3H

Pearson Edexcel
International GCSE

EDEXCEL
IGCSE

MATHEMATICS A
SOLUTIONS

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4MA0/3H

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

The ocean liner Queen Mary 2 is the longest of its type.
It has a length of 345 metres.

A scale model is made of the Queen Mary 2
The scale of the model is 1 : 200

Work out the length of the scale model.
Give your answer in centimetres.

REAL IS 200 × BIGGER!

(M1)

[÷ 200]

$$\frac{345}{200} = 1.725 \text{ METRES}$$

(A1)

$$= 172.5 \text{ cm}$$

(A1)

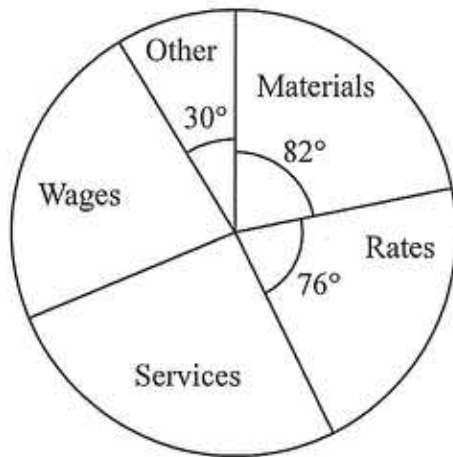The pie chart gives information about the amounts spent by a gas company in one year.

Diagram **NOT** accurately drawn

Other 30°

Materials 82°

Wages

Rates 76°

Services

$82\% \equiv 225.5$ MILLION
$\therefore 1\% \equiv 2.75$ MILLION

(M1)

The amount spent on materials was 225.5 million euros.
The amount spent on services was the same as the amount spent on wages.

Work out the amount spent on services.

SERVICES + WAGES = 360 − (76 + 82 + 30)

= 172

∴ SERVICES = WAGES = 86°   ← (B1)

⇒ AMOUNT SPENT ON SERVICES = 86 × 2.75

= 236.5 MILLION   (A1)

The first four terms of an arithmetic sequence are

$$5 \quad 9 \quad 13 \quad 17$$

$+4$

$\div 4$ $4n + ?$

(a) Write down an expression, in terms of n, for the nth term.

(A1) (A1)

$4n + 1$

(2)

(b) Write down an expression, in terms of n, for the $(n + 1)$th term.

$$4(n+1) + 1$$
$$= 4n + 4 + 1$$
$$= 4n + 5 \quad \text{(A1)}$$

w, x, y and z are 4 integers written in order of size, starting with the smallest.

The mean of w, x, y and z is 13
The sum of w, x and y is 33

$\rightarrow$ TOTAL $= 4 \times 13$
$= 52$ (B1)

(a) Find the value of z.

$$z = 52 - 33$$

$z =$ _____19_____ (A1)

(2)

Given also that the range of w, x, y and z is 10,

(b) work out the median of w, x, y and z.   (1st)

$$\boxed{9} \; \boxed{} \; \boxed{} \; \boxed{19}$$
$$\;\; w \quad x \quad y \quad z$$
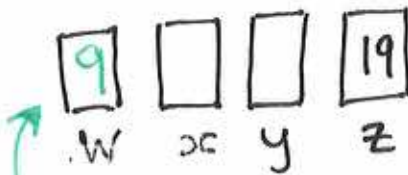
$z - w = 10$
$19 - w = 10$
$\Rightarrow w = 9$ (B1)

$x + y = 52 - (9 + 19)$
$= 24$

$\therefore \dfrac{x+y}{2} = 12$ [MEDIAN] (A1)

On 1st May 2012, the cost of 5.7 grams of gold was 15 960 rupees.

(a) Work out the cost, in rupees, of 4.6 grams of gold on the same day.

5.7 GRAMS COST 15 960

4.6 GRAMS COST $\dfrac{15960 \times 4.6}{5.7}$ (m1)

(A1)

.......12 880....... rupees

(2)

The cost of gold decreased by 7.5% from 1st May 2012 to 1st May 2013

(b) Work out the cost, in rupees, of 5.7 grams of gold on 1st May 2013

92.5%
=0.925

15960 × 0.925

(m1) (B1)

[MULTIPLY]

(A1)

.......14 763....... rupees

(3)

A steam engine for pulling trains has wheels of diameter 1.5 metres.

