4H

Pearson Edexcel
International GCSE

EDEXCEL
IGCSE

MATHEMATICS A
SOLUTIONS

JANUARY 2013   4MA0/4H

<u>Disclaimer</u>

(a) Use your calculator to work out the value of

$$\frac{8.7 + 2.8}{1.4^2}$$

Give your answer as a decimal.
Write down all the figures on your calculator display.

(A2)

5.8673469

(2)

(b) Give your answer to part (a) correct to 2 significant figures.

5.87 (B1)

(1)

A circle has a diameter of 7.6 cm.
Work out the circumference of the circle.
Give your answer correct to 3 significant figures.

$$C = \pi D \quad (D = 7.6)$$

$$= \pi \times 7.6 \quad \text{(M1)}$$

$$= 23.876\ldots$$

23.9 (B1) cm

The table shows information about the marks of 20 students in a science test.

Mark (x)	Frequency (f)	xf
6	2	$6 \times 2 = 12$
7	4	$7 \times 4 = 28$
8	5	40
9	8	72
10	1	10

(m1)

TOTAL = 162

Work out the mean mark of the 20 students.

$$MEAN = \frac{162}{20}$$ (B1)

$$= 8.1$$

8.1 (A1)

Abid is waiting for a bus.
The probability that his bus will be early is 0.2
The probability that his bus will be on time is 0.7

Work out the probability that his bus will be either early or on time.

$$0.2 + 0.7 \quad \text{(M1)}$$

$$0.9 \quad \text{(A1)}$$

$\mathcal{E}$ = {even numbers}
A = {2, 4, 6, 8, 10}

(a) B is a set such that $A \cap B$ = {4, 8}
    The set B has 3 members.

    List the members of one possible set B.

ANY EVEN NUMBER
OTHER THAN
            2, 6, 10
              ↓
   { 4, 8, 12 }
            (2)   Ⓐ2

(b) C is a set such that $A \cap C$ = Ø
    The set C has 3 members.

    List the members of one possible set C.

ANY THREE EVEN
NUMBERS > 10
      ↓ ↓ ↘
   { 12, 14, 16 }
            (1)   Ⓐ1

$\frac{5}{9}$ of the students in a group are male. $\left(\frac{4}{9}\ \text{ARE FEMALE}\right)$

$\frac{5}{6}$ of the **female** students in the group are right-handed.

(a) Work out the fraction of students in the group who are right-handed females.

(B1) → $\frac{4}{9} \times \frac{5}{6} = \frac{20}{54}$

(m1)

$\frac{10}{27}$ (A1)

(3)

(b) Find the smallest possible number of students in the group.

SINCE FRACTION OF RIGHT-HANDED FEMALES

IS $\frac{10}{27}$. THERE MUST BE AT LEAST

27 PEOPLE IN THE GROUP

(B2)

27 (OR 54)

(2)

16 cm

7

6

(B)

10 cm

(A)

4 cm

9 cm
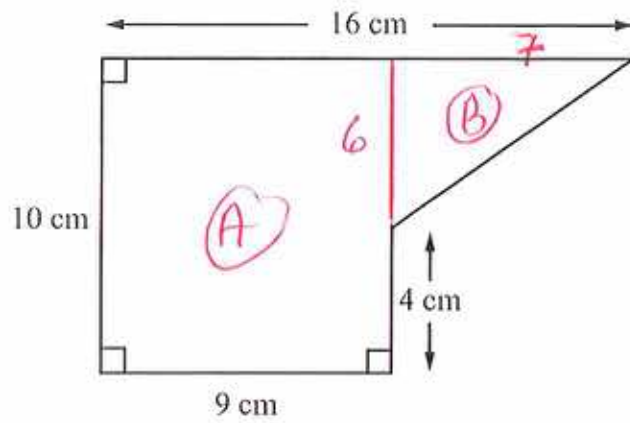
Diagram **NOT** accurately drawn

The diagram shows a shape.

