

4H

Pearson Edexcel
International GCSE

EDEXCEL

IGCSE

MATHEMATICS A

SOLUTIONS

JANUARY 2014

4MA0/4H

Disclaimer

These solutions have been produced by Maths4Everyone Limited. While We have used reasonable endeavours to verify the accuracy of these solutions, these solutions are provided on an “as is” basis and We make no warranties of any kind, whether express or implied, in relation to these solutions.

We make no warranty that these solutions will meet Your requirements or provide the results which You want, or that they are complete, or that they are error-free. If You find anything confusing within these solutions then it is Your responsibility to seek clarification from Your teacher, tutor or mentor.

We request that You use the ‘contact’ link on Our web site to inform Us of any errors or omissions that You find. We will update these solutions and correct errors that We become aware of. We recommend that You check Our web site for the most up-to-date version of these solutions.

The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or ‘best’ method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

A school has 840 pupils and 40 teachers.

- (a) Find the ratio of the number of pupils to the number of teachers.
Give your ratio in the form $n : 1$

$$\begin{array}{l}
 P : T \\
 840 : 40 \\
 \end{array}
 = 21 : 1$$

$(\div 40)$

$$\begin{array}{r}
 21 : 1 \\
 \hline
 (2)
 \end{array}$$

(B2)

In Year 11 at the school, the ratio of the number of pupils who study Chemistry to the number of pupils who study Physics is 3 : 2

- (b) 105 pupils in Year 11 study Chemistry.
Work out the number of pupils in Year 11 who study Physics.

$$\begin{array}{l}
 C : P \\
 3 : 2 \\
 \updownarrow \\
 105
 \end{array}$$

$$\frac{105}{3} = 35 \quad (M1) \rightarrow 2 \times 35 = \underline{\underline{70}}$$

$$\begin{array}{r}
 70 \\
 \hline
 (2)
 \end{array}$$

(A1)

For the 105 pupils who study Chemistry, the ratio of the number of boys to the number of girls is 4 : 3

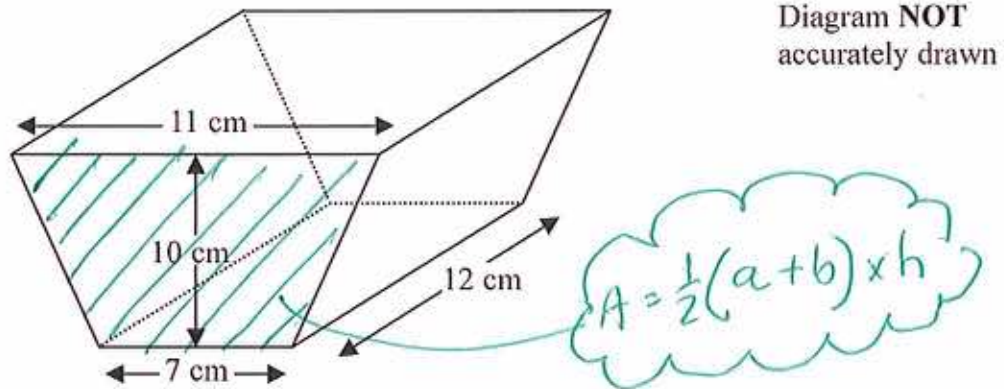
- (c) Work out the number of girls in Year 11 who study Chemistry.

$$\begin{array}{l}
 B : G \\
 4 : 3 \\
 \updownarrow \\
 105
 \end{array}$$

$$\begin{array}{l}
 \text{TOTALS} \\
 7 \\
 \updownarrow \\
 105
 \end{array}
 \left| \frac{105}{7} = 15 \quad (M1) \rightarrow 15 \times 3 = \underline{\underline{45}}
 \right.$$

$$\begin{array}{r}
 45 \\
 \hline
 (2)
 \end{array}$$

(A1)



The diagram shows a solid prism.

The cross section of the prism is a trapezium.

The lengths of the parallel sides of the trapezium are 11 cm and 7 cm.

The perpendicular distance between the parallel sides of the trapezium is 10 cm.

The length of the prism is 12 cm.