(a) Calculate the circumference of a wheel.
Give your answer correct to 3 significant figures.

$C = \pi \times 1.5$ (m1)

$= 4.7123...$

4.71 (A1) m

(2)

The steam engine travels 1000 metres along a test track.

(b) Work out the number of complete turns of a wheel.

(m1) $\dfrac{1000}{4.7123...}$ $= 212.206...$

(A1) $= \underline{212}$ COMPLETE TURNS.

John changes £450 to euros.

The exchange rate is £1 = 1.16 euros.

(a) Change £450 to euros.

$$450 \times 1 \cdot 16 \quad \text{(m1)}$$

522   (A1)   euros

(2)

When in Amsterdam, John uses his credit card to pay for a ring costing 850 euros.

He has to pay a bank charge of £3.50 for using his credit card in addition to the cost of the ring.

(b) Work out the total cost, in pounds (£), of the ring and the bank charge.

$$\text{(m1)} \quad \frac{850}{1 \cdot 16} = 732 \cdot 76$$

$$732 \cdot 76 + 3 \cdot 50 = £736 \cdot 26 \quad \text{(A1)}$$

(m1)

Here is a right-angled triangle.
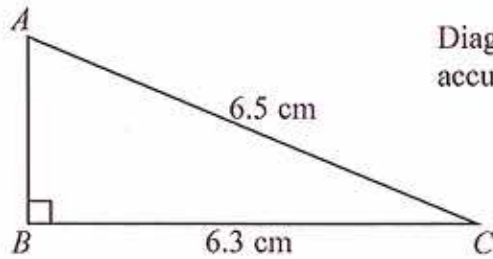
Diagram **NOT**
accurately drawn

$AC = 6.5$ cm.
$BC = 6.3$ cm.
Angle $ABC = 90°$

Calculate the length of AB.

$$AB^2 = 6.5^2 - 6.3^2 \quad \text{(M1)} \; [\text{SUBTRACT SQUARES}]$$

$$= 2.56$$

$$AB = \sqrt{2.56} \quad \text{(M1)} \; [\text{SQUARE ROOT}]$$

$$= 1.6$$

(A1)

............1.6............ cm

(a) Simplify $5y \times 4y^2$

$$\underline{20y^3}$$ (A1) (A1)

(2)

(b) Simplify $\dfrac{15e^2 f}{25ef^3}$

(A1) [FOR $\dfrac{3}{5}$ or 0.6] $\left\{ \dfrac{3e}{5f^2} \right.$ (A1) $\left[\dfrac{e}{f^2} \right]$

(2)

(c) Factorise $6p^2 - 5pq - 6q^2$

$(2p - 3q)(3p + 2q)$ (A2)

(2)

(d) Simplify $(x^{-y})^{-z}$

$$x^{yz}$$ (A1)

(1)

The table shows some information about the five Great Lakes in North America.

Name	Surface area (m^2)	Volume of water (m^3)
Lake Erie	2.57×10^{10}	4.80×10^{11}
Lake Huron	6.01×10^{10}	3.52×10^{12}
Lake Michigan	5.80×10^{10}	4.87×10^{12}
Lake Ontario	1.91×10^{10}	1.64×10^{12}
Lake Superior	8.21×10^{10}	1.22×10^{13}

(a) Work out the total surface area of the five Great Lakes.
Give your answer in standard form.

$$2.57 \times 10^{10} + 6.01 \times 10^{10} + \ldots \quad \text{(M1)}$$

$$\underline{2.45 \times 10^{11}} \text{ (A1)} \quad m^2$$

(2)

Loch Ness is the largest lake in Scotland.
The lake has a volume of water of 7.45×10^9 m^3

The volume of water in Lake Superior is k times the volume of water in Loch Ness.

