3H(R)

Pearson Edexcel International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

JANUARY 2016 | **4MA0/3HR**

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Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

Work out the value of $\dfrac{21.89 - 7.75}{0.65 + 2.85}$

A2

4.04

(a) Factorise fully $18c - 27$

B1 A1

$9(2c-3)$

(2)

(b) Expand and simplify $(t-4)(t+5)$

F O I L

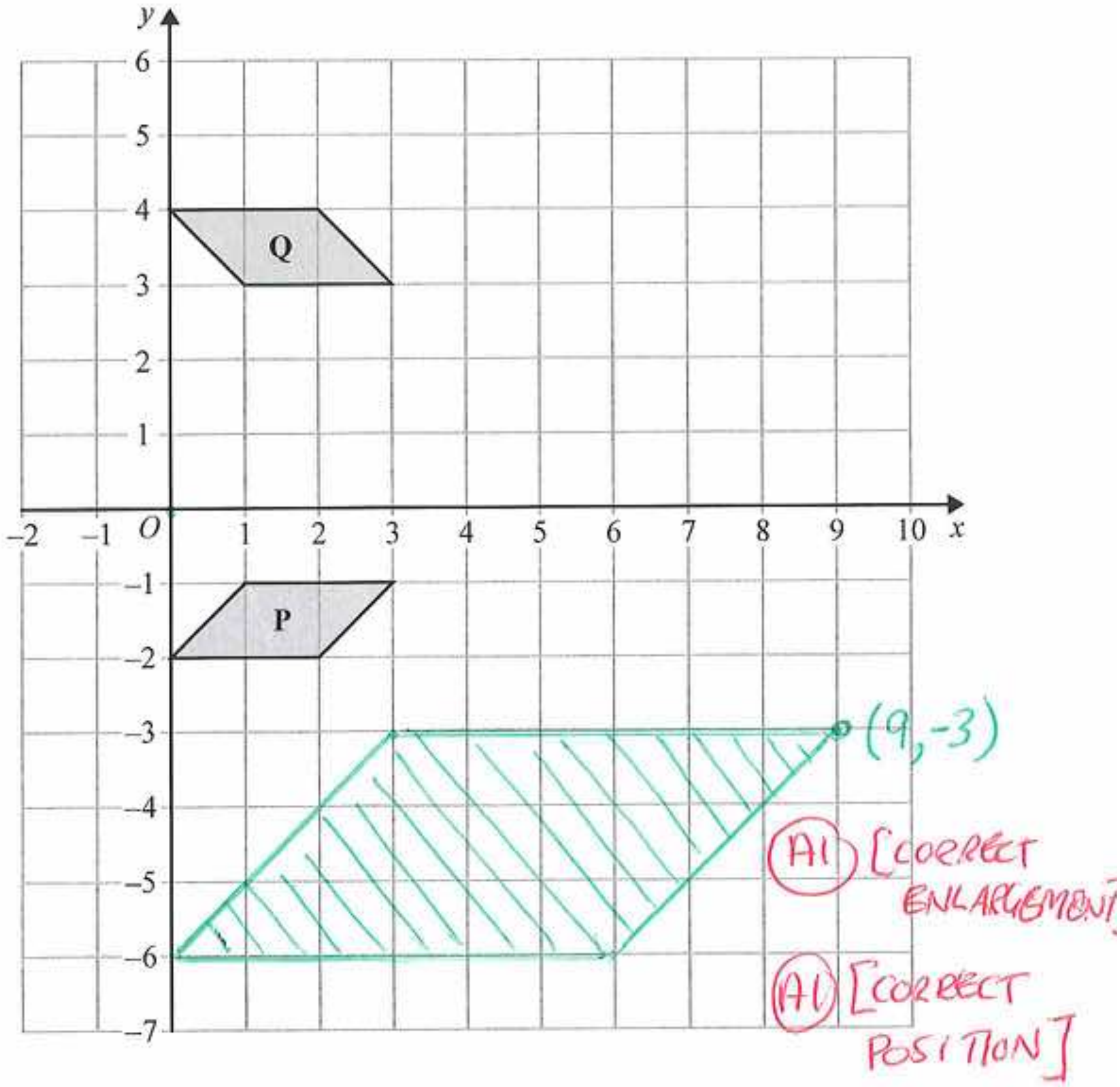
$t^2 + 5t - 4t - 20$

M1

A1

$t^2 + t - 20$

(2)



(a) Describe fully the single transformation that maps shape **P** onto shape **Q**.

