

# 3H(R)

Pearson Edexcel  
International GCSE

# EDEXCEL

# IGCSE

## MATHEMATICS A

# SOLUTIONS

### MAY 2016

### 4MA0/3HR

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Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

Rafael and Roger played tennis against each other 30 times.

Each of the times they played, either Rafael won or Roger won.

The ratio of the number of times Rafael won to the number of times Roger won is 7:3

(a) Work out the number of times Rafael won.

Ra : Ro	TOTAL	
7    3	10	
	$\frac{30}{10} = 3$	$7 \times 3 = \underline{\underline{21}}$
	(B1)	<u>21</u> (A1)
		(2)

In a school, there are 75 girls in the tennis squad.

The ratio of the number of boys in the tennis squad to the number of girls in the tennis squad is 4:3

(b) Work out the number of boys in the tennis squad.

B : G	TOTAL	
4 : 3	7	
	$\frac{75}{3} = 25$	$25 \times 4 = \underline{\underline{100}}$
	(B1)	<u>100</u> (A1)
		(2)

[LOTS OF OTHER METHODS ARE POSSIBLE!]

(a) Factorise fully  $2x^2 - 4x$

$$\frac{2x(x-2)}{(2)} \quad \text{(A2)}$$

$$A = 2p + 3q$$

(b) Work out the value of  $p$  when  $A = 32$  and  $q = 7$

$$2(x^2 - 2x) \text{ AND}$$

$$x(2x - 4) \text{ ARE WORTHY (A1) ONLY}$$

$$(32) = 2p + 3(7) \quad \text{(M1) [SUBSTITUTE]}$$

$$2p = (32) - 3(7) \quad \text{(M1) [REARRANGE]}$$

$$2p = 11$$

$$\Rightarrow p = \underline{\underline{5.5}}$$

$$p = \frac{5.5}{(3)} \quad \text{(A1)}$$

There are 50 marbles in a bag.  
35 of the marbles are brown.

Otti takes at random a marble from the bag.  
He records the colour of the marble and puts the marble back in the bag.

He does this 300 times.

Work out an estimate for the number of brown marbles he takes.

$$300 \times \frac{35}{50} \quad (M1)$$

$$\underline{\quad 210 \quad} \quad (A1)$$

Work out the size of an exterior angle of a regular polygon with 8 sides.

$$\text{EXTERIOR} = \frac{360}{\text{SIDES}}$$

$$\frac{360}{8} \text{ (m)}$$

$$45 \text{ (A)}^\circ$$

In a sale, normal prices are reduced by 8%



(a) The normal price of a jacket is £28

Work out the price of the jacket in the sale.

$$28 \times 0.92 \leftarrow \text{(BI)}$$

(M1)

[MULTIPLY]

$$\begin{array}{r} \text{£ } 25.76 \\ \hline \end{array} \text{(A1)} \\ (3)$$

(b) In the sale, the price of a shirt decreases by £3

£3 IS THE 8% DECREASE!

Work out the normal price of the shirt.

WORKING BACKWARDS  
SO DIVIDE

$$\frac{3}{0.08} \mid \text{(M1) [DIVIDE]}$$

(BI)

$$\begin{array}{r} \text{£ } 37.50 \\ \hline \end{array} \text{(A1)} \\ (3)$$

(a) Solve the inequalities  $-4 < 3x + 5 \leq 11$

$$-9 < 3x \leq 6 \quad \text{(m)} \quad (-5)$$

$$-3 < x \leq 2 \quad \text{(m)} \quad (\div 3)$$

$$\begin{array}{c} \text{(Bi)} \quad \text{[CORRECT SIGNS]} \\ \swarrow \quad \downarrow \\ -3 < x \leq 2 \\ \hline (3) \end{array}$$

(b) Write down the integer values of  $x$  which satisfy  $-4 < 3x + 5 \leq 11$

↙

SAME AS  $-3 < x \leq 2$

$$\begin{array}{c} \text{(Bi)} \quad \text{(Bi)} \\ \hline -2, -1, 0, 1, 2 \\ \hline (2) \end{array}$$

Write 792 as a product of its prime factors.  
Show your working clearly.

