3H(R)

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

MAY 2013	4MA0/3HR

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

A box contains some coloured cards.
Each card is red or blue or yellow or green.
The table shows the probability of taking a red card or a blue card or a yellow card.

Card	Probability
Red	0.3
Blue	0.35
Yellow	0.15
Green	

George takes at random a card from the box.

(a) Work out the probability that George takes a green card.

B1

$$\begin{array}{r} 0.3 \\ +0.35 \\ +0.15 \\ \hline 0.8 \end{array}$$

$1 - 0.8 = 0.2$

0.2 A1

(2)

George replaces his card in the box.
Anish takes a card from the box and then replaces the card.
Anish does this 40 times.

(b) Work out an estimate for the number of times Anish takes a yellow card.

0.15×40

M1

6 A1

(2)

Wendy travelled on the Eurostar train from St Pancras station to the Gare du Nord station.
The Eurostar train travelled a distance of 495 km.
The journey time was 2 hours 15 minutes. → 2.25 HOURS

Work out the average speed of the Eurostar train in kilometres per hour.

$$V = \frac{\text{distance}}{\text{time}}$$

$$= \frac{495}{2.25} \quad \text{M1}$$

B1 → 2.25

$$= \underline{\underline{220}} \text{ km/h} \quad \text{A1}$$

The table shows information about the time, in minutes, spent on homework by each of 32 pupils in one night.

MIDPOINT x	Time (t minutes)	Number of pupils f	$f \times x$
10	$0 < t \leqslant 20$	7	70
30	$20 < t \leqslant 40$	16	480
50	$40 < t \leqslant 60$	3	150
70	$60 < t \leqslant 80$	6	420
		TOTAL	1120

M1 M1

(a) Calculate the percentage of the 32 pupils who spent more than 60 minutes on their homework.

$$\frac{6}{32} \times 100$$ M1

18.75 % A1

(2)

(b) Calculate an estimate for the total time spent on homework by the 32 pupils.

WORKING OUT IS SHOWN IN TABLE ABOVE!

1120 A1 minutes

(3)

(a) Expand $6(3a - 2b + c)$

A1

$18a - 12b + 6c$

(1)

(b) Factorise $t^2 - 10t$

A1 A1

$t(t - 10)$

(2)

(c) Solve $x = \frac{7 - 2x}{3}$

Show clear algebraic working.

$3x = 7 - 2x$ M1

$\Rightarrow 3x + 2x = 7$

$5x = 7$ M1

$\rightarrow x = \frac{7}{5}$

$x =$ 1.4 A1

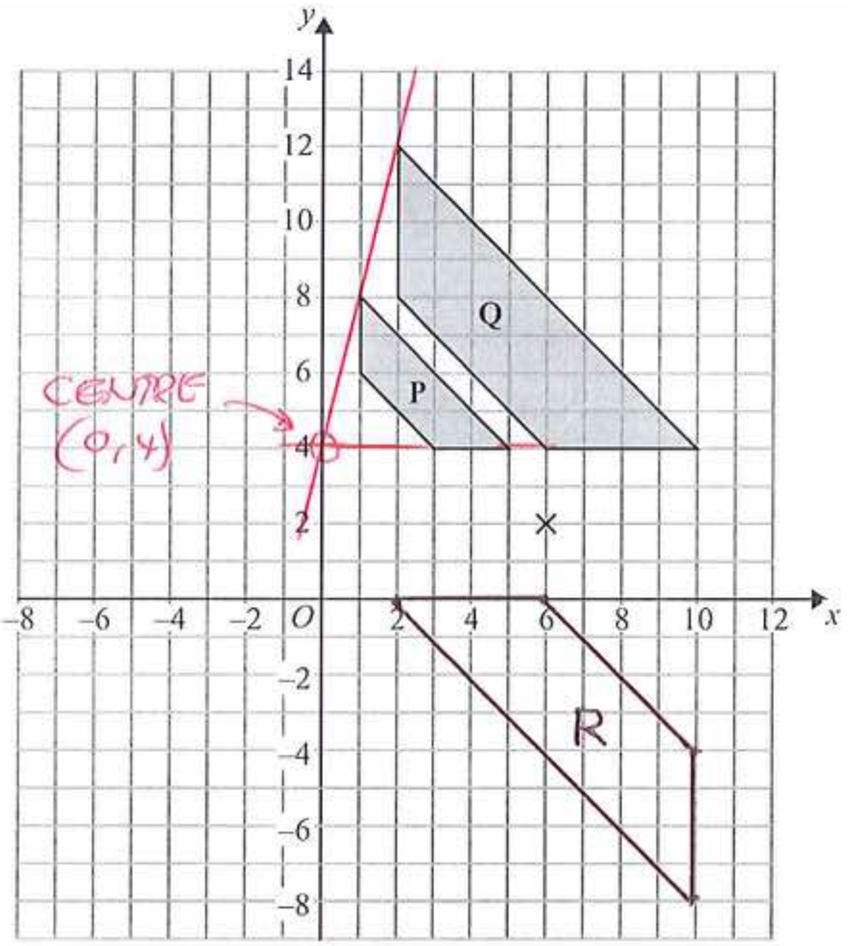
(3)