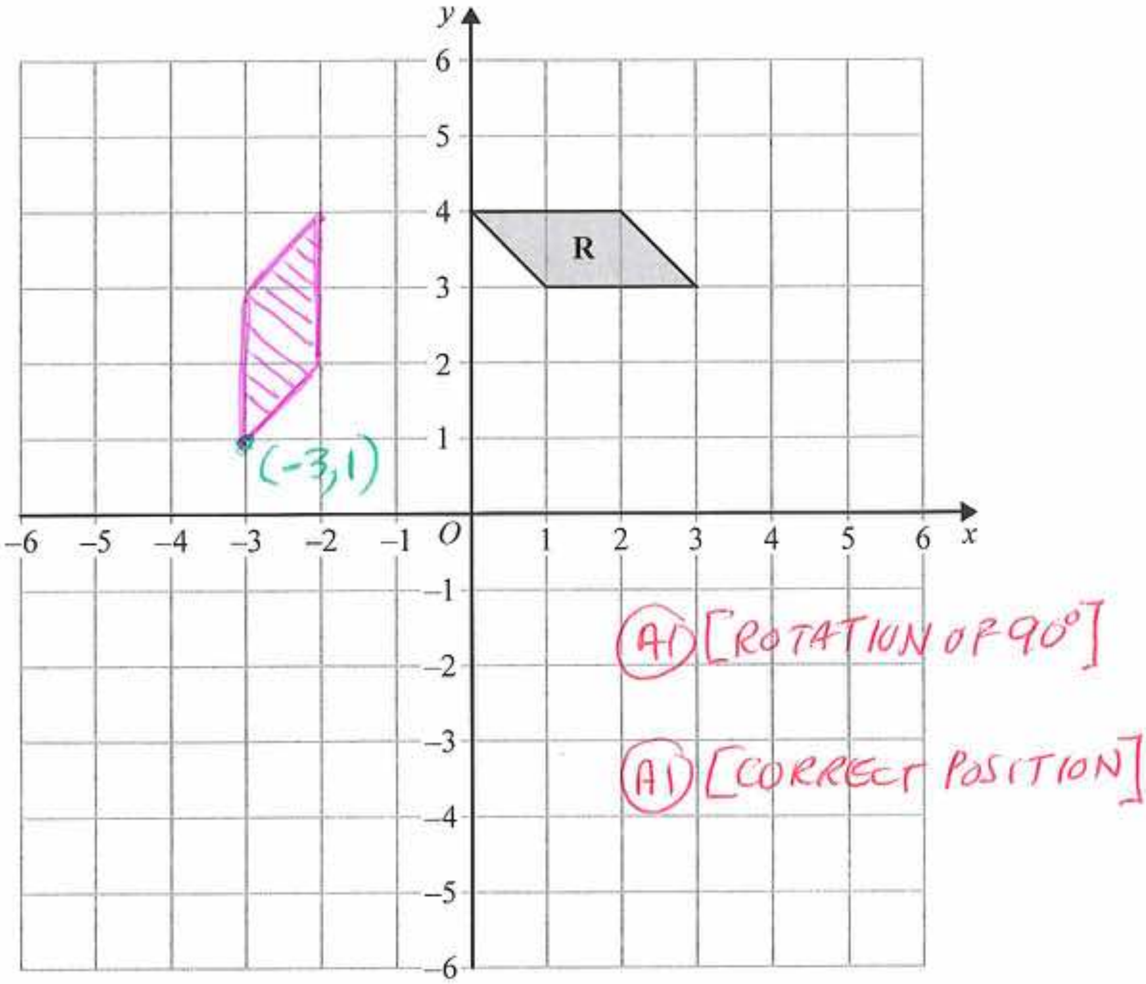
REFLECTION, IN LINE y = 1

A1 A1

(2)

(b) On the grid above, enlarge shape **P** with scale factor 3 and centre *O*.

(2)



(c) On the grid above, rotate shape **R** 90° anticlockwise with centre (0, 1)

(2)

Maisie plays a game.
Each time she plays, she can win a prize of $1 or $5 or $10
When she does not win one of these prizes, she loses.
The table gives the probability of winning each of the prizes.

Prize	Probability
$1	0.50
$5	0.15
$10	0.05

Maisie plays the game once.

(a) Work out the probability that Maisie loses.

$1-(0.5+0.15+0.05)$

$= 1-0.7$ M1

A1 0.3

(2)

(b) Maisie plays the game 40 times.

(i) Work out an estimate for the number of $5 prizes she wins.