(M1) [TREE SEEN]

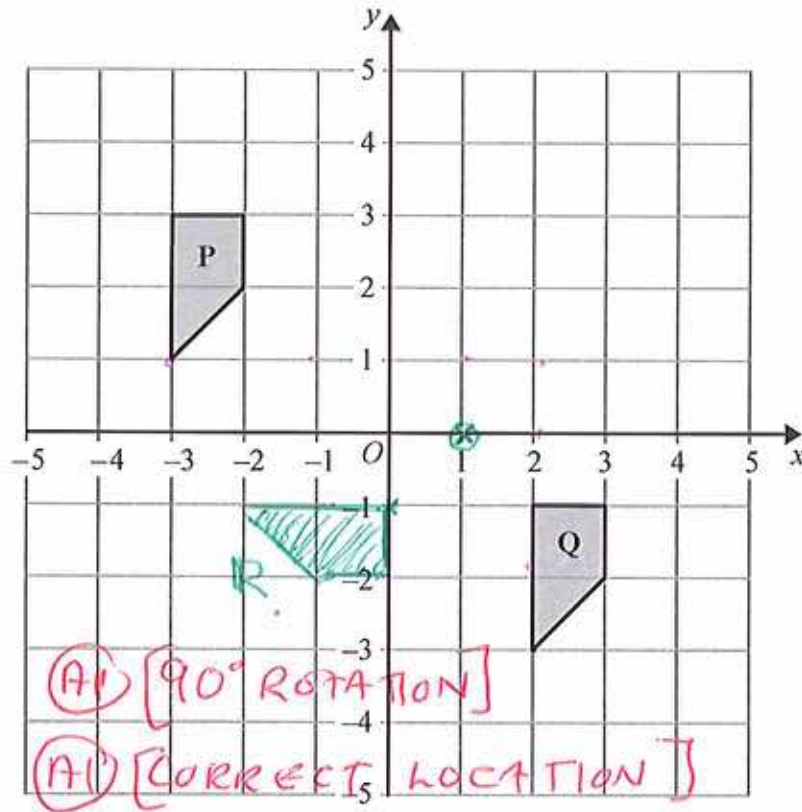
[OR OTHER VALID METHOD]

PRIME FACTOR TREE

REMEMBER TO PUT INTO POWERS

$$2 \times 2 \times 2 \times 3 \times 3 \times 11 \quad (M1)$$
$$= \underline{\underline{2^3 \times 3^2 \times 11}} \quad (A1)$$





(a) Describe fully the single transformation that maps shape P onto shape Q.

TRANSLATION, BY VECTOR  $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$  (B1) (2)

(b) Rotate shape Q 90° clockwise about (1,0).  
Label the new shape R.



(2)

Li throws a 6-sided biased dice once.

The table shows the probability that the dice will land on 1, 2, 3, 5 or 6

Number	1	2	3	4	5	6
Probability	0.15	0.1	0.05		0.2	0.15

(a) Work out the probability that the dice will land on 4

$$1 - (0.15 + 0.1 + 0.05 + 0.2 + 0.15) \quad \text{mi}$$

$$= 1 - 0.65$$

$$\underline{\underline{0.35}} \quad \text{AS}$$

(2)

(b) Work out the probability that the dice will land on an odd number.

ODDS

$$0.15 + 0.05 + 0.2 \quad \text{mi}$$

$$\underline{\underline{0.4}} \quad \text{AI}$$

(2)

Julie asked 50 children how many exercise sessions they each took part in last month. The table shows information about her results.

Number of exercise sessions	Frequency	MID VALUE	$fx$
0 to 6	13	3	39
7 to 13	10	10	100
14 to 20	16	17	272
21 to 27	7	24	168
28 to 34	4	31	124

(M) [MULTIPLY]

Calculate an estimate for the total number of exercise sessions the children took part in last month.