(a) Work out the area of the trapezium.

$$A = \frac{1}{2}(7+11) \times 10 \quad (M1)$$

$$\begin{array}{r} 90 \quad (A1) \\ \hline (2) \end{array} \text{ cm}^2$$

(b) Work out the volume of the prism.

$$90 \times 12 \quad (M1)$$

$$\begin{array}{r} 1080 \quad (A1) \\ \hline (2) \end{array} \text{ cm}^3$$

Solve $6(3y + 5) = 39$
Show clear algebraic working.

$$\begin{aligned}18y + 30 &= 39 && \text{(ml)} \\18y &= 9 && \text{(ml)} \\y &= \frac{9}{18}\end{aligned}$$

$$y = \overset{\text{(Al)}}{0.5}$$

The table gives information about the numbers of goals scored by a football team in 30 matches.

Number of goals scored	Frequency	$x \times f$
0	2	0
1	10	10
2	7	14
3	6	18
4	3	12
5	2	10
TOTAL		64

(M1)

(A1)

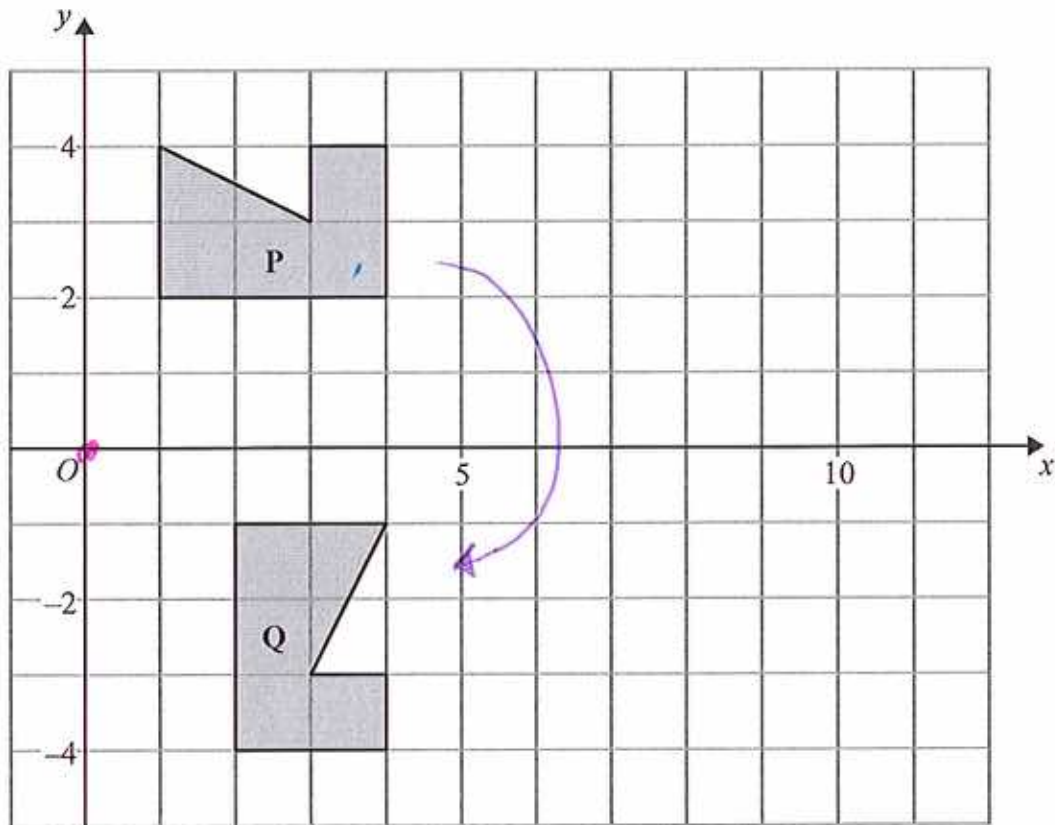
Find the mean number of goals scored.

$$\text{MEAN} = \frac{\text{TOTAL NO. OF GOALS}}{\text{NO. OF MATCHES}}$$

$$= \frac{64}{30}$$

$$= \underline{\underline{2.13}} \quad \text{(A1)}$$

The diagram shows a shape P, and a shape Q.



Describe fully the single transformation which maps shape P onto shape Q.