40×0.15 M1

6 A1

(2)

(ii) Work out an estimate for the total value of the prizes she wins.

$$\left.\begin{aligned} 40 \times 0.5 \times \$1 &= \$20 \\ 40 \times 0.15 \times \$5 &= \$30 \\ 40 \times 0.05 \times \$10 &= \$20 \end{aligned}\right\} \$70$$

M1 [ALL THREE]

M1 [ATTEMPT TO ADD]

$ 70 A1

(3)

The diagram shows a circle with centre O and radius 6.5 cm

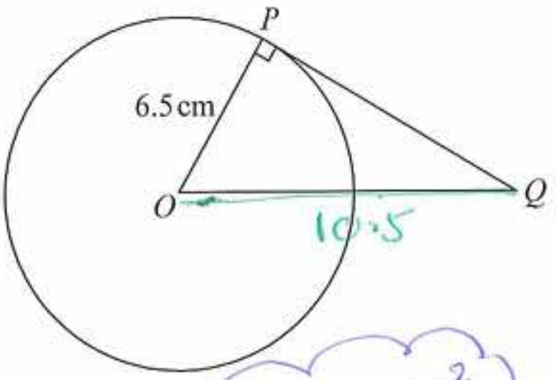


Diagram **NOT** accurately drawn

(a) Work out the area of the circle. USE $A=\pi r^2$
Give your answer correct to 3 significant figures.

$A = \pi \times 6.5^2$ (M1)

$= 132.73...$

133 (A1) cm^2

(2)

PQ is the tangent to the circle at P
OQ = 10.5 cm

(b) Work out the length of PQ USE PYTHAGORAS
Give your answer correct to 3 significant figures.

$PQ^2 = 10.5^2 - 6.5^2$ (M1) [SUBTRACT SQUARES]

$= 68$

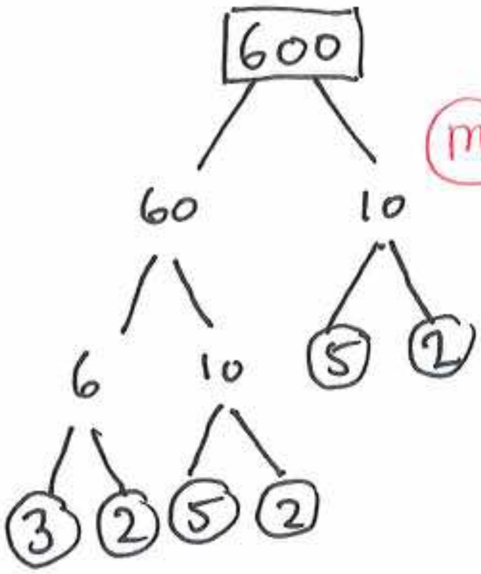
$PQ = \sqrt{68}$ (M1) [SQUARE ROOTING]

$= 8.2462...$

8.25 (A1) cm

(3)

(a) Express 600 as a product of powers of its prime factors.
Show your working clearly.



M1 [OTHER OBVIOUS METHODS ARE FINE]

M1

$= 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$2^3 \times 3 \times 5^2$ A1

(3)

(b) Simplify $\dfrac{5^{12}}{5^2 \times 5}$

Give your answer as a power of 5

M1 $\dfrac{5^{12}}{5^3} = 5^9$

5^9 A1

(2)

(a) Solve the inequality $e - 2 < 0$

B1

$e < 2$

(1)

(b) Solve the inequality $5 - 3e < 4$

*

$-3e < -1$ M1

$e > \frac{-1}{-3}$

INEQUALITY IS REVERSED WHEN DIVIDING BY A NEGATIVE

A1

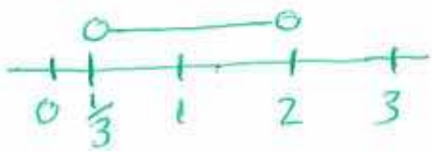
$e > \frac{1}{3}$

(2)

(c) Write down the integer value of e that satisfies both of the inequalities

$e - 2 < 0$ and $5 - 3e < 4$

$[e < 2]$ $[e > \frac{1}{3}]$



A1

1

(1)

In 1981, the population of India was 683 million.
Between 1981 and 1991, the population of India increased by 163 million.

(a) Express 163 million as a percentage of 683 million.
Give your answer correct to 3 significant figures.

$$\frac{163}{683} \times 100 = 23.865\ldots$$

M1

A1

23.9 %
(2)

In 2001, the population of India was 1028 million.
Between 2001 and 2011, the population of India increased by 17.6%

(b) Increase 1028 million by 17.6%
Give your answer to the nearest million.

$$1028 \times 1.176 = 1208.928$$

M1 B1

[MULTIPLY]

A1

1209 million
(3)

In 2001, the population of India was 1028 million.
Between 1971 and 2001, the population of India increased by 87.6%

(c) Work out the population of India in 1971.
Give your answer correct to the nearest million.