(b) Work out the value of k.
Give your answer correct to 3 significant figures.

$$k = \frac{SUPERIOR}{NESS}$$

$$= \frac{1.22 \times 10^{13}}{7.45 \times 10^9} \text{ (M1)} = 1637.58\ldots$$

$$k = \underline{1640} \text{ (A1)}$$
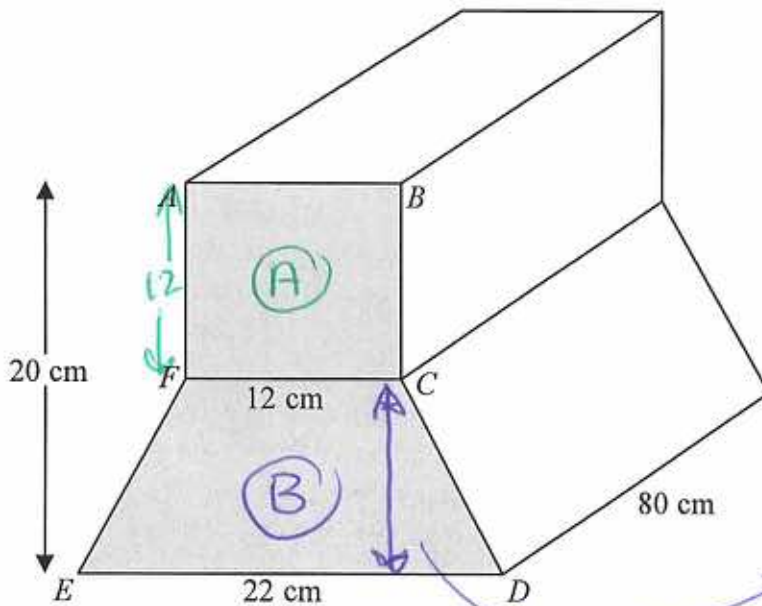
(2)

Here is a prism.

Diagram **NOT** accurately drawn

ABCDEF is a cross section of the prism.

ABCF is a square of side 12 cm.

FCDE is a trapezium.
ED = 22 cm.

The height of the prism is 20 cm.
The length of the prism is 80 cm.

Work out the total volume of the prism.

HEIGHT OF Ⓑ

$= 20 - 12$

$= 8$

AREA OF Ⓐ $= 12 \times 12 = \underline{144}$ Ⓜ

AREA OF Ⓑ $= \frac{1}{2} \times (12 + 22) \times 8 = \underline{136}$ Ⓜ

TOTAL AREA $= 144 + 136$ Ⓜ [ADDING AREAS]

$= \underline{280}$

VOLUME $= 280 \times 80$ Ⓜ [MULTIPLY BY '8']

$\underline{22400}$ Ⓐ cm³

There are 32 students in Mr Newton's class.
20 are boys and 12 are girls.

The mean height of the boys is 151 cm.
The mean height of the girls is 148 cm.

Calculate the mean height of all the students in Mr Newton's class.

TOTAL HEIGHT OF BOYS = 151 × 20 = 3020

" " " GIRLS = 148 × 12 = 1776 (M1)

TOTAL HEIGHT OF EVERYONE

3020 + 1776 = 4796

MEAN HEIGHT = $\frac{4796}{32}$ (M1)

= 149·875 (A1)

(a) Solve

$$3x + 3y = 9 \quad \longrightarrow \text{①} \times 2$$

$$4x + 2y = 13 \quad \longrightarrow \text{②} \times 3$$

Show clear algebraic working.

(m1)
$$6x + 6y = 18 \quad \longrightarrow \text{③}$$
$$12x + 6y = 39 \quad \longrightarrow \text{④}$$ } SUBTRACT

$$-6x = -21 \quad \text{(m1)}$$

$$x = \frac{-21}{-6}$$

$$= 3.5 \quad \text{(A1)}$$

SUBSTITUTE INTO ①

$$3 \times 3.5 + 3y = 9$$

$$\Rightarrow \quad y = \frac{9 - 3 \times 3.5}{3} = -0.5 \quad \text{(A1)}$$

L is a line parallel to the line with equation $4x + 2y = 13$

L passes through the point with coordinates $(3, -1)$

(b) Find an equation for the line L.

$$4x + 2y = 13$$
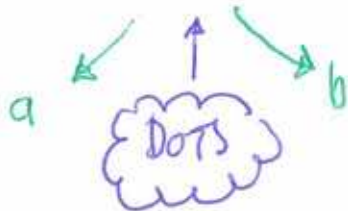
$$\Rightarrow \quad 2y = -4x + 13$$

$$\Rightarrow \quad y = -2x + 6.5$$

GRADIENT $= -2$ (B1)

(m1)
$$(y - -1) - 2(x - 3)$$

$$\Rightarrow y + 1 = -2x + 6$$

$$\Rightarrow y = -2x + 5 \quad \text{(A1)}$$

(a) Factorise $a^2 - b^2$

a {DOTS} b

(A1)

$$(a-b)(a+b)$$

(1)