ROTATION, (B1) 90° CLOCKWISE, (B1)
CENTRE (0,0) (B1)

(a) Simplify $4 + 4 + 4 + 4 + 4$

$$20 \quad (1)$$

(b) Expand $2(7 - 3)$

$$14 - 6 \quad (1)$$

(c) Expand and simplify fully

(i) $4(2y + 6) - 3(2y - 7)$

$$8y + 24 - 6y + 21 \quad (1)$$

$$2y + 45 \quad (1)$$

(ii) $(x - 6)(x - 4)$

F O I L

$$x^2 - 6x - 4x + 24$$

$$x^2 - 10x + 24 \quad (1)$$

(d) Simplify fully $\frac{v^8 \times v^2}{v^5}$

$$= \frac{v^{10}}{v^5} \quad (1)$$

$$v^5 \quad (1)$$

A square hole is cut from a circular piece of card.

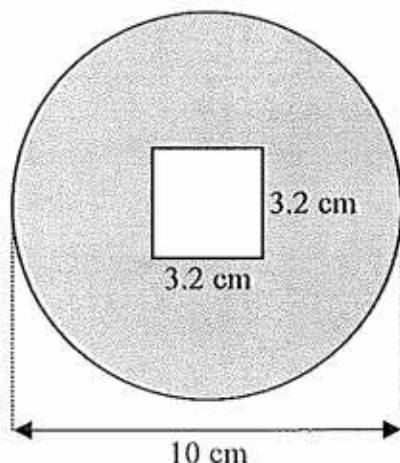


Diagram NOT
accurately drawn

The square has sides of length 3.2 cm.
The diameter of the circular piece of card is 10 cm.

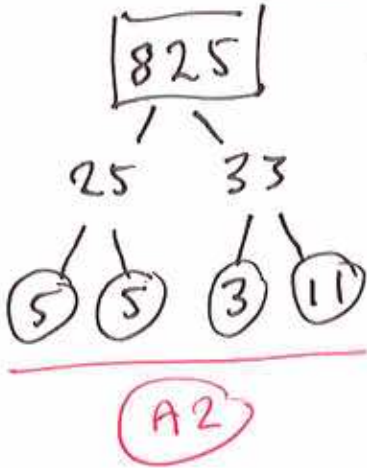
Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{AREA OF} \\ \text{SQUARE} &= 3.2 \times 3.2 \\ &= \underline{10.24} \end{aligned}$$

$$\begin{aligned} \text{AREA OF CIRCLE} &= \pi r^2 \quad (r=5) \\ &= \pi \times 5^2 \\ &= 78.5398 \dots \end{aligned}$$

$$\begin{aligned} \text{AREA OF SHADED REGION} \\ &= 78.5398 - 10.24 \\ &= 68.2998 \end{aligned} \quad \underline{\underline{68.3}} \text{ cm}^2$$

Express 825 as a product of its prime factors.



(M1) ALTERNATIVE METHODS ARE FINE

$$3 \times 5^2 \times 11$$

(A1) ↑ WRITTEN AS SQUARE

$\mathcal{U} = \{\text{positive whole numbers less than 13}\}$

$A = \{\text{even numbers}\}$

$B = \{\text{multiples of 3}\}$

$C = \{\text{prime numbers}\}$

(a) List the members of the set

(i) $A \cap B$

$\{6, 12\}$

(AI)

(ii) $B \cup C$

$\{2, 3, 5, 6, 7, 9, 11, 12\}$

(AI)

(2)

(b) Is it true that $14 \in A$?

Tick (\checkmark) the appropriate box.

Yes

No

Explain your answer.

THE UNIVERSAL SET ONLY HAS NUMBERS

LESS THAN 13

(AI)

(1)

(ANY EQUIVALENT STATEMENT)

The mean of four numbers is 2.6
One of the four numbers is 5

$$\longrightarrow \text{TOTAL} = 4 \times 2.6$$
$$= 10.4 \text{ (B1)}$$

Find the mean of the other three numbers.

$$10.4 - 5 = 5.4$$

(M1)

$$\frac{5.4}{3} = \underline{\underline{1.8}} \text{ (A1)}$$

The table shows the land areas, in km^2 , of four countries.

Country	Land area (km^2)
Ethiopia	1.13×10^6
Algeria	2.38×10^6
Nigeria	9.24×10^5
Kenya	5.83×10^5

(a) Which country has the largest land area?

