3H(R)

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

MAY 2014

4MA0/3HR

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Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

(a) Complete the table to show each number written correct to 1 significant figure.

Number	42.37	58.92	21.04
Number written correct to 1 significant figure	40	60	20

B2

(2)

(b) Use the approximations in part (a) to work out an estimate for the value of

$$\frac{42.37 + 58.92}{21.04}$$

Show clearly how you obtain your answer.

M1 $\frac{40 + 60}{20} = \frac{100}{20}$

$= 5$ A1

$\mathscr{E}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {1, 3, 5, 7}
B = {2, 4, 6, 8}

(a) Explain why $A \cap B = \varnothing$

THERE ARE NO NUMBERS THAT ARE IN BOTH SET A AND SET B (A1)

(1)

$x \in \mathscr{E}$ and $x \notin A \cup B$

(b) Write down the value of x.

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$x =$ 9 (A1)

(1)

$A \cap C = \{3, 7\}$, $B \cap C = \{8\}$ and $A \cup B \cup C = \mathscr{E}$

(c) List all the members of C.

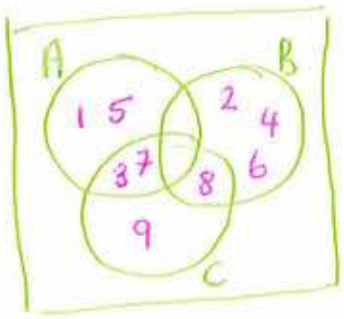
$3, 7 \in C$

$8 \in C$

$9 \in C$

(A2)

$C = \{3, 7, 8, 9\}$



The diagram shows a rectangle $PQRS$.
$PQ = 14$ cm and $QR = 9$ cm.
The point A lies on PS so that $PA = 5$ cm.
The point B lies on SR so that $BR = 8$ cm.

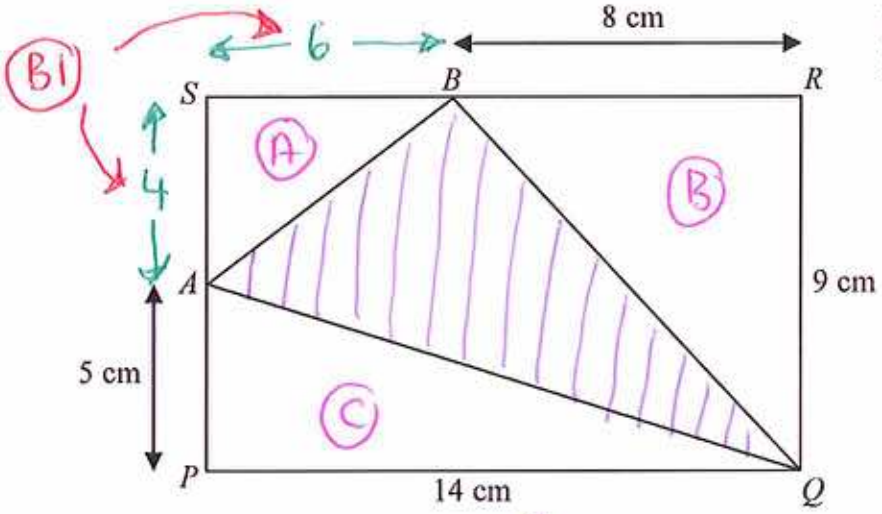
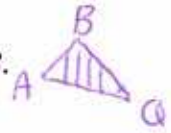


Diagram **NOT** accurately drawn

(a) Work out the area of triangle AQB.



AREA OF PQRS − AREA OF TRIANGLES A, B AND C

$= 9 \times 14 - \left(\frac{6 \times 4}{2} + \frac{8 \times 9}{2} + \frac{14 \times 5}{2}\right)$ M1 M1

$= 63\text{ cm}^2$ A1

(b) Work out the length of AQ.
Give your answer correct to 3 significant figures.