$N = 2^{22} - 1$

(b) Write N as the product of two integers, both of which are greater than 1000

$$2^{22} - 1$$

2^{11} {DOTS} 1
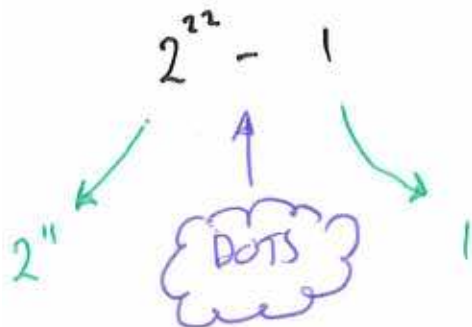
$$(2^{11} - 1)(2^{11} + 1) \quad \text{(M1)}$$

$$= (2048 - 1)(2048 + 1)$$

(A1)

$$2047 \times 2049$$

(2)

ABCD is a trapezium.

(OPP) 15

25 cm

E

10

x (

(ADJ)

D 90°

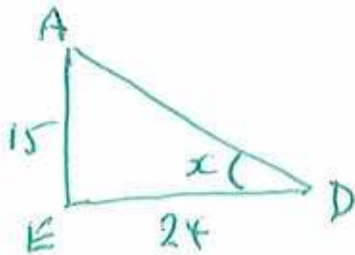10 cm

B   24 cm   *C*

Diagram **NOT**
accurately drawn
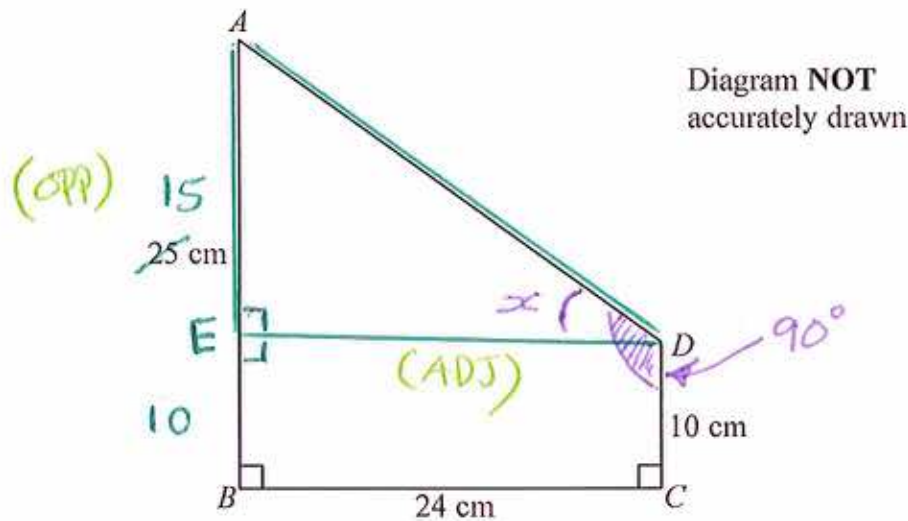
$AB = 25$ cm.
$BC = 24$ cm.
$CD = 10$ cm.

Angle ABC = angle $BCD = 90°$

Calculate the size of angle CDA.
Give your answer correct to 3 significant figures.