Show that $\frac{4}{9} - \frac{1}{6} = \frac{5}{18}$

$$\frac{4}{9} - \frac{1}{6} = \underbrace{\frac{8}{18} - \frac{3}{18}}_{\text{B1}} \leftarrow \text{B1} \quad (\text{ACCEPT } 36)$$

LCM = 18

$$= \frac{5}{18} \quad \underline{\underline{\text{QED}}}\,!$$



(a) Describe fully the single transformation that maps shape **P** onto shape **Q**.

ENLARGEMENT, SCALE FACTOR 2

CENTRE (0, 4)

(3)

(b) On the grid, rotate shape **Q** 180° about the point (6, 2).
Label the new shape **R**.

(2)

$M = 3x^2 - nx$

(a) Work out the value of M when

$x = -2$ and $n = 5$

$M = 3\times(-2)^2 - 5\times(-2)$ M1

$= 3\times 4 + 10$

$= \underline{\underline{22}}$

$M =$ 22 A1

(2)

(b) Work out the value of n when

$M = 12$ and $x = 4$

$12 = 3\times 4^2 - n\times 4$ M1

$12 = 3\times 16 - 4n$

$12 = 48 - 4n$

$-36 = -4n$ M1

$n = \frac{-36}{-4}$

$= \underline{\underline{9}}$

$n =$ 9 A1

(3)

(a) $A = \{s, u, p, e, r\}$
$B = \{c, o, m, p, u, t, e, r\}$

List the members of the set

(i) $A \cap B$

$\{u, p, e, r\}$ A1

(ii) $A \cup B$

$\{s, c, o, m, p, u, t, e, r\}$

(2) A1

(b) $X = \{\text{prime numbers}\}$
$Y = \{\text{factors of } 12\}$ → $\{1, 2, 3, 4, 6, 12\}$

Is it true that $X \cap Y = \varnothing$?

Tick (✓) the appropriate box.

Yes	No
	✓

Explain your answer.

2 AND 3 ARE FACTORS OF 12 AND PRIME NUMBERS

A1 (1)

(a) Simplify, leaving your answers in index form,

(i) $6^5 \times 6^2 \times 6$

6^8 (B1)

(ii) $(9^7)^2$

9^{14} (B1)

(2)

(b) $\dfrac{5^n \times 5^3}{5^6} = 5^4$

Find the value of n.

$5^n \times 5^3 = 5^4 \times 5^6$ (M1)

$5^n \times 5^3 = 5^{10}$ $\rightarrow$ $n + 3 = 10$

$n =$ 7 (A1)

(2)

The diagram shows the path of an athlete on a running track.