Work out the area of the shape.

$$(A) = 9 \times 10 = 90$$

$$(B) = \frac{6 \times 7}{2} = 21$$

TOTAL $\overline{111}$ cm^2

111 cm^2

(a) Factorise $n^2 + 8n$

$$n(n+8)$$

(A1)   (A1)

(2)

(b) Expand and simplify $3(2x - 5) - 4(x + 3)$

$$6x - 15 \quad - 4x - 12 \quad \text{(M1)}$$

$$2x - 27 \quad \text{(A1)}$$

(2)

(c) Expand and simplify $(y + 7)(y + 2)$

$$y^2 + 2y + 7y + 14 \quad \text{(M1)}$$

$$\text{F} \quad \text{O} \quad \text{I} \quad \text{L}$$

$$y^2 + 9y + 14 \quad \text{(A1)}$$

(2)

Diagram **NOT** accurately drawn

8.6 cm

x cm

6.9 cm
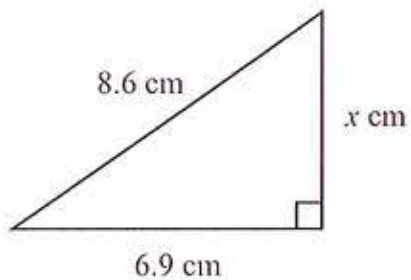
Work out the value of x.
Give your answer correct to 3 significant figures.

$$x^2 = 8.6^2 - 6.9^2 \quad \text{(m1)}$$

$$= 26.35$$

$$\Rightarrow x = \sqrt{26.35} \quad \text{(m1)}$$

$$\approx 5.1332\ldots$$

$x = \underline{5.13}$ (B1)

Solve $3x + 16 = 1 - 2x$
Show clear algebraic working.

$$3x + 16 = 1 - 2x$$
$$3x + 2x = 1 - 16 \quad \text{(M1)}$$
$$5x = -15 \quad \text{(M1)}$$
$$x = -\frac{15}{5}$$

$x = \underline{-3}$ (A1)

Jack, Kate and Lila share some money in the ratios 5 : 9 : 6
In total, Jack and Kate receive £56

Work out the amount of money Lila receives.

$$J : K : L \qquad \text{TOTAL PARTS} = 20$$
$$5 : 9 : 6$$

£56   (B1)

$$\frac{14}{20} = £56$$

$$\Rightarrow \frac{1}{20} = 4 \quad (B1)$$

$$\text{LILA} = 4 \times 6 \qquad £ \underline{} 24 \quad (A1)$$
$$= 24$$

M]ETHOD 2

$$\frac{56}{14} = 4 \quad (B1) \qquad , \quad 6 \times 4 = \underline{24} \quad (A1)$$

(B1)

Use ruler and compasses to construct the bisector of angle *ABC*.
You must show all your construction lines.

(B1) CONSTRUCTION
LINES
(BOTH)

(A1)
BISECTOR
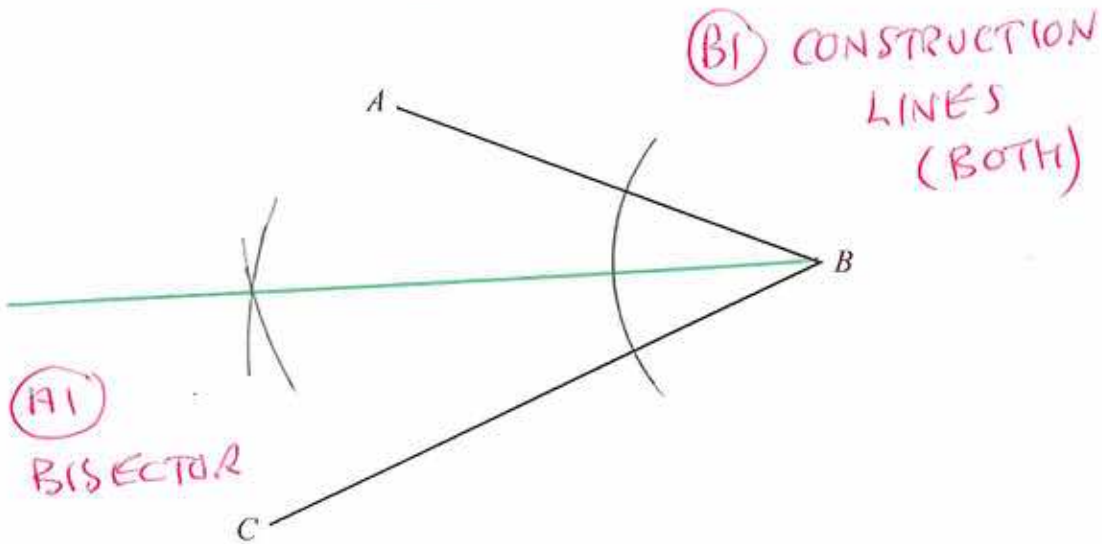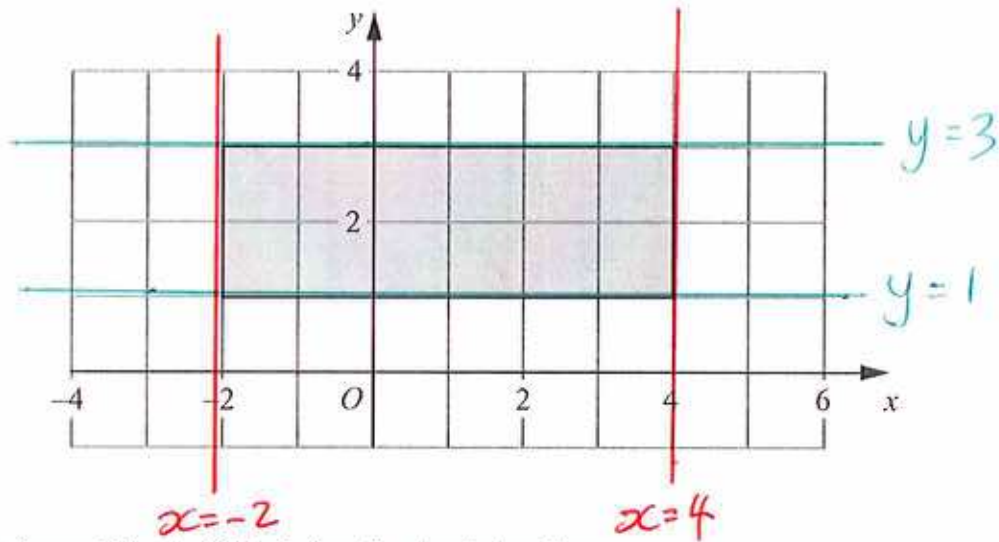
Write down inequalities to fully define the shaded region.