A

15

x (

E   24   D

$$TAN \, x = \frac{OPP}{ADJ}$$

$$TAN \, x = \frac{15}{24} \quad \text{(m1)}$$

$$\therefore \quad x = TAN^{-1}\left(\frac{15}{24}\right)$$

$$= 32.005... \quad \text{(A1)}$$

$$\therefore \quad CDA = 32.005 + 90 \quad \text{(m1)}$$

$$= 122° \quad \text{(A1)}$$

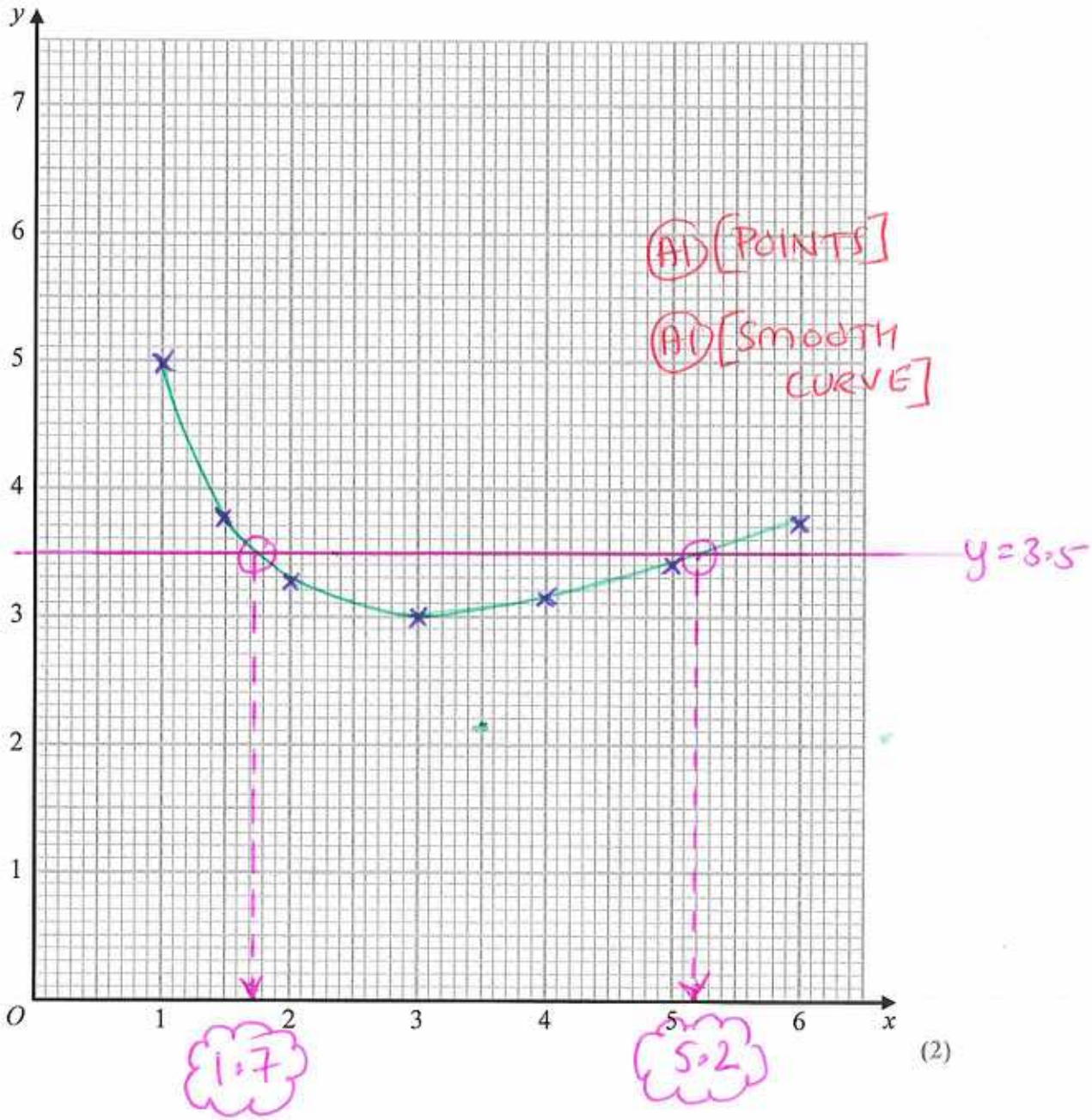(a) Complete the table of values for $y = \frac{1}{2}\left(x + \frac{9}{x}\right)$

$$y = 0.5 \times \left[(1.5) + \frac{9}{(1.5)}\right]$$

x	1	1.5	2	3	4	5	6
y	5	3.75	3.25	3	3.125	3.4	3.75

(A2)

(2)

(b) Draw the graph of $y = \frac{1}{2}\left(x + \frac{9}{x}\right)$ for values of x from 1 to 6

(A1) [POINTS]

(A1) [SMOOTH CURVE]

$y = 3.5$

{1.7}

{5.2}

(2)

(c) Use the graph to find estimates for the solutions of the equation $x + \dfrac{9}{x} = 7$

$$x + \frac{9}{x} = 7$$

$$\Rightarrow \underline{\frac{1}{2}\left[x + \frac{9}{x}\right]} = 3.5$$

$\downarrow$

CURVE

$x = 1.7, \quad x = 5.2$

(2)

(A1) [BOTH]

DRAW LINE
$y = 3.5$

(m1) [FOR LINE ON GRAPH]

$f(x) = \dfrac{3}{x+1} + \dfrac{1}{x-2}$

(a) State one value of x which cannot be included in any domain of f.