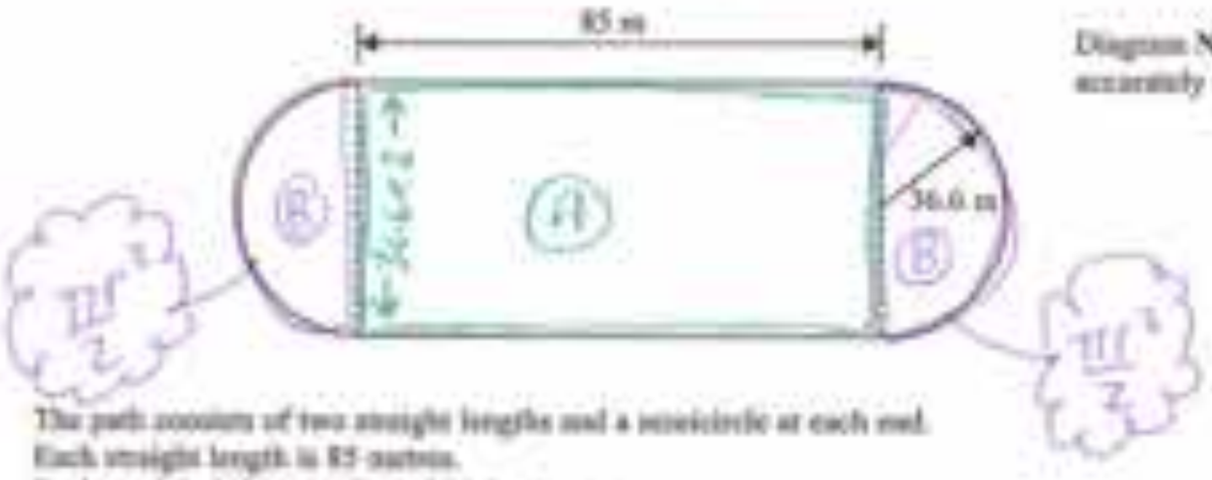


Diagram **NOT** accurately drawn

The path consists of two straight lengths and a semicircle at each end.
Each straight length is 85 metres.
Each semicircle has a radius of 36.6 metres.

Calculate the area enclosed by the path.
Give your answer correct to 3 significant figures.

A $\quad 85 \times 73.2 = \underline{6222}$ (M1)

B $\quad \pi \times 36.6^2 = \underline{4208.35}$ (M1)

TOTAL

$6222 + 4208.35 = 10430.35\ldots$ (M1)

$= \underline{10400}\ m^2$ (A1)

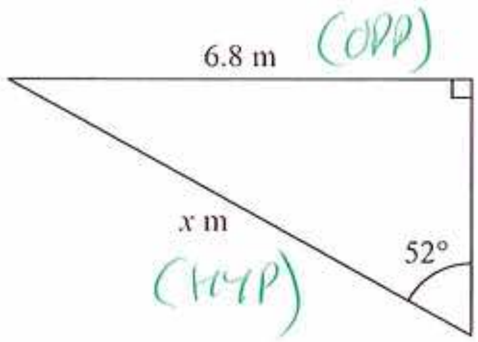


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 3 significant figures.

SOH CAH TOA

$\sin 52 = \frac{OPP}{HYP}$

$\sin 52 = \frac{6.8}{x}$ (M1)

$x = \frac{6.8}{\sin 52}$ (M1)

$= 8.6293\ldots$

$x =$ 8.63 (A1)

(a) Write as an ordinary number

(i) 4.2×10^6

4 200 000 (A1)

(ii) 3.82×10^{-4}

0.000382 (A1)

(2)