(B) [MID VALUES]

$$13 \times 3 + 10 \times 10 + 16 \times 17 + 7 \times 24 + 4 \times 31$$

$$= \underline{\underline{703}} \quad (A)$$

The line L passes through the point  $(3, 1)$  and is parallel to the line with equation  $y = \frac{7}{2} - 2x$ .

Find an equation for the line L.

USE  $y - y_1 = m(x - x_1)$

$m = -2$  (B1)

$$y - 1 = -2(x - 3) \quad (M1)$$

$$y - 1 = -2x + 6$$

$$y = -2x + 7 \quad (A1)$$

(a) Simplify fully  $\frac{a^{11}}{a^2 \times a^5} = \frac{a^4}{a^7} \quad (M1)$

$$\frac{a^4}{a^7} \quad (A1)$$

(2)

(b) Make  $p$  the subject of  $p + 4q = 3p + 5$

$$3p - p = 4q - 5 \quad (M0)$$

$$2p = 4q - 5 \quad \rightarrow$$

$$p = \frac{4q - 5}{2} \quad (A1)$$

(2)

(c) Expand and simplify  $(2y + 3)(4y - 1)$

$$\begin{array}{cccc} F & O & I & L \\ 8y^2 & - 2y & + 12y & - 3 \quad (M1) \end{array}$$

$$8y^2 + 10y - 3 \quad (A1)$$

(2)

(d) Simplify  $(8a^6b^3)^{\frac{1}{3}}$

$$8^{\frac{1}{3}} = 2 \quad (B1)$$

$$(a^6)^{\frac{1}{3}} = a^2 \quad \left. \vphantom{(a^6)^{\frac{1}{3}} = a^2} \right\} (B1)$$

$$(b^3)^{\frac{1}{3}} = b$$

$$2a^2b$$

(2)

Here is the quadrilateral  $ABCD$ .

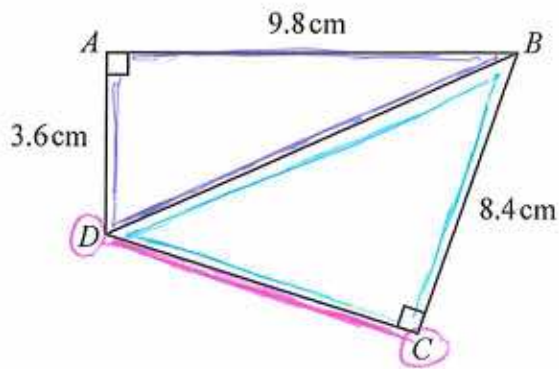


Diagram NOT  
accurately drawn

Angle  $BAD = 90^\circ$  and angle  $BCD = 90^\circ$

$AB = 9.8\text{ cm}$

$AD = 3.6\text{ cm}$

$BC = 8.4\text{ cm}$

Calculate the length of  $DC$ .

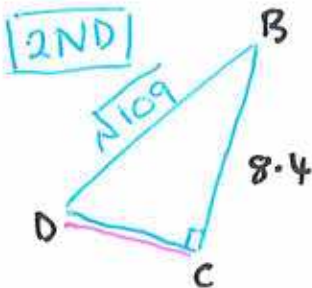


**1ST**

$$BD^2 = 9.8^2 + 3.6^2$$

$$= 109 \quad \text{(BI)}$$

**2ND**



$$DC^2 = BD^2 - 8.4^2 \quad \text{(MI)}$$

$$= 38.44 \quad \text{(MI)}$$

$$\Rightarrow DC = \sqrt{38.44}$$

$$\rightarrow 6.2 \quad \text{(AI)} \quad \text{cm}$$

Linford and Alan race against each other in a competition.

If one of them wins a race, he wins the competition.