$x \geqslant -2$, $x \leqslant 4$, $y \geqslant 1$, $y \leqslant 3$

OR

$-2 \leqslant x \leqslant 4$, $1 \leqslant y \leqslant 3$

Solve $\dfrac{2}{5x-2} = \dfrac{3}{6x+1}$

Show clear algebraic working.

$2(6x+1) = 3(5x-2)$ (m1)

$12x + 2 = 15x - 6$ (m1)

$12x - 15x = -6 - 2$

$\qquad -3x = -8$ (m1)

$\qquad\quad x = -\dfrac{8}{-3}$

$\qquad\qquad = 2\frac{2}{3}$

$x = \underline{\qquad 2\frac{2}{3} \qquad}$ (A1)

(a) Simplify $\dfrac{5x^5 y^6}{x^2 y^4}$

$5x^3 y^2$

(2)

(b) Simplify $(2n^4)^3$

$2^3 \times n^{4 \times 3}$

$8n^{12}$

(2)

The grouped frequency table gives information about the lengths of 160 pythons.

Length (x metres)	Frequency
$0 < x \leqslant 1$	4
$1 < x \leqslant 2$	8
$2 < x \leqslant 3$	16
$3 < x \leqslant 4$	32
$4 < x \leqslant 5$	72
$5 < x \leqslant 6$	28

(a) Complete the cumulative frequency table.

Length (x metres)	Cumulative frequency
$0 < x \leqslant 1$	4
$0 < x \leqslant 2$	12
$0 < x \leqslant 3$	28
$0 < x \leqslant 4$	60
$0 < x \leqslant 5$	132
$0 < x \leqslant 6$	160

(B1)

(b) On the grid, draw a cumulative frequency graph for your table.

(B1) POINTS (B1) SMOOTH CURVE

(B1) LINE FROM 80

Length (x metres) (4·3)