$AQ^2 = 14^2 + 5^2$ M1

$= 221$

$AQ = \sqrt{221}$ M1

$= 14.866\ldots$ → 14.9 A1 cm

Freya keeps hens.
The table shows information about the number of boxes of eggs she sold in each of 52 weeks.

Number of boxes sold in a week	Number of weeks f	MID VALUE (x)	fx
0 to 4	2	2	4
5 to 9	6	7	42
10 to 14	20	12	240
15 to 19	13	17	221
20 to 24	8	22	176
25 to 29	3	27	81
	52	TOTAL	764

(a) Write down the modal class.

↑ HIGHEST FREQUENCY (M1)

10 TO 14

(1)

(b) Work out an estimate for the mean number of boxes of eggs that Freya sold each week.
Give your answer correct to 3 significant figures.

$$\text{MEAN NO. OF BOXES} = \frac{\text{TOTAL NO. OF BOXES}}{\text{NO. OF WEEKS}}$$

$$= \frac{764}{52} \quad \text{(B1) (M1)}$$

$$= 14.6923...$$

14.7 (A1)

(4)

(a) Factorise $7dg - 9de$

A1 A1

$d(7g - 9e)$

(2)

(b) Expand and simplify $(x + 2)(x + 5)$

F O I L

$x^2 + 5x + 2x + 10$ M1

A1

$x^2 + 7x + 10$

(2)

Solve $3(2z - 5) = 4z + 11$
Show clear algebraic working.

$6z - 15 = 4z + 11$ M1 [EXPAND BRACKETS]

$6z - 4z = 11 + 15$ M1 [Z -TERMS TOGETHER]

$2z = 26$

$z = 13$ A1

The table gives some information about the average price of a litre of petrol in England.

	January 2007	January 2012
Average price of a litre of petrol (pence)	87.3	133.3

(a) Work out the percentage increase in the average price of a litre of petrol in England between January 2007 and January 2012.
Give your answer correct to 3 significant figures.

INCREASE IS $133.3 - 87.3 = 46$ (M1)