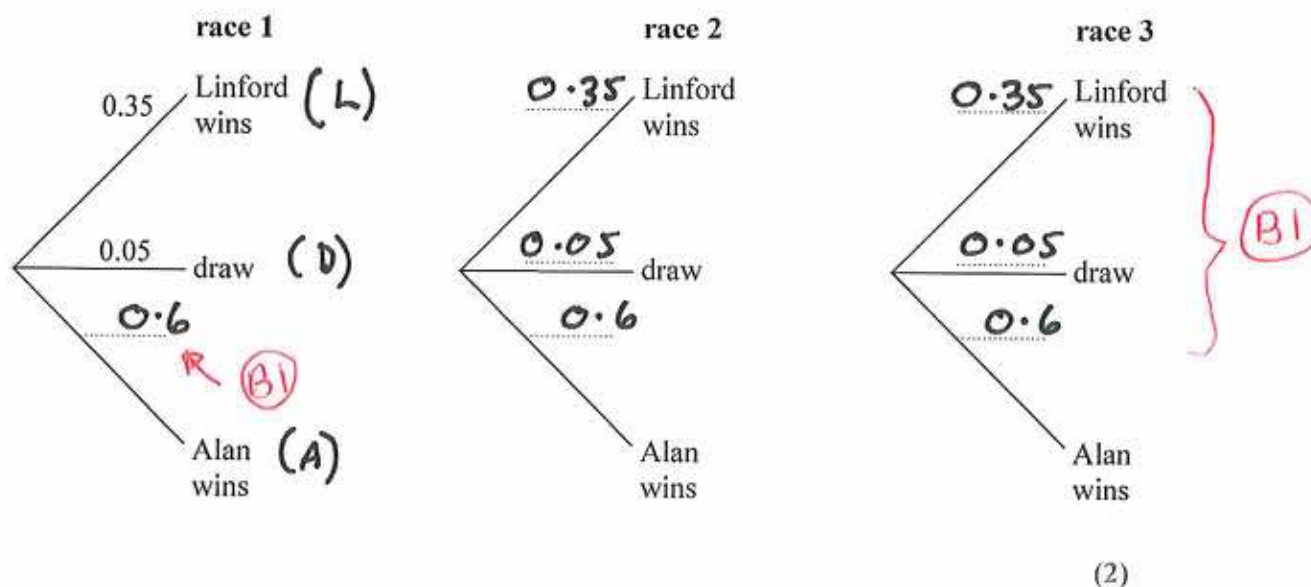
If the race is a draw, they run another race.

They run a maximum of three races.

Each time they race, the probability that Linford wins is 0.35

Each time they race, the probability that there is a draw is 0.05

(a) Complete the probability tree diagram.



(b) Calculate the probability that Linford wins the competition.

$$P(L) = \underline{0.35}$$

$$P(DL) = 0.05 \times 0.35 = \underline{0.0175} \text{ (m1)}$$

$$P(DDL) = 0.05 \times 0.05 \times 0.35 = \underline{0.000875} \text{ (m1)}$$

$$\text{TOTAL} = 0.35 + 0.0175 + 0.000875$$

$$= \underline{0.368375} \text{ (A1)}$$

(3)

[0.368 IS OKAY]

$$y = x^3 - \frac{9}{2}x^2 - 54x + 10$$

(a) Find  $\frac{dy}{dx} = 3x^2 - 2 \times \frac{9}{2}x - 54$

(M1) [FOR ANY TWO TERMS]

$$3x^2 - 9x - 54$$

(2)

The curve with equation  $y = x^3 - \frac{9}{2}x^2 - 54x + 10$  has two turning points.

(b) Find the x coordinate of each of these two points.

$$\frac{dy}{dx} = 0$$

$$3x^2 - 9x - 54 = 0$$

(M1) [RHS = 0]

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

(M1) [FACTORISING - ALLOW USE OF FORMULA]