(2)
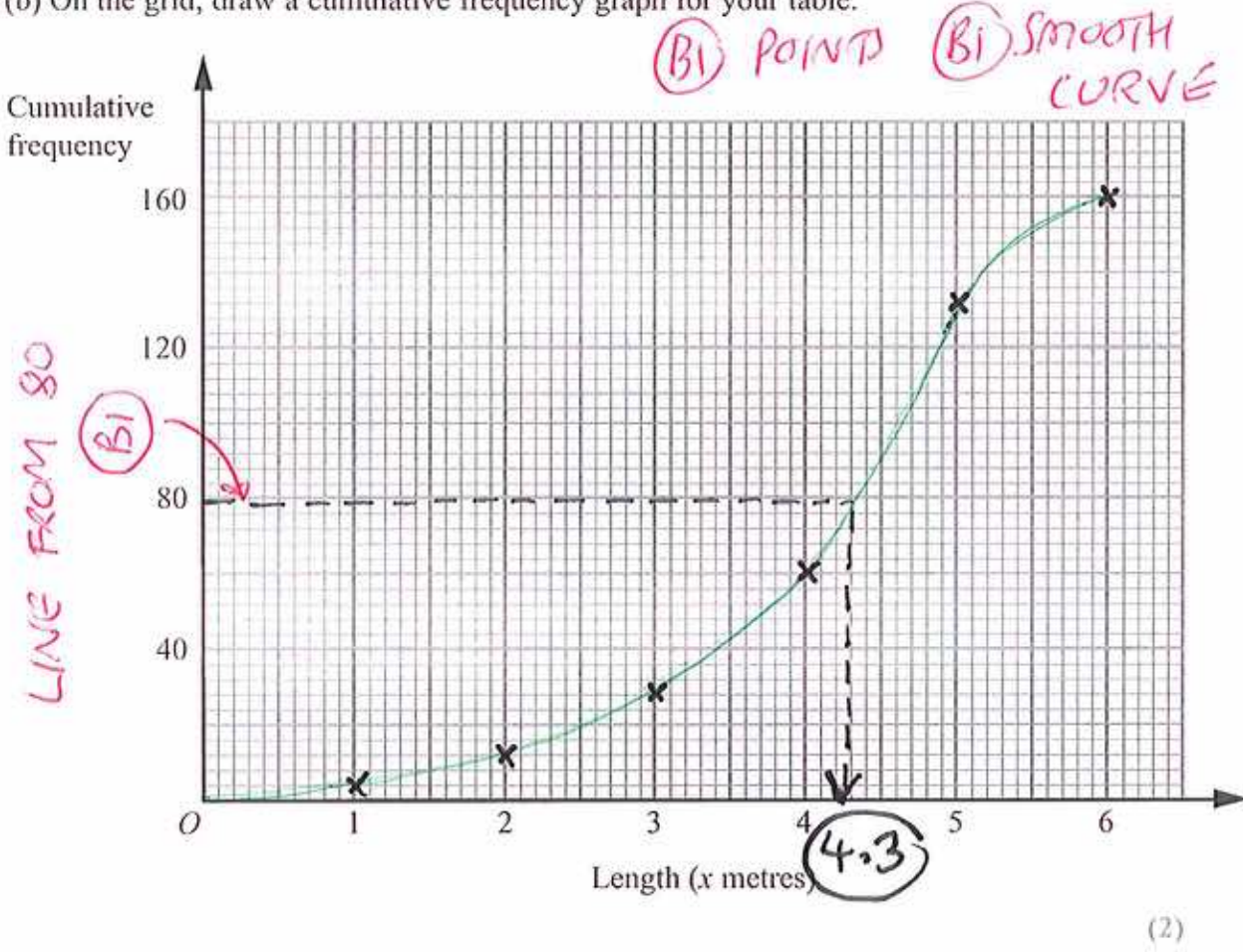
(c) Use your graph to find an estimate for the median length of the pythons.

MAX = 160 ⇒ MEDIAN = $\frac{160}{2}$

= 80th VALUE

4·3 (A1) metres

(2)
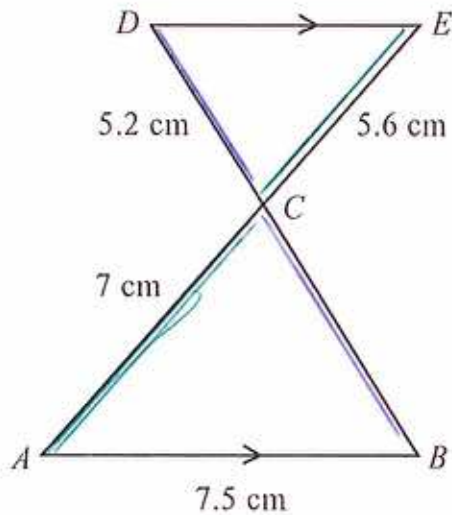
Diagram **NOT** accurately drawn

$$SF = \frac{7}{5.6} = 1.25$$

AB is parallel to DE.