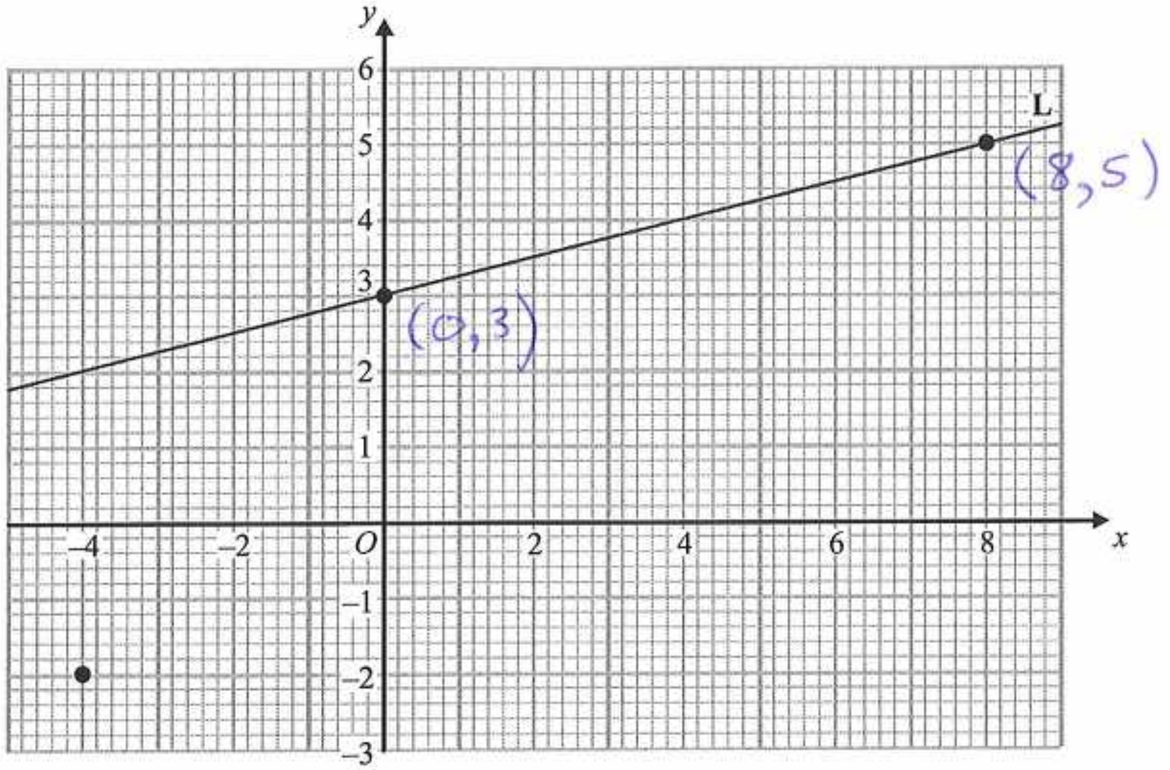
(b) Here are three numbers written in standard form.
Arrange these numbers in order of size.
Start with the smallest number.

5.6×10^{-7} 8.6×10^{-9} 5.64×10^{-8}

8.6×10^{-9}, 5.64×10^{-8}, 5.6×10^{-7} (A2)

(2)

The points with coordinates (0, 3) and (8, 5) lie on the straight line **L**.



(a) Work out the gradient of **L**.

$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5-3}{8-0} = \frac{2}{8}$ (M1)

(A1) $\frac{1}{4}$ $(= 0.25)$

(2)

(b) Write down an equation of **L**.

(A1) $y = \frac{1}{4}x + 3$

(1)

(c) Find an equation of the line which is parallel to **L** and which passes through the point (–4, –2)

{SAME GRADIENT}

$y - y_1 = \frac{1}{4}(x - x_1)$

$y - -2 = \frac{1}{4}(x - -4)$ (M1)

$y = \frac{1}{4}(x+4) - 2$

(A1) $y = \frac{1}{4}x - 1$

Triangles ABC and ACD are similar.

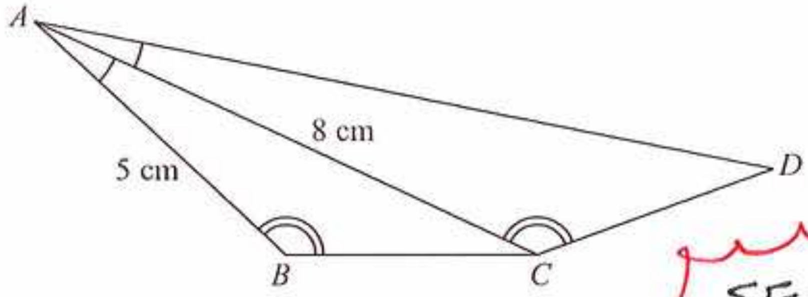


Diagram **NOT** accurately drawn

SF = $\frac{8}{5}$ = 1.6 B1

Angle BAC = angle CAD.
Angle ABC = angle ACD.
AB = 5 cm and AC = 8 cm.

(a) Calculate the length of AD.

8 × 1.6

12.8 A1 cm
(2)

The area of triangle ABC is 12 cm²

(b) Calculate the area of triangle ACD.

12 × 1.6^2

B1

30.72 A1 cm²
(2)

The table shows information about the times, in minutes, that some people took to complete a sudoku puzzle.

Time (t minutes)	$0 < t \leq 5$	$5 < t \leq 20$	$20 < t \leq 30$	$30 < t \leq 50$
Number of people	4	18	34	30

← AREAS!