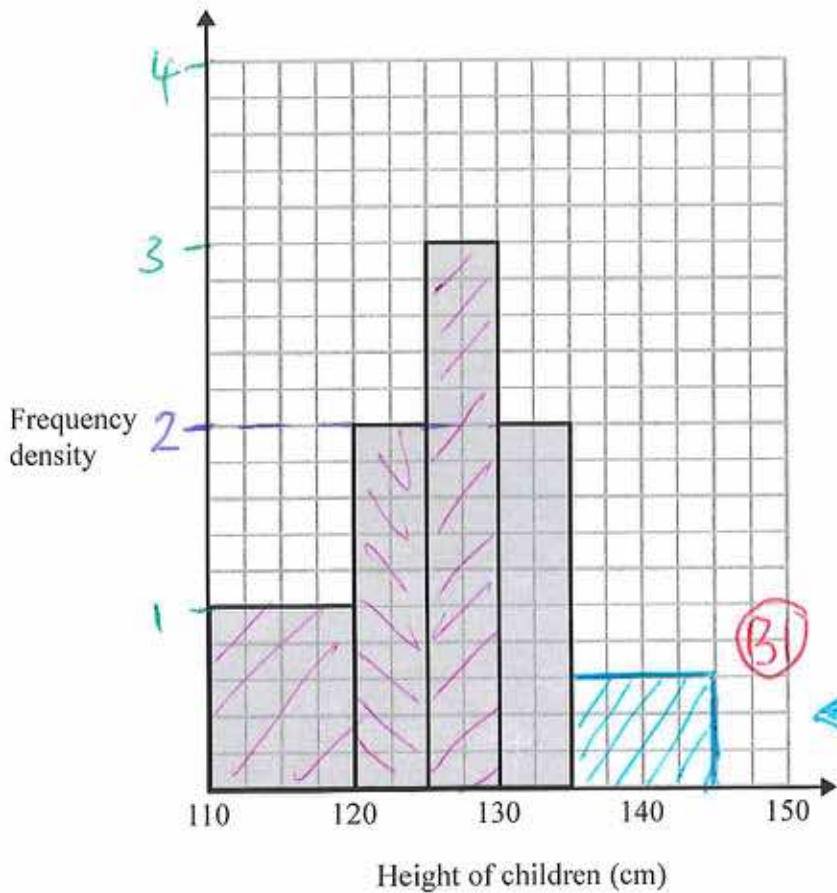
$$x = \underline{\underline{6}}$$

$$x = \underline{\underline{-3}}$$

(A1)



The incomplete histogram shows information about the heights of a group of children.



There were 10 children with heights between 130cm and 135cm. → Area = 10!

(a) How many children had heights between 110cm and 130cm?

1st bar: 130-135, height 10

2nd bar: 135-140, height 2

3rd bar: 140-145, height 3

Height =  $\frac{10}{5} = 2$

$10 \times 1 + 5 \times 2 + 5 \times 3 = 35$

[ALLOW COUNTING SQUARES]

35 (3)

There were 6 children with heights between 135cm and 145cm.

(b) Show this information on the histogram.

WIDTH =  $145 - 135 = 10$

HEIGHT OF BAR =  $\frac{6}{10}$

(1)

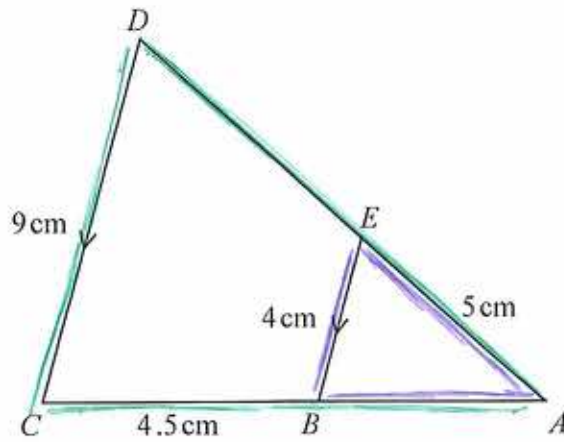


Diagram NOT accurately drawn

SCALE =  $\frac{9}{4}$   
 FACTOR =  $\frac{9}{4}$   
 $= 2.25$

Triangle  $ABE$  is similar to triangle  $ACD$ .  
 $AED$  and  $ABC$  are straight lines.  
 $EB$  and  $DC$  are parallel.  
 $AE = 5\text{ cm}$ ,  $BC = 4.5\text{ cm}$ ,  $BE = 4\text{ cm}$ ,  $CD = 9\text{ cm}$

(a) Calculate the length of  $AD$ .