(A1) [EITHER]

$x = -1, \quad x = 2$

(1)

(b) Find the value of $f(0)$

$\dfrac{3}{0+1} + \dfrac{1}{0-2} = 3 - \dfrac{1}{2}$

2.5 (A1)

(1)

(c) Find the value of x for which $f(x) = 0$
Show clear algebraic working.

$\dfrac{3}{x+1} + \dfrac{1}{x-2} = 0$ (M1) [CORRECT EQUATION]

$\Rightarrow 3(x-2) + (x+1) = 0$ (M1) [CLEARING FRACTIONS]

$\Rightarrow 3x - 6 + x + 1 = 0$

$\Rightarrow 4x - 5 = 0$

$4x = 5$

$x = \dfrac{5}{4}$ (A1)

$y = \dfrac{2a}{b-c}$

$a = 42$ correct to 2 significant figures.
$b = 24$ correct to 2 significant figures.
$c = 14$ correct to 2 significant figures.

Work out the lower bound for the value of y.
Give your answer correct to 2 significant figures.
Show your working clearly.

42 ± 0.5
24 ± 0.5 (m1) [FOR ANY CORRECT BOUNDS]
14 ± 0.5

$y = \dfrac{2a}{b-c}$ [SMALLEST] [LARGEST]

$= \dfrac{2 \times 41.5}{24.5 - 13.5}$ (m1)

$= \underline{\underline{7.5}}$ (A1)

The table gives information about the areas of some farms in France.

Area (A hectares)	Frequency	WIDTH	FREQ. DEN
$0 < A \leqslant 20$	50	20	2·5
$20 < A \leqslant 50$	90	30	3
$50 < A \leqslant 100$	120	50	2·4
$100 < A \leqslant 300$	160	200	0·8

USE {FREQUENCY DENSITY = FREQUENCY / WIDTH.}

On the grid, draw a histogram to show this information.
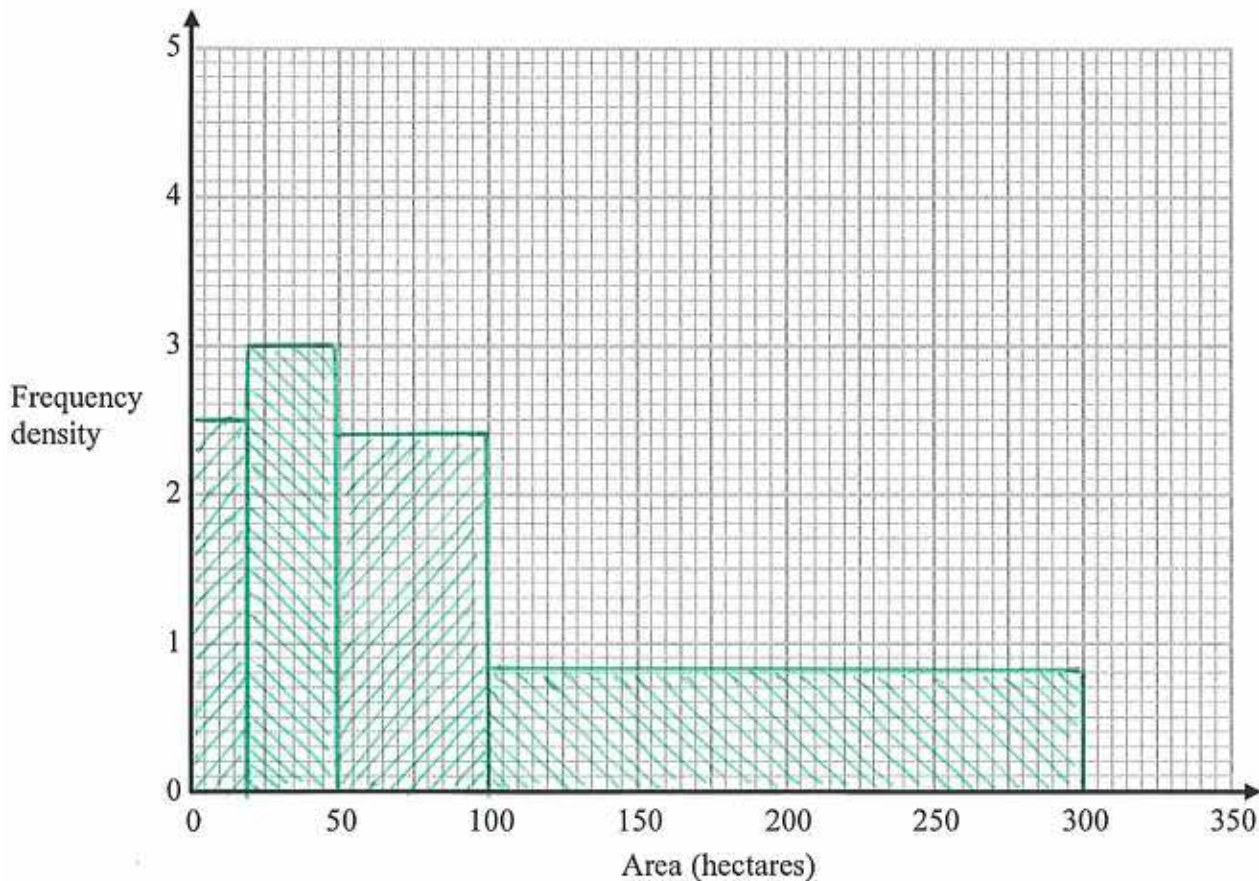
TWO CORRECT BARS ⇒ A1
THREE " " ⇒ A2
ALL FOUR CORRECT ⇒ A3