The lines AE and BD intersect at C.

$AB = 7.5$ cm, $AC = 7$ cm, $CD = 5.2$ cm, $CE = 5.6$ cm.

(a) Calculate the length of BC.

$$5.2 \times 1.25 \quad \text{(B1)}$$

6.5 cm (A1)

(2)

(b) Calculate the length of DE.

$$\frac{7.5}{1.25} \quad \text{(M1) DIVIDE}$$

6 cm (A1)

(2)

(c) The area of triangle ABC is 21 cm²
Calculate the area of triangle EDC.

(M1) DIVIDE

$$\frac{21}{1.25^2} = 13.44 \quad \text{(M1)}$$

13.44 cm² (A1)

Solve $5x^2 + 2x - 4 = 0$
Give your solutions correct to 3 significant figures.
Show your working clearly.

USE FORMULA !

$a = 5, \quad b = 2, \quad c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4 \times 5 \times (-4)}}{2 \times 5} \quad \text{(M1)}$$

$$= \frac{-2 \pm \sqrt{4 + 80}}{10}$$

$$x = \frac{-2 + \sqrt{84}}{10}$$

$$= 0.717 \quad \text{(A1)}$$

$$x = \frac{-2 - \sqrt{84}}{10}$$

$$= -1.12 \quad \text{(A1)}$$

$(3 + \sqrt{a})(4 + \sqrt{a}) = 17 + k\sqrt{a}$ where a and k are positive integers.

Find the value of a and the value of k.

$$(3 + \sqrt{a})(4 + \sqrt{a}) = 12 + 3\sqrt{a} + 4\sqrt{a} + a \quad \boxed{M1}$$

$$= 12 + a + 7\sqrt{a}$$

$$\underbrace{}_{17} \quad \underbrace{\phantom{7\sqrt{a}}}_{k\sqrt{a}}$$

$$12 + a = 17$$
$$a = 5$$

$a =$ _____5_____ $\boxed{A1}$

$k =$ _____7_____ $\boxed{B1}$

Box X   Box Y
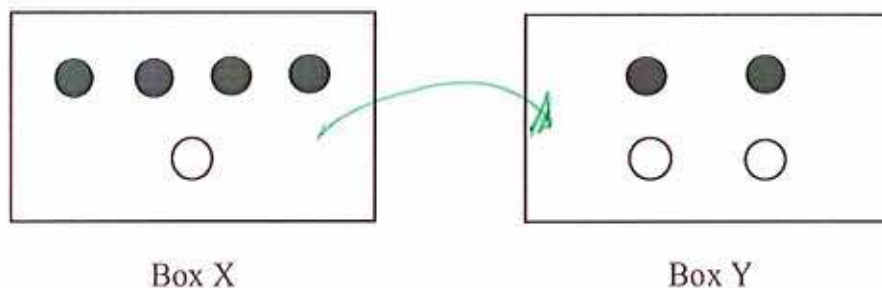
In Box X, there are 4 black discs and 1 white disc.
In Box Y, there are 2 black discs and 2 white discs.

Vikram takes at random a disc from Box X and puts it in Box Y.
He then takes at random a disc from Box Y.

(a) Calculate the probability that the disc he takes from Box X and the disc he takes
from Box Y will both be black discs.

$$P(BB) = \frac{4}{5} \times \frac{3}{5} \quad \text{(m1)}$$

$$= \frac{12}{25}$$

$$\frac{12}{25} \quad \text{(A1)}$$

(2)

(b) Calculate the probability that the disc he takes from Box Y will be a white disc.

$$P(BW) = \frac{4}{5} \times \frac{2}{5} = \frac{8}{25}$$

$$\text{(m1) EITHER}$$

$$P(WW) = \frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$$

$$\frac{11}{25} \quad \text{(A1)}$$

(m1) FOR ADDING.

When a photograph is taken, the exposure time, t, is directly proportional to the square of the size, f, of the opening in the camera lens.

$t = 0.02$ when $f = 8$

(a) Find a formula for t in terms of f.   (M1)

$$t = k \times f^2 \quad (t = 0.02, \, f = 8)$$

$$\Rightarrow 0.02 = k \times 8^2$$

$$\Rightarrow k = \frac{0.02}{8^2}$$

$$= \frac{1}{3200} \quad \text{(A1)}$$

$$t = \frac{1}{3200} \times f^2 \quad \text{(A1)}$$

....