$$\% \text{ INCREASE} = \frac{46}{87.3} \times 100 \quad \text{(M1)}$$

$$= 52.7\% \quad \text{(A1)}$$

(a) Complete the table of values for $y = x^2 - 5x + 4$

x	0	1	2	3	4	5
y	4	0	−2	−2	0	4

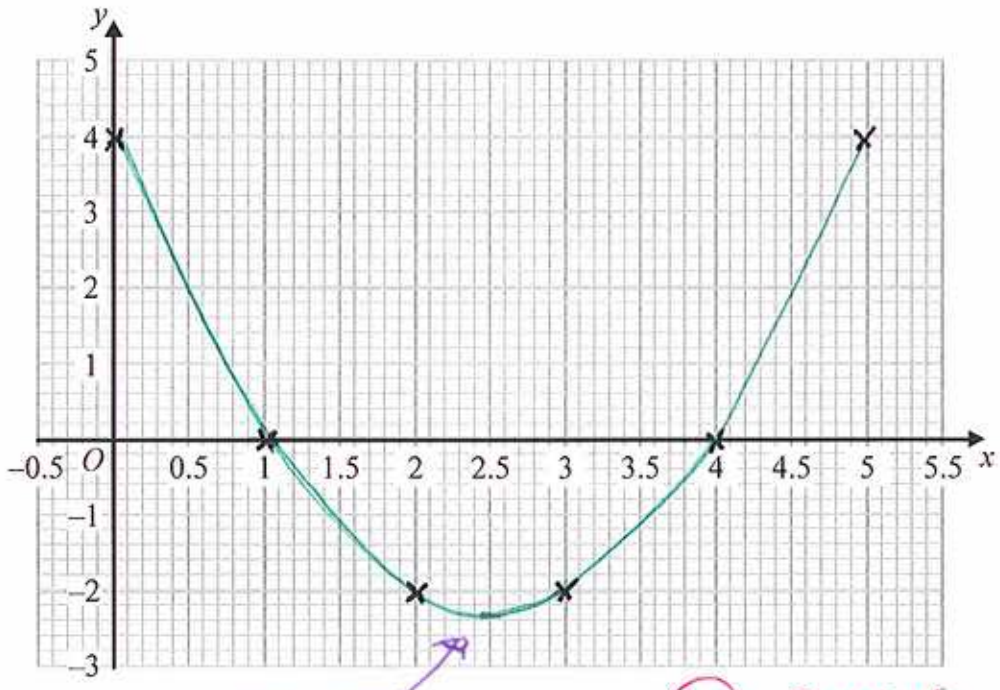
A2

$y = (4)^2 - 5 \times (4) + 4$ ETC

(2)

(b) On the grid, draw the graph of $y = x^2 - 5x + 4$ for all values of x from $x = 0$ to $x = 5$

(2)



NOTE THAT THE TURNING POINT DROPS BELOW $y = -2$!

A1 POINTS

A1 SMOOTH CURVE

A cylinder has diameter 12 cm and length 30 cm.

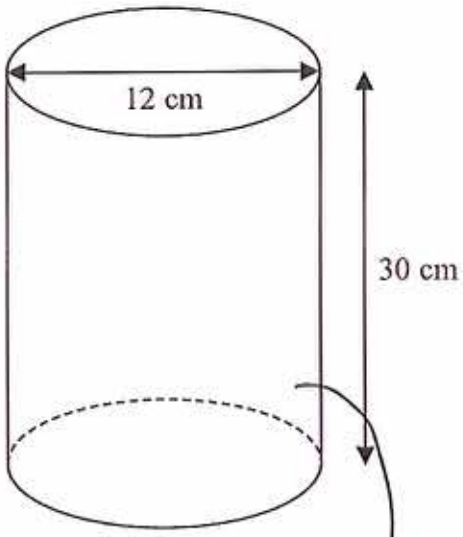


Diagram **NOT** accurately drawn

Work out the curved surface area of the cylinder.
Give your answer correct to 3 significant figures.

CURVED SURFACE $= 2\pi rh$ (B1)