*

M1

[DIVIDE]

$$\frac{1028}{1.876} = 547.97\ldots$$

B1

A1

548 million
(3)

The point A has coordinates (0, 2)
The point B has coordinates (–4, –1)

(a) Find the coordinates of the midpoint of AB.

USE $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\left(\frac{0+-4}{2}, \frac{2+-1}{2}\right) = (-2, 0.5)$$

M1

A1

$(-2, 0.5)$
(2)

(b) Work out the gradient of the line AB.

USE $\frac{y_1-y_2}{x_1-x_2}$

$$\frac{2--1}{0--4} = \frac{2+1}{0+4}$$ M1

$$= \frac{3}{4}$$

A1

$\frac{3}{4}$
(2)

(c) Find an equation of the line AB.

$y-y_1=m(x-x_1)$

AND CHANGE TO $y=mx+c$

$$y-2 = \frac{3}{4}(x-0)$$

$$\Rightarrow y-2 = \frac{3}{4}x \Rightarrow y = \frac{3}{4}x+2$$

SHORTCUT:
NOTICE A IS (0,2)
[SINCE $x=0$, $c=2$!]

A1 B1

$y=\frac{3}{4}x+2$
ft (2)

The diagram shows a circle with centre O.
The points A, B and C lie on the circle.

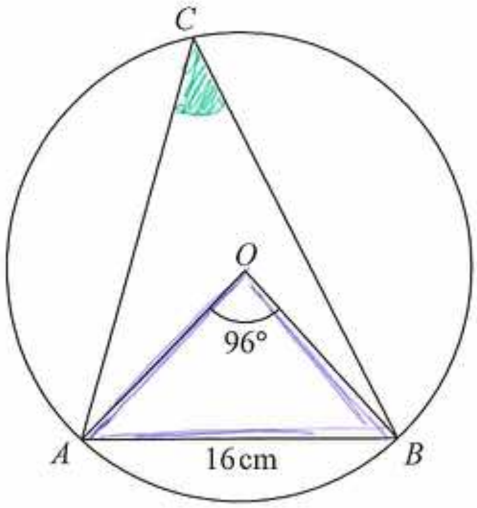


Diagram **NOT** accurately drawn

Angle $AOB = 96°$

(a) Work out the size of angle ACB.

$\frac{96}{2}$

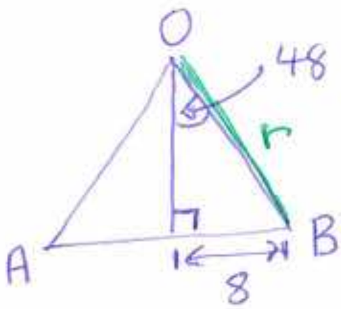
A1

48 °

(1)

$AB = 16$ cm

(b) Work out the radius of the circle.
Give your answer correct to 3 significant figures.



SOH CAH TOA

$\sin 48 = \frac{8}{r}$ M1 [SIN...]

B1 ['48']

$\Rightarrow r = \frac{8}{\sin 48}$ M1 [r = ...]

$= 10.765...$

A1

10.8 cm

(4)

Solve the simultaneous equations

$$c + 5d = -13 \quad (1)$$
$$4c - 5d = 48 \quad (2)$$

ADD

Show clear algebraic working.

$5c = 35$ M1

$c = \frac{35}{5}$

$c = 7$

SUBSTITUTE INTO (1)

$7 + 5d = -13$

$5d = -20$

$d = -4$

$c =$ 7 A1

$d =$ −4 A1

A stone is thrown vertically upwards from a point O.

The height above O of the stone t seconds after it was thrown from O is h metres, where $h = 17t - 5t^2$ — QUADRATIC!

Work out the values of t when the height of the stone above O is 12 metres.
Show your working clearly.

$12 = 17t - 5t^2$

1×12
2×6
3×4

$\Rightarrow 5t^2 - 17t + 12 = 0$

M1 [EITHER]

$(5t - 12)(t - 1) = 0$ M1 [FACTORISING]

[OK USING FORMULA!]

$t = \frac{12}{5}$ $\quad t = 1$

$= 2.4$

A1 [BOTH]

1 AND 2.4

(a) Simplify $\left(4h^{\frac{2}{3}}\right)^3$

$= 4^3 \times h^{\frac{2}{3}\times 3}$

$= 64 \times h^2$

B1 B1

$64h^2$

(2)

$$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = a^k$$

(b) Work out the value of k.