Leonidas has a fair dice.

He throws the dice twice.
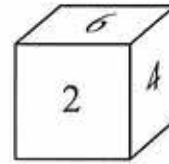
(a) Work out the probability that he gets the number 5 both times.

$$P(5,5) = \frac{1}{6} \times \frac{1}{6}　\text{(m1)}$$

$$\frac{1}{36}　\text{(A1)}$$

(2)

Alicia has a fair dice.

She throws the dice 3 times.

(b) Work out the probability that she gets the number 5 exactly once.

(m1) [THREE POSSIBILITIES]

$$P(5,\bar{5},\bar{5}) = \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{25}{216}$$

$$P(\bar{5},5,\bar{5}) = \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{25}{216}$$

$$P(\bar{5},\bar{5},5) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$\frac{75}{216}$$

(m1) [MULTIPLYING]

$$\frac{25}{72}　\text{(A1)}$$

(3)
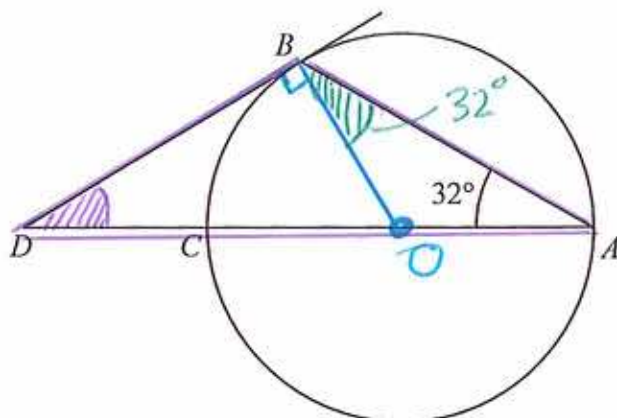
Diagram **NOT**
accurately drawn

A, B and C are three points on a circle.

DCA is a straight line.
CA is a diameter of the circle.
DB is a tangent to the circle.

Calculate the size of angle CDB.

1ST

DRAW LINE OB; ANGLE OBD = 90° **B1**
          [TANGENT MEETS A RADIUS]

2ND

ANGLE ABO = 32° **B1**
[TRIANGLE OAB IS ISOSCELES]

3RD

CDB = 180 − (32 + 122) = 26°

[ANGLES IN A TRIANGLE ADD TO 180°]

26 ° **A1**

A, r and T are three variables.

A is proportional to T^2
A is also proportional to r^3

$T = 47$ when $r = 0.25$

Find r when $T = 365$
Give your answer correct to 3 significant figures.