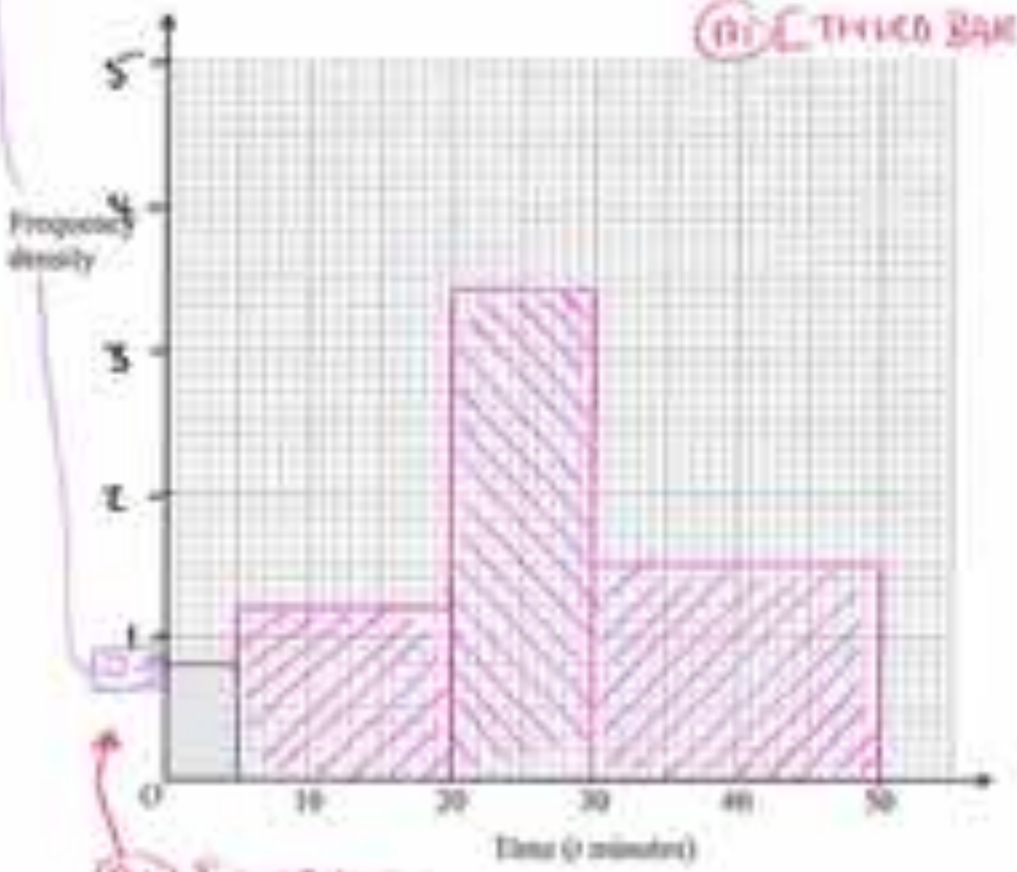
Complete the histogram for this information.

WIDTH	5	15	10	20
HEIGHT	$\frac{4}{5} = \boxed{0.8}$	$\frac{18}{15} = 1.2$	$\frac{34}{10} = 3.4$	$\frac{30}{20} = 1.5$

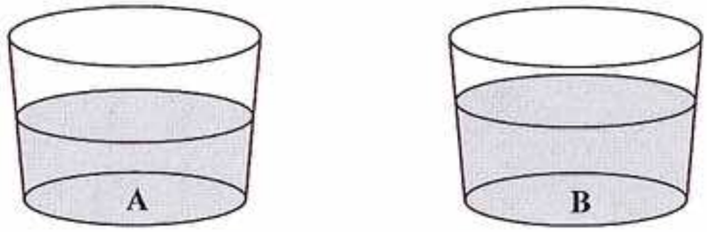
ALREADY ON GRAPH!

B1 [TWO BARS CORRECT]

B1 [THIRD BAR CORRECT]



B1 [CORRECT SCALE]



Glass **A** contains 122 millilitres of water, correct to the nearest millilitre. — 122 ± 0.5
Glass **B** contains 168 millilitres of water, correct to the nearest millilitre. — 168 ± 0.5

Calculate the upper bound of the difference, in millilitres, between the volume of water in glass **A** and the volume of water in glass **B**.