$\dfrac{a^1 \times a^{\frac{1}{2}}}{a^{\frac{2}{3}}} = \dfrac{a^{\frac{3}{2}}}{a^{\frac{2}{3}}} = a^{\frac{3}{2}-\frac{2}{3}} = a^{\frac{5}{6}}$

M1

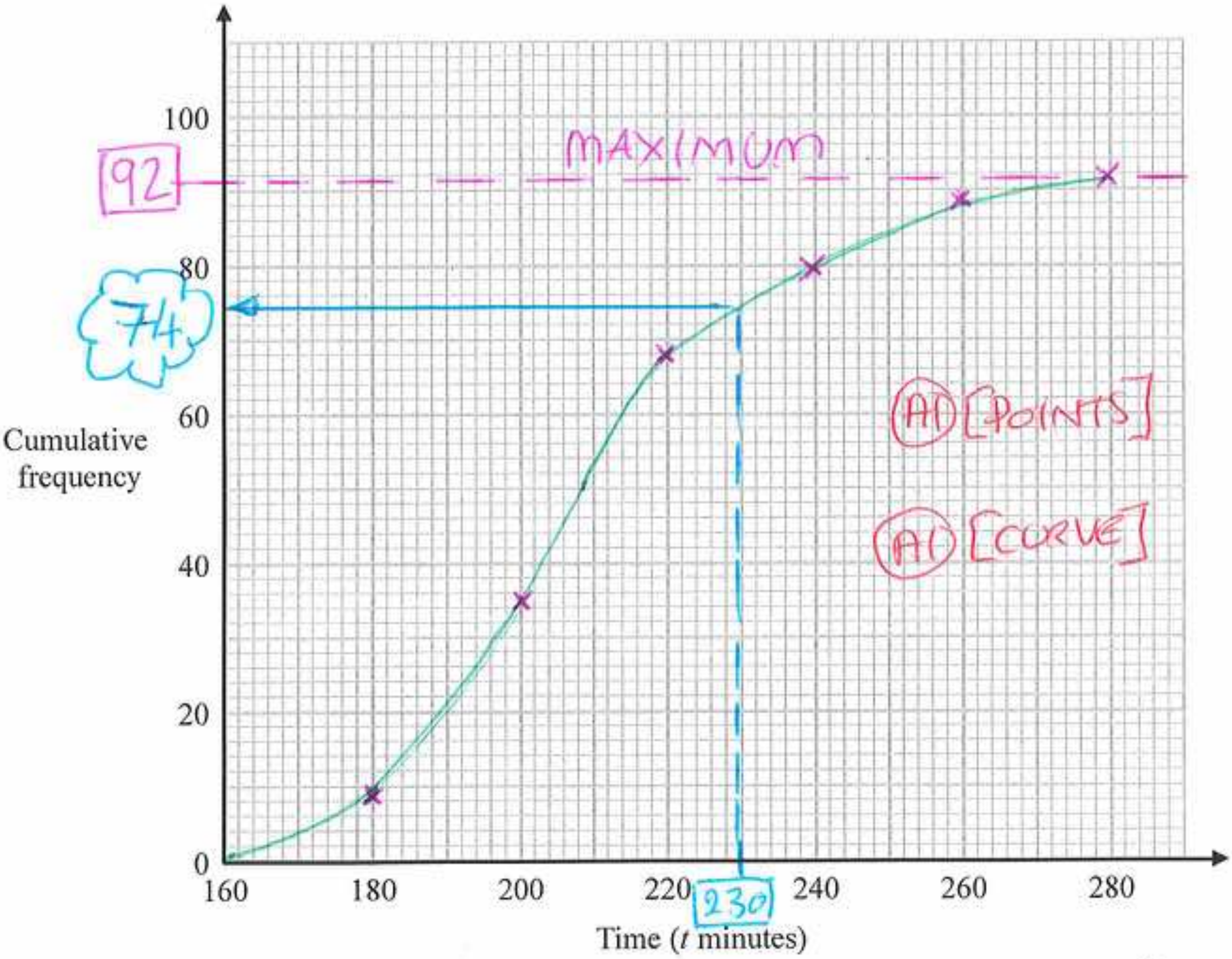
$k =$ $\frac{5}{6}$ A1

(3)

The cumulative frequency table shows information about the times taken by 92 runners to complete a marathon.