$$5 \times 2.25$$

(M1)

$$\underline{11.25} \text{ cm}$$

(2)

(b) Calculate the length of  $AB$ .

$$2.25x = x + 4.5$$

$$1.25x = 4.5$$

(M1)

$$x = \frac{4.5}{1.25}$$

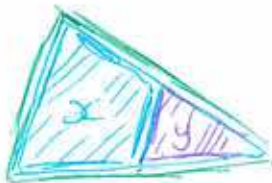
$$= \underline{3.6} \text{ cm}$$

(2)



The area of quadrilateral  $BCDE$  is  $x\text{ cm}^2$   
 The area of triangle  $ABE$  is  $y\text{ cm}^2$

(c) Find an expression for  $y$  in terms of  $x$ .  
 Give your answer as simply as possible.



$$x + y = 2.25^2 y$$

(M1)

$$x + y = 5.0625y$$

$$x = 4.0625y$$

(M1)

$$\Rightarrow y = \frac{x}{4.0625} \left[ = \frac{16}{65} x \right]$$

(A1)

$f$  is the function such that

$$f(x) = \frac{x}{3x+1}$$

(a) Find  $f(0.5)$

$$f(0.5) = \frac{(0.5)}{3(0.5)+1}$$

$$\frac{0.2}{(1)} \quad \text{(B)}$$

(b) Find  $ff(-1)$   $\rightarrow$  do it  $f[f(-1)]$

$$f(-1) = \frac{(-1)}{3(-1)+1} \quad \text{(M)}$$

$$= 0.5$$

$$\rightarrow f(0.5) = \underline{0.2} \quad \text{(A)}$$

$$\frac{(2)}{(2)}$$

(c) Find the value of  $x$  that cannot be included in any domain of  $f$

$$3x+1 \neq 0$$

$$3x \neq -1$$

$$\rightarrow x \neq -\frac{1}{3} \quad \text{(B)}$$

(1)

(d) Express the inverse function  $f^{-1}$  in the form  $f^{-1}(x) = \dots$

Show clear algebraic working.

$$y = \frac{x}{3x+1}$$

$$\Rightarrow x = \frac{y}{3y+1} \quad \text{(M)}$$

$$\Rightarrow 3xy + x = y$$

$$\Rightarrow 3xy - y = -x \quad \text{(M)}$$

$$\Rightarrow y(3x-1) = -x$$

$$\Rightarrow y = \frac{-x}{3x-1}$$

$$f^{-1}(x) = \frac{x}{1-3x} \quad \text{(A)} \quad \text{[EITHER]}$$

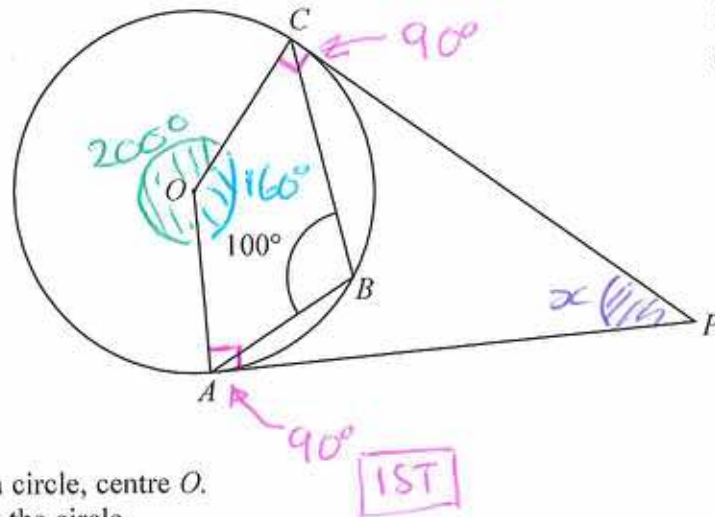


Diagram NOT  
accurately drawn

$A$ ,  $B$  and  $C$  are points on a circle, centre  $O$ .  
 $PA$  and  $PC$  are tangents to the circle.  
Angle  $ABC = 100^\circ$

Calculate the size of angle  $APC$ .