(3)

(b) Calculate the value of f when $t = 0.0098$

$$t = \frac{f^2}{3200} \quad (t = 0.0098)$$

$$0.0098 = \frac{f^2}{3200}$$

(M1) FOR f^2 STATEMENT

$$f^2 = 0.0098 \times 3200$$

$$= 31.36$$

$$f = \sqrt{31.36}$$

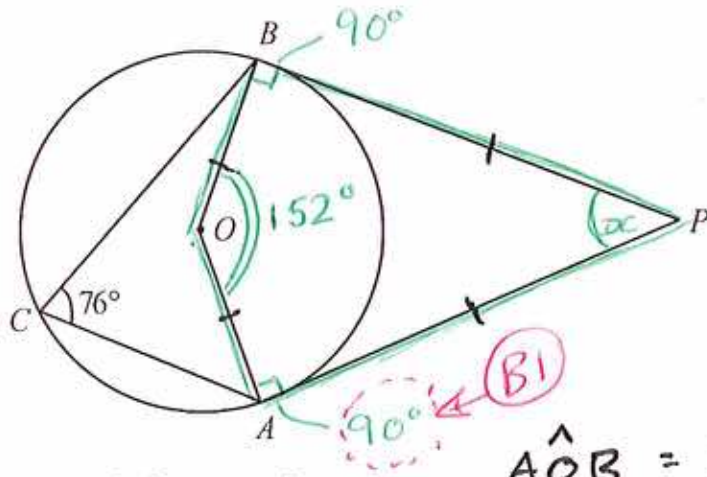$$= 5.6$$

(A1)

$f = $ ......5.6......

(2)

Diagram **NOT** accurately drawn

A, B and C are points on a circle, centre O.

Angle $ACB = 76°$

PA and PB are tangents to the circle.

Calculate the size of angle APB.

$A\hat{O}B = 152$ B1

[ANGLE AT CENTRE =
2 × ANGLE AT CIRC.]

$360 - [152 + 90 + 90]$

M1

28 °   A1

f is a function such that

$$f(x) = \frac{1}{x^2 + 1}$$

(a) Find $f(\frac{1}{2})$ $= \dfrac{1}{(0.5)^2 + 1}$

$$= \frac{1}{0.25 + 1}$$

$$\frac{1}{1.25} = \underline{\underline{0.8}}$$

                                              0.8 (B1)

                                                      (1)

g is a function such that

$$g(x) = \sqrt{x-1} \quad x \geqslant 1$$

(b) Find $fg(x)$

    Give your answer as simply as possible.

$$\frac{1}{(\sqrt{x-1})^2 + 1} = \frac{1}{(x-1) + 1}$$

        (B1) EITHER

                                      $fg(x) = \dfrac{1}{x}$ (A1)

                                                        (2)

On Monday, Nalim made a journey.
On Tuesday, she made the same journey.
Her average speed on Tuesday was 25% greater than her average speed on Monday.

Calculate the percentage reduction in the time her journey took on Tuesday compared with Monday.