Time (t minutes)	**Cumulative frequency**
$160 < t \leqslant 180$	9
$160 < t \leqslant 200$	35
$160 < t \leqslant 220$	68
$160 < t \leqslant 240$	80
$160 < t \leqslant 260$	89
$160 < t \leqslant 280$	92

(a) On the grid, draw a cumulative frequency graph for the information in the table.



(2)

(b) Use the graph to find an estimate for the number of runners who took more than 230 minutes to complete the marathon.

LESS THAN 230 MINUTES IS 74

B1 [73 → 75]

$\therefore$ MORE THAN $= 92 - 74$

$= 18$

18 A1

(2)

The diagram shows a cylinder inside a cone on a horizontal base.
The cone and the cylinder have the same vertical axis.
The base of the cylinder lies on the base of the cone.
The circumference of the top face of the cylinder touches the curved surface of the cone.

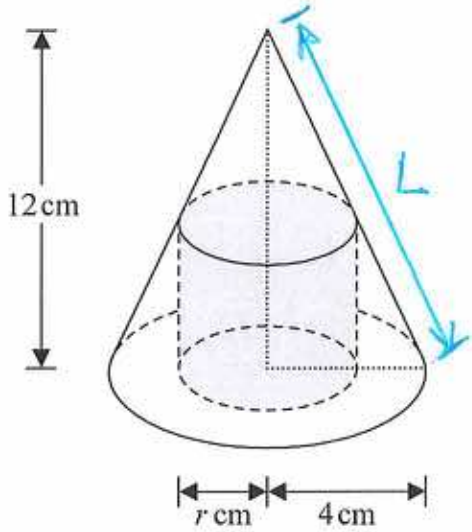


Diagram **NOT** accurately drawn

1ST

$L = \sqrt{12^2 + 4^2}$

$= 12.649...$ M1

The height of the cone is 12 cm and the radius of the base of the cone is 4 cm.

(a) Work out the curved surface area of the cone. USE $\{A = \pi r L\}$
Give your answer correct to 3 significant figures.

$A = \pi \times 4 \times 12.649$ M1

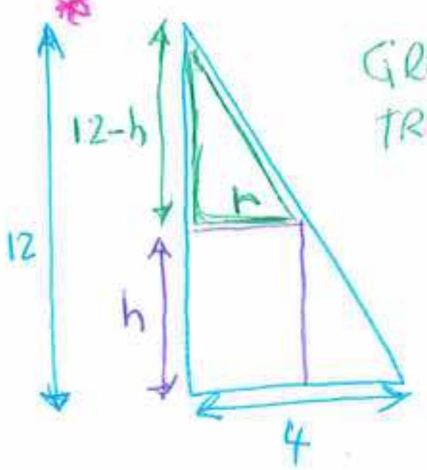
$= 158.95...$

159 A1 cm²

(3)

The cylinder has radius r cm and $\boxed{\text{volume } V \text{ cm}^3}$ – USE $\{V = \pi r^2 h\}$

$\boxed{\text{(b)}}$ Show that $V = 12\pi r^2 - 3\pi r^3$



GREEN TRIANGLE AND BLUE TRIANGLE ARE 'SIMILAR'

$$\therefore \quad \underbrace{\frac{r}{12-h} = \frac{4}{12}}_{\text{M1}} \Rightarrow 12r = 48 - 4h$$

$$\Rightarrow 3r = 12 - h$$

$$\Rightarrow h = 12 - 3r \quad \text{M1}$$

$$\therefore \quad V = \pi r^2(12 - 3r) \quad \text{M1}$$

$$= 12\pi r^2 - 3\pi r^3$$

(3)

(c) $V = 12\pi r^2 - 3\pi r^3$

Find the value of r for which V is a $\boxed{\text{maximum.}}$ – DIFFERENTIATE AND PUT $\frac{dV}{dr} = 0$

$$\frac{dV}{dr} = \underbrace{24\pi r}_{\text{B1}} - \underbrace{9r^2}_{\text{B1}}$$

$$\Rightarrow 24\pi r - 9\pi r^2 = 0 \quad \text{M1} \left[\text{SETTING } \frac{dv}{dr} = 0\right]$$

$$\Rightarrow 3\pi r(8 - 3r) = 0$$

$$r = 0 \qquad 8 - 3r = 0$$

$$\Rightarrow 3r = 8$$

$$r = \frac{8}{3}$$

$r =$ $\frac{8}{3}$ A1

(4)

$$f(x) = \frac{2x}{\boxed{x-1}}$$

DENOMINATOR CANNOT BE ZERO!