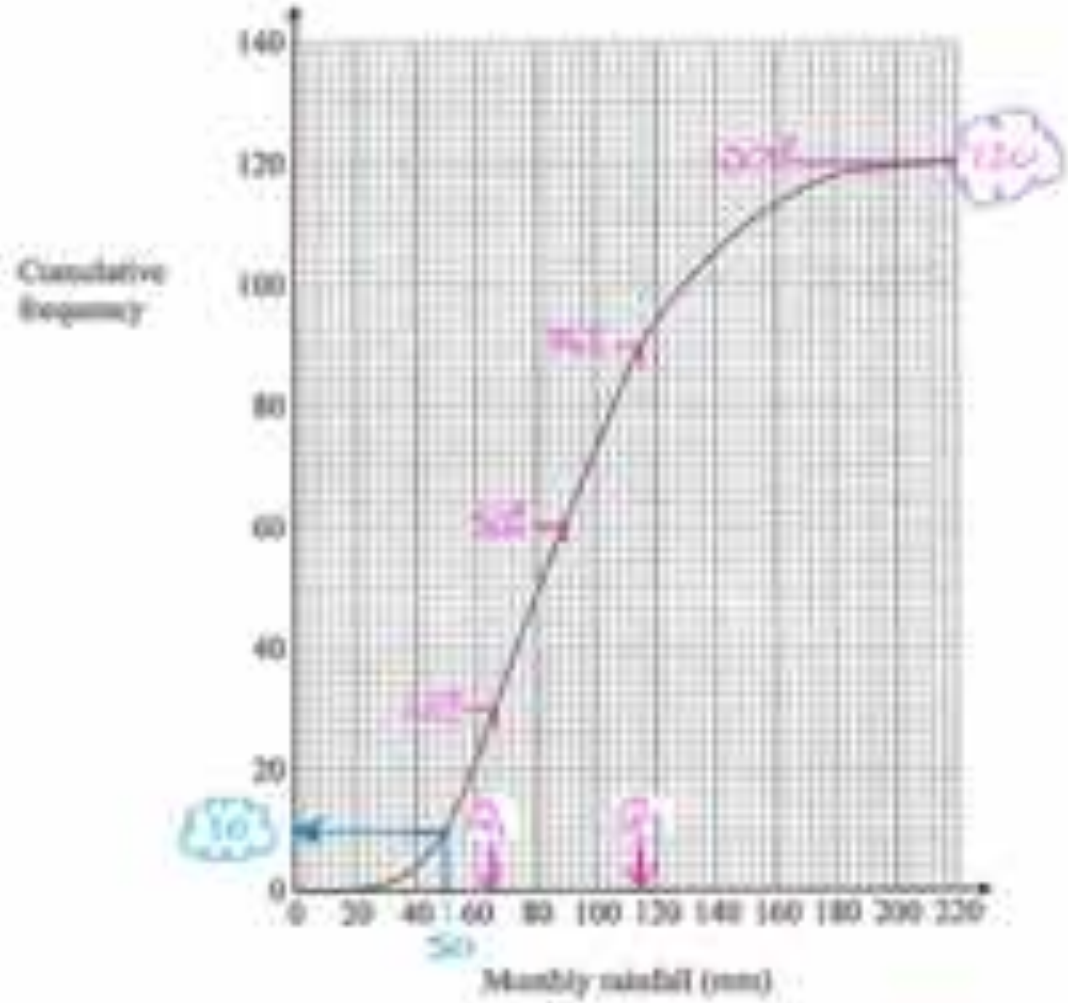
$= 2 \times \pi \times 6 \times 30$ (M1)

$= 1130.97...$

$= \underline{1130}\ cm^2$

(A1)

The cumulative frequency graph gives information about the monthly rainfall, in millimetres, in the United Kingdom during 120 months in the years 2001 to 2010.



(a) Use the graph to estimate the number of months for which rainfall was less than 50 mm.

10

(1)

(b) Use the graph to find an estimate for the median monthly rainfall.

$\frac{120}{2}$ = 60th VALUE

90 mm

(1)

(c) Use the graph to find an estimate for the interquartile range of the monthly rainfall.

IQR = $Q_3 - Q_1$

= 116 − 66

50 mm

(2)

The functions f and g are defined as

$$f(x) = \frac{1}{2}x + 4$$

$$g(x) = \frac{2x}{x+1}$$

(a) Work out f(6)

$\frac{1}{2} \times 6 + 4$

7 (A1)

(1)

(b) Work out fg(–3)

(B1)

$g(-3) = \frac{2\times(-3)}{(-3)+1} = 3$, $f(3) = \frac{1}{2}\times 3 + 4$

$= 5.5$ (A1)

(c) $g(a) = -2$

Work out the value of a.

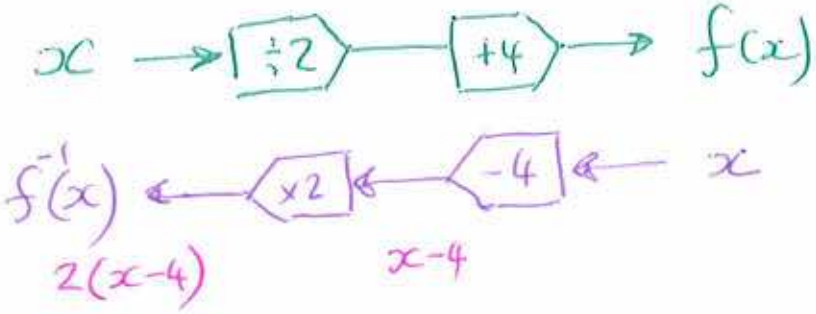
(M1)