[handwritten annotation in cloud:] $A \propto T^2$ AND $A \propto r^3 \Rightarrow T^2 \propto r^3$

[handwritten annotation:] USE THIS!

$$T^2 = kr^3 \qquad (T = 47, \ r = 0.25)$$

$$\Rightarrow 47^2 = k \times 0.25^3 \quad \text{(m1)}$$

$$\Rightarrow k = \frac{47^2}{0.25^3}$$

$$= 141376 \quad \text{(m1)}$$

$$\Rightarrow T^2 = 141376 r^3 \qquad \text{OR} \qquad r^3 = \frac{T^2}{141376}$$

[brace:] (m1) [EITHER]

$$\Rightarrow r = \sqrt[3]{\frac{T^2}{141376}}$$

$$\Rightarrow r = \sqrt[3]{\frac{365^2}{141376}}$$

$$= 0.98040\ldots$$

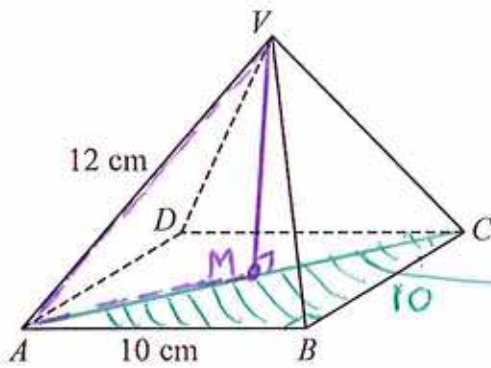$$= 0.980 \quad \text{(A1)}$$

Diagram **NOT**
accurately drawn

$AC = \sqrt{10^2 + 10^2}$

ABCD is the square base of the pyramid *VABCD*.

$AB = BC = CD = DA = 10$ cm.
$VA = VB = VC = VD = 12$ cm.

Calculate the height of the pyramid.
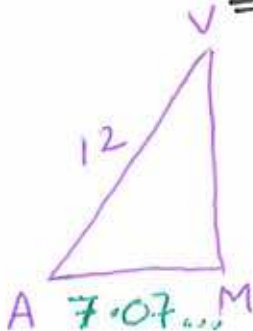Give your answer correct to 3 significant figures.

IF 'M' IS MIDPOINT OF AC
THEN HEIGHT IS MV

$AC = \sqrt{10^2 + 10^2}$

$= 14.142\ldots$ (m1)

$AM = \dfrac{14.142\ldots}{2}$

$= 7.07106\ldots$ (m1)

$VM = \sqrt{12^2 - 7.07\ldots^2}$ (m1)

$= 9.695\ldots$

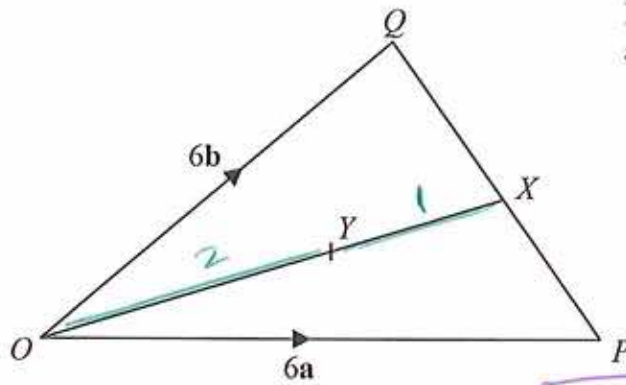$= 9.70$ cm   (A1)

12

A   7.07...   M

Diagram **NOT** accurately drawn

In triangle OPQ, $\vec{OP} = 6\mathbf{a}$ and $\vec{OQ} = 6\mathbf{b}$

X is the midpoint of PQ.

(a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector $\vec{OX}$
    Give your answer in its simplest form.

$$\boxed{1ST}$$

$$\vec{QP} = \vec{QO} + \vec{OP}$$
$$= -6b + 6a$$

$$\vec{OX} = \vec{OQ} + \frac{1}{2}\vec{QP}$$
$$= 6b + \frac{1}{2}(-6b + 6a)$$
$$= 6b - 3b + 3a$$

(m1) [EITHER]

$$\underline{3a + 3b} \quad \text{(A1)}$$

(2)

Y is the point on OX such that $OY : YX = 2 : 1$

(b) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector $\vec{QY}$
    Give your answer in its simplest form.

$$\vec{QY} = \vec{QO} + \vec{OY}$$
$$= \vec{QO} + \frac{2}{3}\vec{OX}$$
$$= -6b + \frac{2}{3}(3a + 3b)$$
$$= -6b + 2a + 2b$$

(m1) [EITHER]

$$\underline{2a - 4b} \quad \text{(A1)}$$

(2)