HIGHEST − LOWEST

$168.5 - 121.5$

B1 CORRECT BOUNDS

47 A1 .... millilitres

Make n the subject of the formula

$$t = \sqrt{\frac{n+3}{n}}$$

$$t^2 = \frac{n+3}{n}$$ M1 [SQUARING]

$$nt^2 = n + 3$$
$$nt^2 - n = 3$$
M1 [EITHER]

$$n(t^2 - 1) = 3$$ M1 [FACTORISING]

$$n = \frac{3}{t^2 - 1}$$ A1

Boris and Nigel play games of chess against each other in a match.
In each game, Boris wins or Nigel wins or the game is a draw.

When a player wins a game, he wins the match.
When a game is a draw, the players play another game against each other.
Boris and Nigel play a maximum of 3 games.

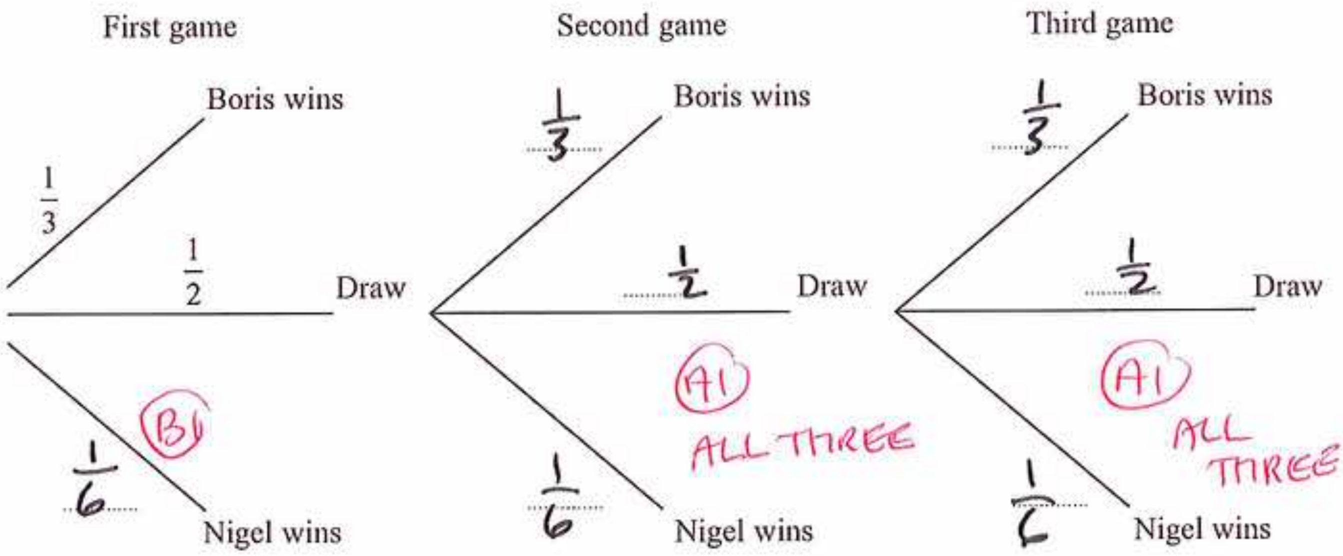
The probability that Boris wins a game is $\frac{1}{3}$

The probability that a game is a draw is $\frac{1}{2}$

$$\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\therefore \text{P(NIGEL WINS)} = \frac{1}{6}$$

(a) Complete the probability tree diagram.



(3)

(b) Calculate the probability that Boris wins the match.

$$P(B) = \frac{1}{3}$$

$$P(DB) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \text{M1}$$

$$P(DDB) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12} \quad \text{M1}$$

$$\text{TOTAL} = \frac{7}{12} \quad \text{A1}$$

A particle is moving in a straight line which passes through a fixed point O.
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = 18 + 9t^2 - t^3$$

(a) Find an expression for the velocity, v m/s, of the particle at time t seconds.

DIFFERENTIATE

B1 B1

$v = 18t - 3t^2$

(2)

(b) Find the time at which the acceleration of the particle is zero.

DIFFERENTIATE AGAIN!