$\frac{2a}{a+1} = -2 \Rightarrow 2a = -2a - 2$

$\Rightarrow 4a = -2$

$a = -\frac{1}{2}$ (A1)

(d) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$



(A1) (A1)

$f^{-1}(x) = 2(x-4)$

In the diagram, $DAPS$ and $CBQR$ are straight lines.
AB is parallel to QP and DC is parallel to RS.
$AD = 11$ cm, $BC = 5$ cm, $PS = 27.5$ cm and $RS = 42.5$ cm.

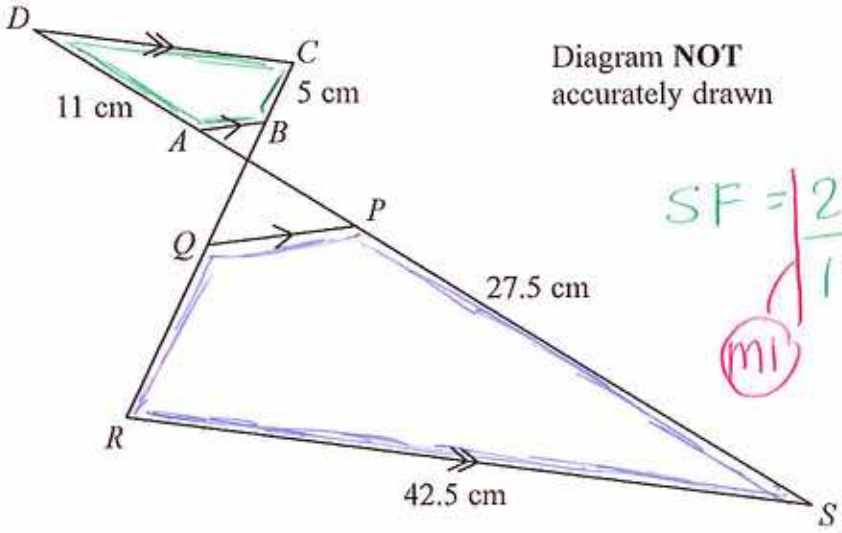


Diagram **NOT** accurately drawn

SF = 27.5 / 11 = 2.5 M1

Quadrilateral $ABCD$ is similar to quadrilateral $PQRS$.

(a) Find the ratio of the length of AB to the length of PQ.
Give your answer in the form $1 : n$

AB : PQ = 1 : 2.5 B1

1 : 2.5

(2)

(b) Work out the length of RQ.

RQ = CB × 2.5

= 5 × 2.5 M1

B1

12.5 cm

(2)

(c) Work out the length of CD.

CD = RS / 2.5 = 42.5 / 2.5 M1 = 17 cm B1

Solve the simultaneous equations

$3x + 4y = 6$ — (1) × 3

$5x + 6y = 11$ — (2) × 2

Show clear algebraic working.

M1 $9x + 12y = 18$ — (3)

$10x + 12y = 22$ — (4)

} SUBTRACT

$-x = -4$

$\Rightarrow x = 4$ A1

SUBSTITUTE $x = 4$ INTO EQ (1)

$3 \times 4 + 4y = 6$ M1

$4y = 6 - 12$

$4y = -6$

$y = -\frac{6}{4}$

$= -1.5$ A1

$x =$ 4

$y =$ −1·5

(a) $y = 2x^3 + 3x^2 + 2$

Find $\frac{dy}{dx}$

A1 A1

$6x^2 + 6x$

(2)

(b) The point P lies on the curve with equation $y = 2x^3 + 3x^2 + 2$

The gradient of the curve at P is $-\frac{3}{2}$

Find the coordinates of P.