**1ST** ANGLE BETWEEN TANGENTS AND RADII =  $90^\circ$

**2ND** ANGLE  $AOC = 200^\circ$  [ANGLE AT CENTRE  
=  $2 \times$  CIRCUMFERENCE]

**3RD**  $AOC = 160^\circ$  [ANGLES AROUND A POINT]

**4TH**  $x = 360 - (90 + 90 + 160)$

$= 20^\circ$

(a) Simplify fully  $\frac{50x^2 - 8}{10x - 4}$

Show clear algebraic working.

FACTORISE THEN CANCEL

$$\frac{50x^2 - 8}{10x - 4} = \frac{25x^2 - 4}{5x - 2} \quad [\text{CANCEL COMMON FACTORS}]$$

$$= \frac{(5x + 2)(5x - 2)}{5x - 2} \quad [\text{'DOTS'}]$$

$$\frac{5x + 2}{(3)}$$

(b) Given that  $a$  is a positive integer, show that

$$\sqrt{3a}(\sqrt{12a} + a\sqrt{3a})$$

is always a multiple of 3

SHOW THAT 3 IS A FACTOR!

$$\begin{aligned} \sqrt{3a}(\sqrt{12a} + a\sqrt{3a}) &= \sqrt{3a}\sqrt{12a} + a\sqrt{3a}\sqrt{3a} \quad (m) \\ &= \sqrt{36a^2} + a \times 3a \\ &= 6a + 3a^2 \quad (m) \\ &= 3(2a + a^2) \quad (m) \end{aligned}$$

SINCE 3 IS A FACTOR THEN THE EXPRESSION IS ALWAYS  
A MULTIPLE OF 3.

Solve  $3 \times 4^{2k+8} = 24$

Show your working clearly.

$$3 \times 4^{2k+8} = 24 \Rightarrow 4^{2k+8} = 8 \quad (m1)$$

$$\Rightarrow (2^2)^{2k+8} = 2^3$$

$$\Rightarrow 2^{4k+16} = 2^3$$

$$\Rightarrow 4k+16 = 3 \quad (m1)$$

$$4k = -13$$

$$k = -\frac{13}{4}$$

$$= -\underline{\underline{3.25}} \quad (A1)$$

FOR TRIANGLE PRC

$$\text{AREA} = \frac{1}{2} ab \sin C$$

$\uparrow$   
 $r^2$

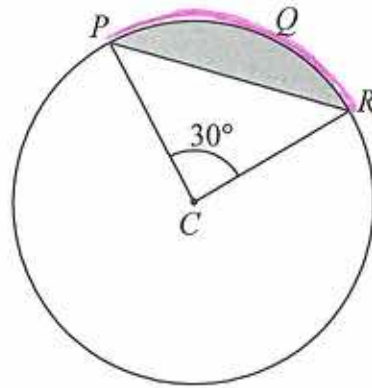


Diagram NOT accurately drawn

FOR SECTOR

$$\text{AREA} = \frac{30}{360} \pi r^2$$

$$\text{ARC} = \frac{30}{360} \times 2\pi r$$

The diagram shows a circle, centre C.  
PR is a chord of the circle.

The area of the shaded region is  $100 \text{ cm}^2$   
Angle  $PCR = 30^\circ$

Calculate the length of the arc PQR.  
Give your answer correct to 3 significant figures.

1ST

$$\frac{30}{360} \pi r^2 - \frac{1}{2} r^2 \sin 30 = 100 \quad \text{(M1) [EQUATION]}$$

$$\Rightarrow \left( \frac{30}{360} \pi - \frac{1}{2} \sin 30 \right) r^2 = 100$$

$$\Rightarrow 0.011799 r^2 = 100$$

$$r^2 = \frac{100}{0.011799}$$

$$r = 92.0598 \dots \quad \text{(A1)}$$

(M1) [ANY]

2ND

$$\text{ARC} = \frac{30}{360} \times 2\pi \times 92.0598 \quad \text{(M1)}$$

$$\underline{48.2} \quad \text{(A1) cm}$$