$a = 18 - 6t$

WHEN a = ZERO

$18 - 6t = 0$ (M1)

$\Rightarrow -6t = -18$

$t = \frac{-18}{-6}$

$= 3$ SECS

(A1)

PTR and *QTS* are chords of a circle.

$PT = 3$ cm.
$ST = 10$ cm.
$RT = 15$ cm.
$QT = x$ cm.

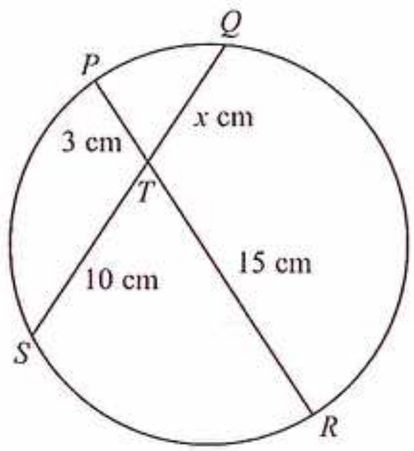


Diagram **NOT** accurately drawn

Calculate the value of *x*.

$$ST \times QT = PT \times RT$$
$$10 \times x = 3 \times 15 \quad \text{M1}$$
$$x = \frac{3 \times 15}{10}$$

$x =$ 4.5 A1

A bag contains x counters.
7 of the counters are blue.

$P(B) = \frac{7}{x}$

Sam takes at random a counter from the bag and does not replace it.
Jill then takes a counter from the bag.
The probability they both take a blue counter is 0.2

PROBABILITIES CHANGE

(a) Form an equation involving x.
Show that your equation can be expressed as $x^2 - x - 210 = 0$

$P(BB) \quad \frac{7}{x} \times \frac{6}{x-1} = 0.2$ B1

$\Rightarrow 42 = 0.2x(x-1)$ M1 [TWO+ STEPS OF WORKING]

$\Rightarrow 42 = 0.2x^2 - 0.2x \Rightarrow 210 = x^2 - x$

$\Rightarrow x^2 - x - 210 = 0$

(b) Solve $x^2 - x - 210 = 0$
Show clear algebraic working.

$x^2 - x - 210 = 0$

$(x+14)(x-15) = 0$ M1

$x = -14 \qquad x = 15$ A1 [FOR TWO SOLUTIONS]

[NOT POSSIBLE]

A1 [FOR SELECTING $x = 15$ AS ACTUAL SOLUTION]

$$\left(\sqrt{a}+\sqrt{8a}\right)^2 = 54 + b\sqrt{2}$$

a and b are positive integers.
Find the value of a and the value of b.
Show your working clearly.

$$(\sqrt{a}+\sqrt{8a})(\sqrt{a}+\sqrt{8a})$$

$$= a + \sqrt{a}\sqrt{8a} + \sqrt{8a}\sqrt{a} + 8a$$

$$= a + 2\sqrt{8a \times a} + 8a$$

$$= a + 2a\sqrt{8} + 8a$$

$$= 9a + 2a\sqrt{8}$$

$$= 9a + 2a \times 2\sqrt{2}$$

$$= 9a + 4a\sqrt{2}$$ A1

M1 ANY CORRECT EXPANSION

COMPARING

$9a + 4a\sqrt{2}$

$54 + b\sqrt{2}$

$9a = 54$

$a = \underline{6}$

$b = 4a$

$= 4 \times 6$

$= \underline{24}$

A1 FOR BOTH

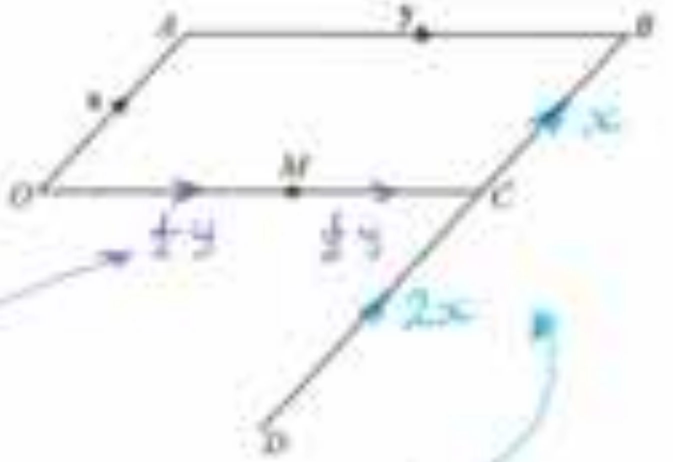


Diagram **NOT** accurately drawn

OABC is a parallelogram.
BCD is a straight line.
BD = 3*BC*
M is the midpoint of *OC*.