$\frac{dy}{dx} = -\frac{3}{2}$

$6x^2 + 6x = -\frac{3}{2}$ M1

$\Rightarrow 12x^2 + 12x = -3$

$\Rightarrow 12x^2 + 12x + 3 = 0$

$\Rightarrow 4x^2 + 4x + 1 = 0$ M1

$(2x+1)(2x+1) = 0$

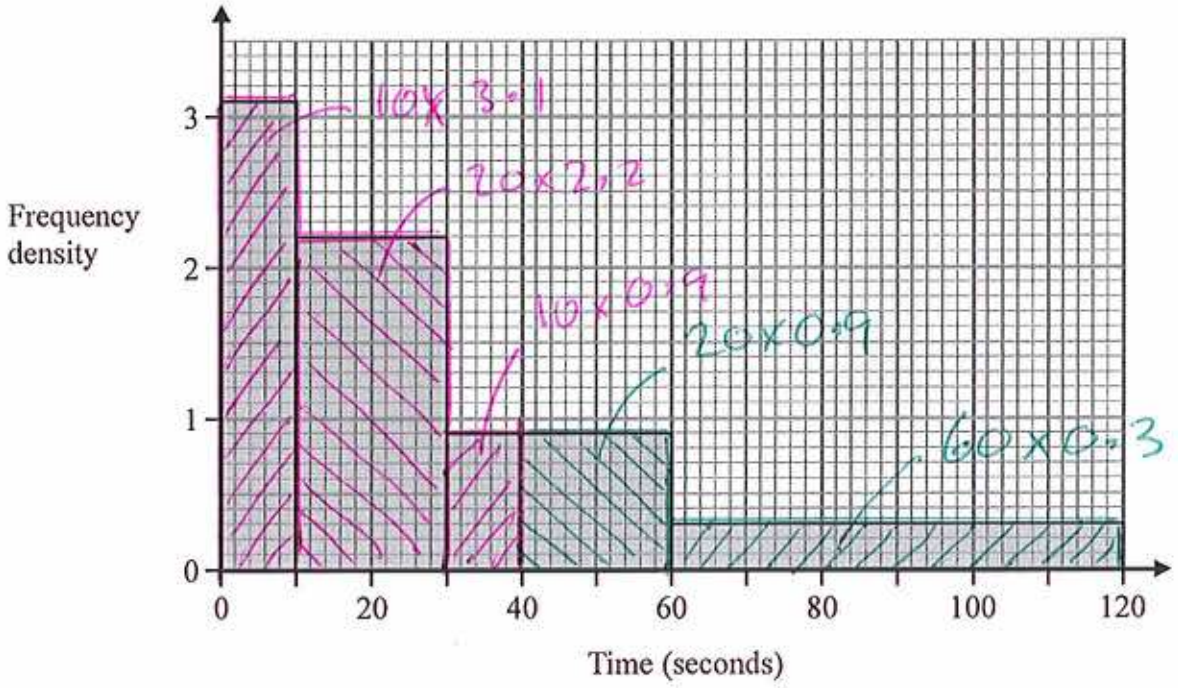
$x = -\frac{1}{2}$ A1

$\Rightarrow y = 2\left(-\frac{1}{2}\right)^3 + 3\times\left(-\frac{1}{2}\right)^2 + 2$ M1

$= 2.5$ A1

$(-0.5, 2.5)$

The histogram shows information about the times taken by a telephone call centre to answer incoming calls.



Work out an estimate for the percentage of calls that are answered in less than 40 seconds.

CALLS ANSWERED IN LESS THAN 40 SECONDS

$10 \times 3.1 + 20 \times 2.2 + 10 \times 0.9 = 84$ (B1)

TOTAL CALLS ANSWERED

$10 \times 3.1 + \ldots + \ldots + 60 \times 0.3 = 120$ (B1)

$$\% = \frac{84}{120} \times 100$$

$= 70\%$ (A1)

(a) Expand $(5 + 3\sqrt{2})^2$

Give your answer in the form $(a + b\sqrt{2})$, where a and b are integers.
Show your working clearly.

