR } \vec{MC} = k \times \vec{PC}$$

$$\vec{AB} = -6a + 4b$$

$$\therefore \vec{AM} = \frac{1}{2}(-6a + 4b)$$

$$= -3a + 2b$$

(A1) [for \vec{AM} or \vec{BM}]

$$\vec{PM} = \vec{PA} + \vec{AM}$$

$$= 2a + (-3a + 2b)$$

$$= \underline{\underline{-a + 2b}} \quad \text{(M1)}$$

$$\vec{PC} = -4a + 8b$$

$$= 4(-a + 2b) \quad \text{(M1)}$$

$$= \underline{\underline{4\vec{PM}}}$$

SINCE $\vec{PC} = k\vec{PM}$
THEY ARE PARALLEL

SINCE THEY ALSO BOTH
GO THROUGH POINT P
THEY MUST FORM A
STRAIGHT LINE. (A1)