$\overrightarrow{OA} = \mathbf{x}$ $\overrightarrow{AB} = \mathbf{y}$

(a) Find, in terms of $\mathbf{x}$ and $\mathbf{y}$,

(i) $\overrightarrow{AM}$

$= \overrightarrow{AO} + \overrightarrow{OM}$

$= -x + \frac{1}{2}y$

A1

$-x + \frac{1}{2}y$

$= \frac{1}{2}(-2x + y)$

(ii) $\overrightarrow{OD}$

$= \overrightarrow{OC} + \overrightarrow{CD}$

$= y - 2x$

A1

$-2x + y$

[2]

(b) Use your answers to (a)(i) and (ii) to write down two different geometric facts about the lines *AM* and *OD*.

SINCE $\overrightarrow{AM} = \frac{1}{2}\overrightarrow{OD}$, AM AND OD ARE PARALLEL

AM IS HALF THE LENGTH OF OD

B1

B1

[2]

The diagram shows a cube *ABCDEFGH*.
The sides of the cube are of length 5 cm.

Calculate the size of the angle between the diagonal *AH* and the base *EFGH*.
Give your answer correct to 1 decimal place.

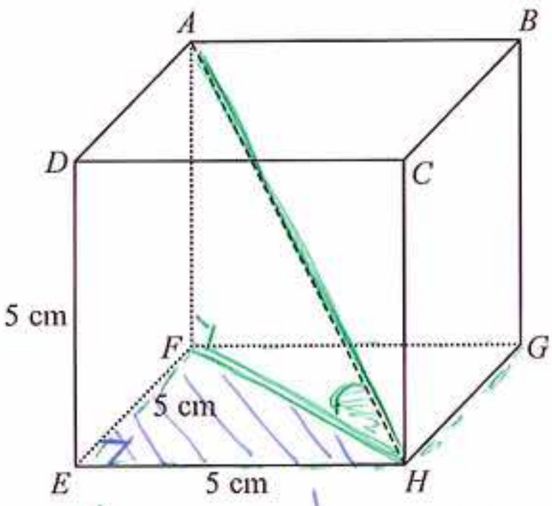


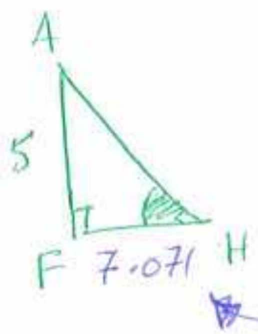
Diagram **NOT** accurately drawn

1ST $FH^2 = 5^2 + 5^2$ M1

$= 50$

$FH = \sqrt{50}$ A1

$= 7.07106..$



2ND

$\tan H = \frac{OPP}{ADJ}$

$\tan H = \frac{5}{7.071}$

$H = \tan^{-1}\left(\frac{5}{7.071}\right)$

M1 EITHER

$= 35.264$ A1

$= 35.3$ cm

Solve the simultaneous equations

$x^2 + y^2 = 26$ —— ①

$y = 3 - 2x$ —— ②

Show clear algebraic working.

SUBSTITUTE ② INTO ①

$x^2 + (3-2x)^2 = 26$

$x^2 + (3-2x)(3-2x) = 26$

$x^2 + 9 - 6x - 6x + 4x^2 = 26$

F O I L

$5x^2 - 12x + 9 = 26$

$5x^2 - 12x - 17 = 0$

$(5x - 17)(x + 1) = 0$

$5x_1 - 17 = 0$

$5x_1 = 17$

$x_1 = \frac{17}{5}$

$= 3.4$

$y_1 = 3 - 2 \times 3.4$

$= 3 - 6.8$

$= -3.8$

$x_2 + 1 = 0$

$x_2 = -1$

$y_2 = 3 - 2 \times (-1)$

$= 3 + 2$

$= 5$