$(5+3\sqrt{2})(5+3\sqrt{2}) = 25 + 15\sqrt{2} + 15\sqrt{2} + 9 \times 2$

$= 43 + 30\sqrt{2}$ (A1) (A1)

(b) $(5 + 3\sqrt{2})^2 = p + \frac{q}{\sqrt{8}}$, where p and q are integers.

Find the value of q.

COMPARING SURD PARTS :-

$\frac{q}{\sqrt{8}} = 30\sqrt{2}$ (M1) $\Rightarrow q = 30\sqrt{2} \times \sqrt{8}$ (M1)

$= 30 \times \sqrt{16}$

$= 120$ (A1)

The table shows information about the 40 coins in Karam's money box.

	Bronze coins		Silver coins			
Value of coin (pence)	1	2	5	10	20	50
Number of coins	6	8	12	7	3	4

Karam shakes his money box until a coin falls out at random.
He does not replace the coin in the money box. — PROBABILITIES CHANGE!
Karam shakes his money box again until a second coin falls out at random.

(a) Work out the probability that both the coins that fall out are silver coins.

$P(5,5) = \frac{26}{40} \times \frac{25}{39}$ M1

$= \frac{650}{1560}$

A1

$\frac{5}{12}$

(2)

(b) Work out the probability that the total value of the two coins that fall out is 60 pence or more.

$P(10,50) = \frac{7}{40} \times \frac{4}{39} = \frac{28}{1560}$

$P(50,10) = \frac{4}{40} \times \frac{7}{39} = \frac{28}{1560}$

$P(20,50) = \frac{3}{40} \times \frac{4}{39} = \frac{12}{1560}$

$P(50,20) = \frac{4}{40} \times \frac{3}{39} = \frac{12}{1560}$

$P(50,50) = \frac{4}{40} \times \frac{3}{39} = \frac{12}{1560}$

TOTAL $= \frac{92}{1560}$

M1 [ALL COMBINATIONS]

M1 [ADDING TO GET TOTAL]

A1

$\frac{23}{390}$

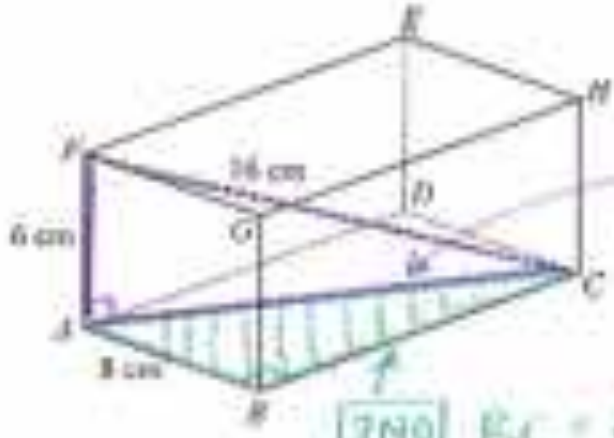


Diagram **NOT** accurately drawn

The diagram shows a cuboid $ABCDEFGH$.
$AB = 8$ cm, $BF = 6$ cm and $FC = 16$ cm.

(a) Find the length of BC.
Give your answer correct to 3 significant figures.

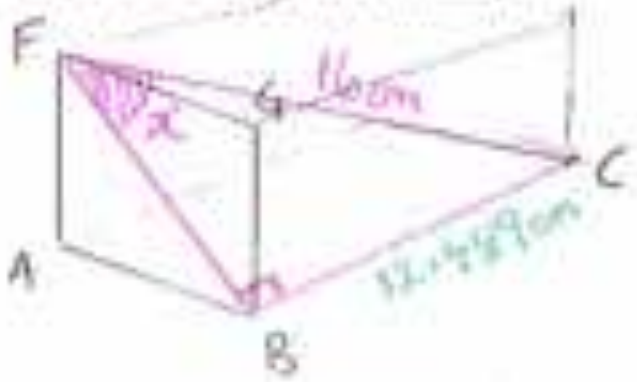
1ST
$AC = \sqrt{16^2 - 6^2}$ M1
$= 14.85$ M1