(AI)
ALGERIA
(1)

(b) Calculate the total land area, in km^2 , of all four countries.
Give your answer in standard form.

5017000 (ml)
↑
USED A CALCULATOR
TO GET THIS

(AI)
 5.017×10^6 km^2
(2)

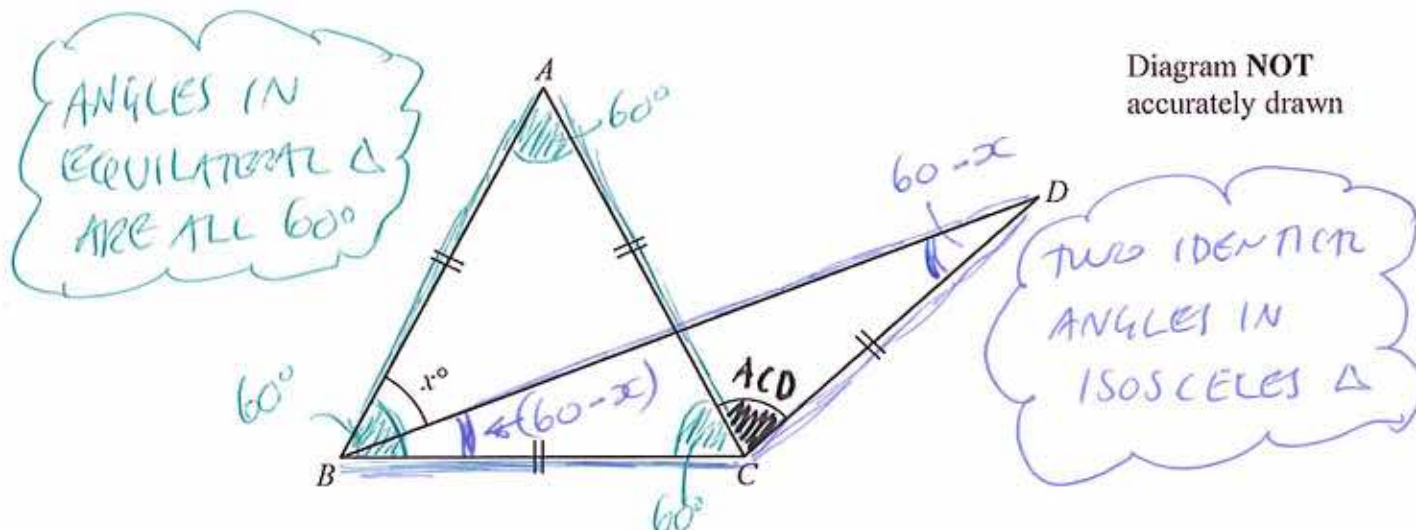
Population density is calculated by the formula

$$\text{Population density} = \text{Population} \div \text{Land area}$$

(c) In one year, the population of Ethiopia was 7.91×10^7
Calculate the population density of Ethiopia for that year.

$$\frac{7.91 \times 10^7}{1.13 \times 10^6} \quad (\text{ml})$$

(AI)
70 people / km^2
(2)



The diagram shows an equilateral triangle ABC and an isosceles triangle BCD .
 $AB = AC = BC = CD$.
 Angle $ABD = x^\circ$

Express the size of angle ACD in terms of x° , giving your answer as simply as possible.
 Give a reason for each step in your working.

$$\text{SUM OF ANGLES} = 180$$

SO IN ISOSCELES Δ .

$$(60-x) + \overset{\text{(BI)}}{\underbrace{(60-x)}} + 60 + \hat{A}CD = 180 \quad \text{(M1)}$$

$$\Rightarrow \hat{A}CD = 180 - (60-x) - (60-x) - 60 \quad \text{(M1)}$$

$$= 180 - 60 + x - 60 + x - 60$$

$$= 180 - 180 + 2x$$

$$= \underline{\underline{2x}} \quad \text{(A1)}$$

Factorise fully $4(x-5)^2 + 3(x-5)$

$$\begin{aligned} & (x-5) \times [4(x-5) + 3] \\ = & (x-5) \times [4x - 20 + 3] \\ = & (x-5)(4x - 17) \end{aligned}$$

(A1) (A1)