(a) Find the value of f(11)

$$= \frac{2(11)}{(11)-1} = \frac{22}{10}$$

$2 \cdot 2$ (B1)
(1)

(b) State which value of x must be excluded from any domain of f

$x = 1$ (B1)
(1)

(c) Find $f^{-1}(x)$

$$y = \frac{2x}{x-1}$$

$$\Rightarrow x = \frac{2y}{y-1}$$ (M1) WILL FIND INVERSE

$$\Rightarrow xy - x = 2y$$

$$\Rightarrow xy - 2y = x$$

(M1) [ANY CORRECT REARRANGING]

$$y(x-2) = x$$

$$\Rightarrow y = \frac{x}{x-2}$$

$\frac{x}{x-2}$ (A1)
(3)

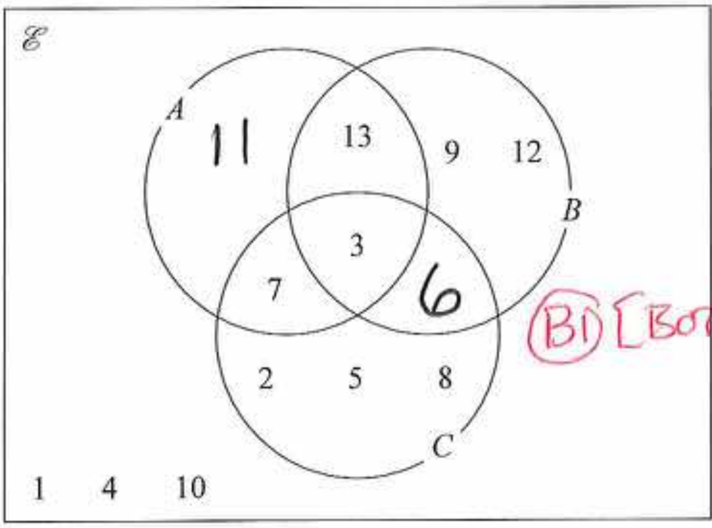
(d) State the value which cannot be in any range of f

* RANGE OF f = DOMAIN OF f^{-1}

$x = 2$ (B1)
(1)

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
$A = \{3, 7, 11, 13\}$
$B = \{3, 6, 9, 12, 13\}$
$C = \{2, 3, 5, 6, 7, 8\}$

(a) Complete the Venn diagram. [MISSING ARE 6, 11]



B1 [BOTH 6 AND 11]

(1)

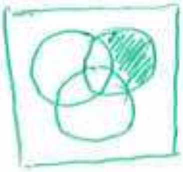
(b) List the members of the set $B' \cap C$

[TWO REGIONS]

{7, 2, 5, 8} B1

(1)

(c) List the members of the set $(A \cup C)' \cap B$



[ONE REGION]

{9, 12} B1

(1)

HOW MANY!

(d) Find $n(A' \cap B')$

[TWO REGIONS]

{1, 4, 10, 2, 5, 8}

6 B1

(1)

There are 100 tiles in a bag.
Each tile is marked with a number.
The table shows information about the tiles.

Number on tile	Frequency
0	2
1	68
2	7
3	13
4	10

Carmen takes at random a tile from the bag.
She records the number on the tile and then $\boxed{\text{replaces}}$ the tile in the bag.
Pablo takes at random a tile from the bag.

PROBABILITIES STAY THE SAME

(a) Work out the probability that Carmen takes a tile with the number 0 or the number 1 and Pablo takes a tile with a number greater than 1

$P(\text{'0' or '1'}) = \frac{70}{100}$

$P(>1) = \frac{30}{100}$

$\therefore P(\text{'0' or '1', THEN } >1) = \frac{70}{100} \times \frac{30}{100}$ (M1)

0.21 (A1)

(2)

All 100 tiles are in the bag.
Juan takes at random a tile from the bag without replacing it.
He then takes a second tile from the bag.

PROBABILITIES CHANGE

(b) (i) Work out the probability that the number on each tile is 4

$$P(4,4) = \frac{10}{100} \times \frac{9}{99} \quad \text{(M1)}$$

$$= \frac{1}{110} \quad (0.0\dot{0}\dot{9})$$

$\frac{1}{110}$ (A1)