2ND $BC = \sqrt{14.85^2 - 8^2}$
$= 12.4899\ldots$
$= 12.5$ A1

WORKING OUT IS BY DIAGRAM

$BC = 12.5$ cm (3)

(b) Find the size of the angle between the line FC and the plane $ABGF$. } DIFFICULT !!
Give your answer correct to 1 decimal place.



$\sin x = \frac{OPP}{HYP}$

$\sin x = \frac{12.4899}{16}$ M1

$x = \sin^{-1}\left(\frac{12.4899}{16}\right)$

$= 51.3°$ A1

Solve the inequality $3x^2 + 5 < 53$

QUADRATICS HAVE TWO SOLUTIONS

$3x^2 + 5 < 53$

$3x^2 < 48$ (M1)

$x^2 < 16$

POSITIVE SOLUTION

$x < +\sqrt{16}$

$x < 4$ (A1)

$x > -\sqrt{16}$

$x > -4$ (A1)

SIGN SWITCHES FOR NEGATIVE SOLUTION!

NOTE: $x > -4$ AND $x < 4$

CAN COMBINE INTO $-4 < x < 4$

[THIS CANNOT ALWAYS BE DONE!]

Solve the equation $\frac{2^{(n^2)}}{2^n \times 2^6} = 1$

Show clear algebraic working.

$2^{n^2} = 2^n \times 2^6$

$\Rightarrow 2^{n^2} = 2^{n+6} \longrightarrow n^2 = n + 6$ (M1)

$\Rightarrow n^2 - n - 6 = 0$

$(n-3)(n+2) = 0$

$n = 3$ (A1) $\quad n = -2$ (A1)

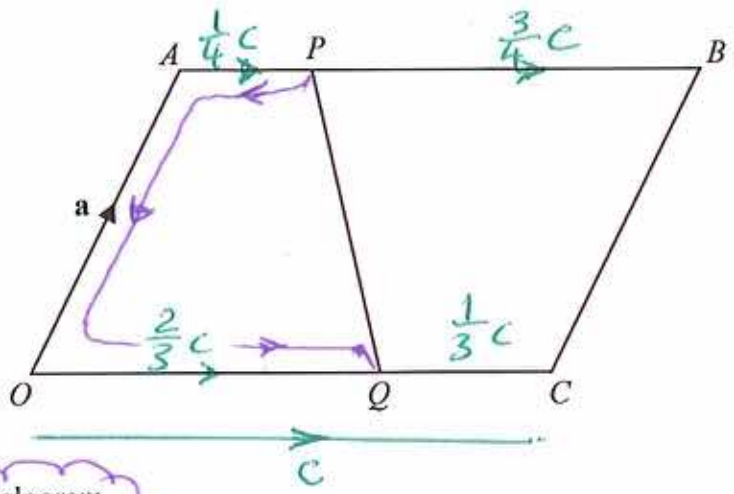


Diagram **NOT** accurately drawn

OABC is a parallelogram.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

P is the point on *AB* such that $AP = \frac{1}{4}AB$.

Q is the point on *OC* such that $OQ = \frac{2}{3}OC$.

Find, in terms of **a** and **c**, $\overrightarrow{PQ}$.
Give your answer in its simplest form.

$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AO} + \overrightarrow{OQ}$

$= -\frac{1}{4}\mathbf{c} - \mathbf{a} + \frac{2}{3}\mathbf{c}$ M1

$= \left(\frac{2}{3} - \frac{1}{4}\right)\mathbf{c} - \mathbf{a}$

$= \frac{5}{12}\mathbf{c} - \mathbf{a}$ A1

A1