$$S_m = \frac{d}{t_m} \qquad S_T = \frac{d}{t_T}$$
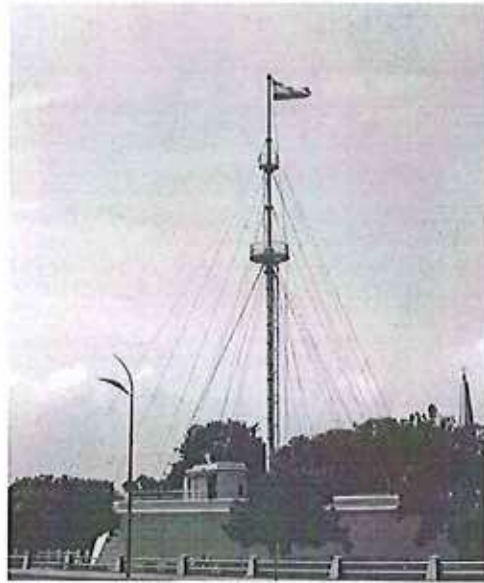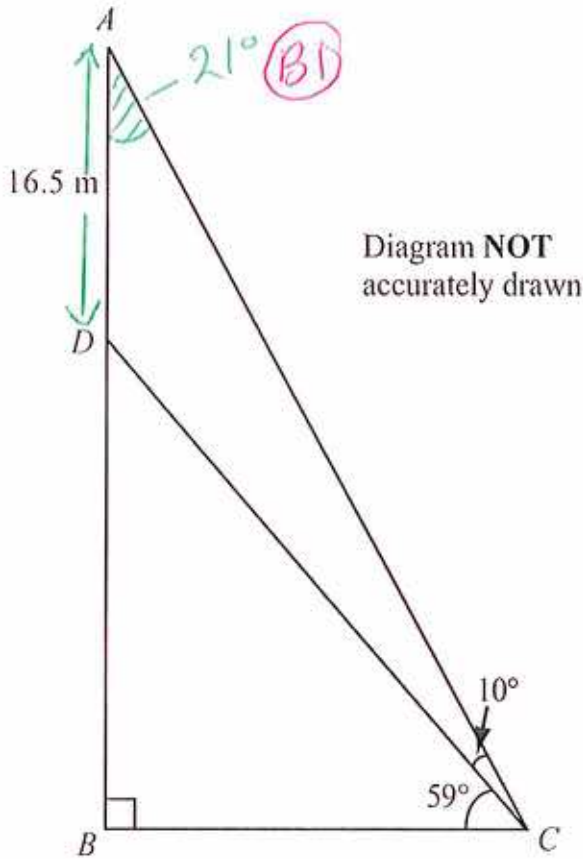
$$S_T = 1.25 \times S_m$$

$$\Rightarrow \frac{d}{t_T} = 1.25 \times \frac{d}{t_m} \quad \text{(M1)}$$

$$\Rightarrow t_m = 1.25 \times t_T$$

$$\Rightarrow t_T = \frac{t_m}{1.25} \quad \Big\} \text{(M1) [ANY]}$$

$$= 0.8 \times t_m$$

$$\therefore \text{ TIME IS REDUCED BY } 0.2$$

$$= \underline{\underline{20\%}} \quad \text{(A1)}$$

A

$-21°$ (BD)

16.5 m

D

Diagram **NOT** accurately drawn

$10°$

$59°$

B C

The diagram shows a vertical flagpole in Chennai, India.
The point A is at the top of the flagpole.
The point B is at the foot of the flagpole.
There is a platform at the point D on the flagpole.
B and C are points on horizontal ground.
$AD = 16.5$ m
The angle of elevation of A from C is 69°
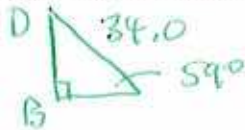The angle of elevation of D from C is 59°

Calculate the height, AB, of the flagpole.
Give your answer correct to 3 significant figures.

① $\dfrac{CD}{\sin 21} = \dfrac{16.5}{\sin 10}$ (m1)

$CD = \dfrac{16.5}{\sin 10} \times \sin 21$ (m1)

$= 34.052...$ (A1)

D 34.0
   59°
B

② $\sin 59 = \dfrac{OPP}{HYP}$

$\sin 59 = \dfrac{BD}{34}$

→ $BD = 34 \times \sin 59$
$= 29.188...$ (A1)

③ $AB = 29.188... + 16.5$
$= 45.688...$
$= 45.7$ m (B1)

Solve the simultaneous equations

$$y = 3x + 2 \quad \longrightarrow \quad ①$$
$$x^2 + y^2 = 20 \quad \longrightarrow \quad ②$$

Show clear algebraic working.

SUBSTITUTE EQ. ① INTO EQ. ②

$$x^2 + (3x+2)^2 = 20 \quad \text{(m1)}$$

$$x^2 + (3x+2)(3x+2) = 20$$

$$x^2 + 9x^2 + 6x + 6x + 4 = 20 \quad \text{(m1)}$$

$$10x^2 + 12x + 4 = 20$$

$$10x^2 + 12x - 16 = 0$$

$$5x^2 + 6x - 8 = 0 \quad \text{(B1)}$$

$$(5x - 4)(x + 2) = 0 \quad \text{(m1)}$$

$$x_1 = \frac{4}{5} \qquad\qquad x_2 = -2 \quad \text{(A1)}$$
$$\text{BOTH}$$

- -

SUBSTITUTE INTO EQ. ①

$$y_1 = 3 \times \frac{4}{5} + 2 \qquad\qquad y_2 = 3 \times (-2) + 2$$

$$= 4.4 \qquad\qquad\qquad = -4$$

$$\text{(A1) BOTH}$$