(ii) Work out the probability that the total of the numbers on the two tiles is 2

$$P(0,2) = \frac{2}{100} \times \frac{7}{99} = \frac{14}{9900}$$

$$P(2,0) = \frac{7}{100} \times \frac{2}{99} = \frac{14}{9900}$$

$$P(1,1) = \frac{68}{100} \times \frac{67}{99} = \frac{4556}{9900}$$

TOTAL $\frac{4584}{9900}$

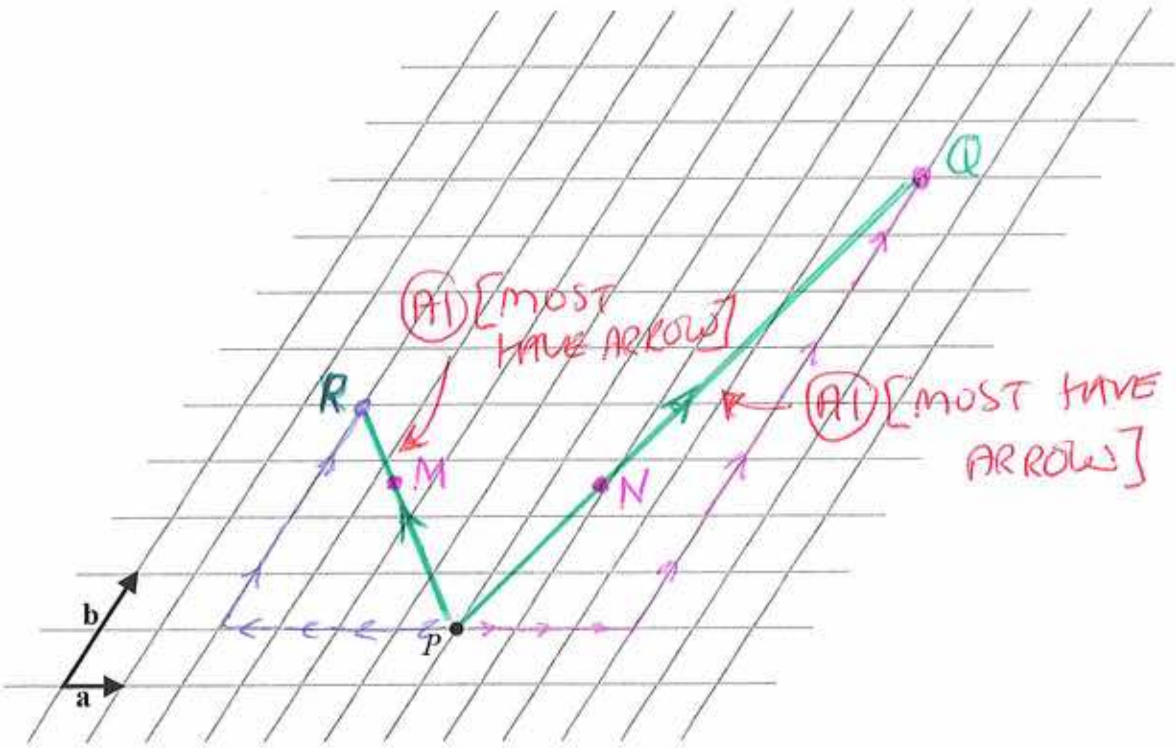
(M1) [ALL THREE OPTIONS]

(M1) [ATTEMPTS TO MULTIPLY AT LEAST ONE OPTION]

$\frac{382}{825}$ (A1)

(5)

The diagram shows a grid of equally spaced parallel lines.
The point P and the vectors **a** and **b** are shown on the grid.



$\overrightarrow{PQ} = 3\mathbf{a} + 4\mathbf{b}$

(a) On the grid, mark the vector $\overrightarrow{PQ}$

(1)

$\overrightarrow{PR} = -4\mathbf{a} + 2\mathbf{b}$

(b) On the grid, mark the vector $\overrightarrow{PR}$

(1)

(c) Find, in terms of **a** and **b**, the vector $\overrightarrow{QR}$

$\overrightarrow{QR} = -4b - 3a - 4a + 2b$

$= -7a - 2b$

$\overrightarrow{QR} = -7a - 2b$ A1

(1)

The point M lies on PR such that $PM = \frac{2}{3}PR$

The point N lies on PQ such that $PN = \frac{1}{3}PQ$

(d) Show that $\overrightarrow{MN} = k\mathbf{a}$ where k is a constant.
State the value of k.

$\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$ M1 [ANY USE OF EITHER $\frac{2}{3}$ PR OR $\frac{1}{3}$ PQ]

$= -\frac{2}{3}\overrightarrow{PR} + \frac{1}{3}PQ$

$= -\frac{2}{3}(-4a + 2b) + \frac{1}{3}(3a + 4b)$ M1 [SUBSTITUTES]

$= \frac{8}{3}a - \frac{4}{3}b + a + \frac{4}{3}b$

$= \frac{8}{3}a + a$

$= \underline{\underline{\frac{11}{3}a}}$

$k = \frac{11}{3}$ A1

(3)