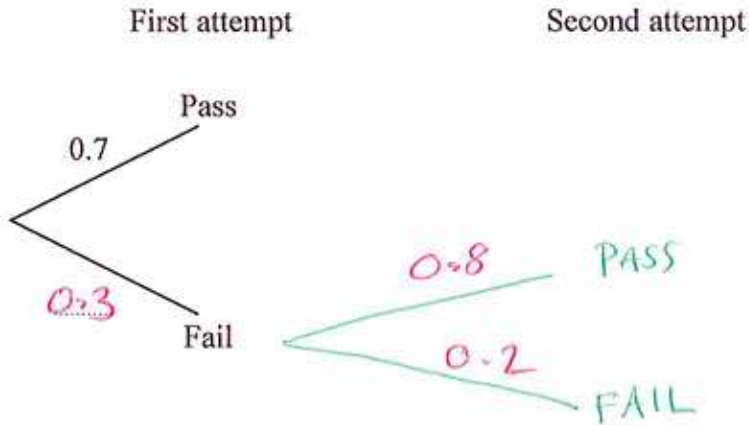
Natasha wants to pass her driving test.

The probability that she passes at her first attempt is 0.7

When Natasha passes her driving test, she does not take it again.

If she fails, the probability that she passes at the next attempt is 0.8.

(i) Complete the probability tree diagram for Natasha's first two attempts.



(ii) Calculate the probability that Natasha needs exactly two attempts to pass her driving test.

(iii) Calculate the probability that Natasha passes her driving test at her third or fourth attempt.

$$(ii) P(FP) = 0.3 \times 0.8 = \underline{\underline{0.24}}$$

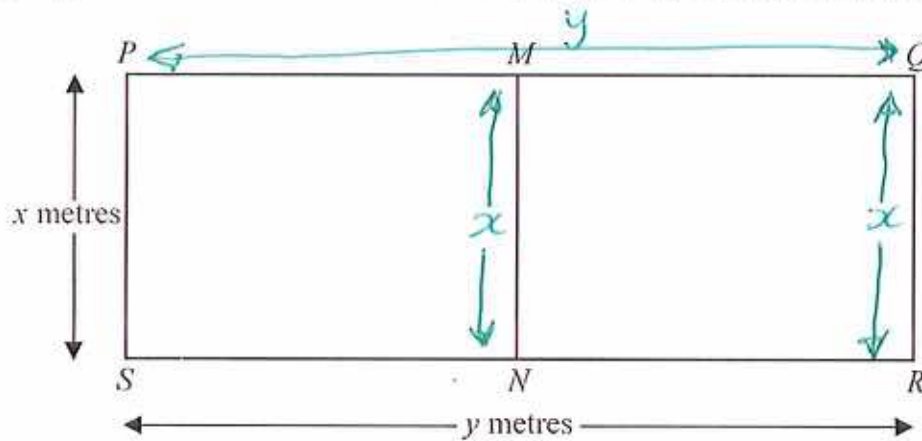
$$(iii) P(FFP) = 0.3 \times 0.2 \times 0.8 = 0.048$$

$$P(FFFP) = 0.3 \times 0.2 \times 0.2 \times 0.8 = 0.0096$$

TOTAL

$$\underline{\underline{0.0576}}$$

A farmer has 120 metres of fencing.
 He is going to make a rectangular enclosure $PQRS$ with the fencing.
 He is also going to divide the enclosure into two equal parts by fencing along MN .



TOTAL OF
FENCING
= 120

The width of the enclosure is x metres.
 The length of the enclosure is y metres.

(a) (i) Show that $y = 60 - 1.5x$

$$2y + 3x = 120 \Rightarrow 2y = 120 - 3x$$

$$\Rightarrow y = 60 - 1.5x$$

(B1)

The area of the enclosure $PQRS$ is A m²

(ii) Show that $A = 60x - 1.5x^2$

$$A = x \times y = x \times (60 - 1.5x)$$

$$= 60x - 1.5x^2$$

(B1)

(b) Find $\frac{dA}{dx}$

$$= 60 - 3x$$

(B1) (B1)

(c) Find the maximum value of A .

$$60 - 3x = 0$$

$$\Rightarrow -3x = -60$$

$$x = \frac{-60}{-3}$$

$$= \underline{\underline{20}}$$

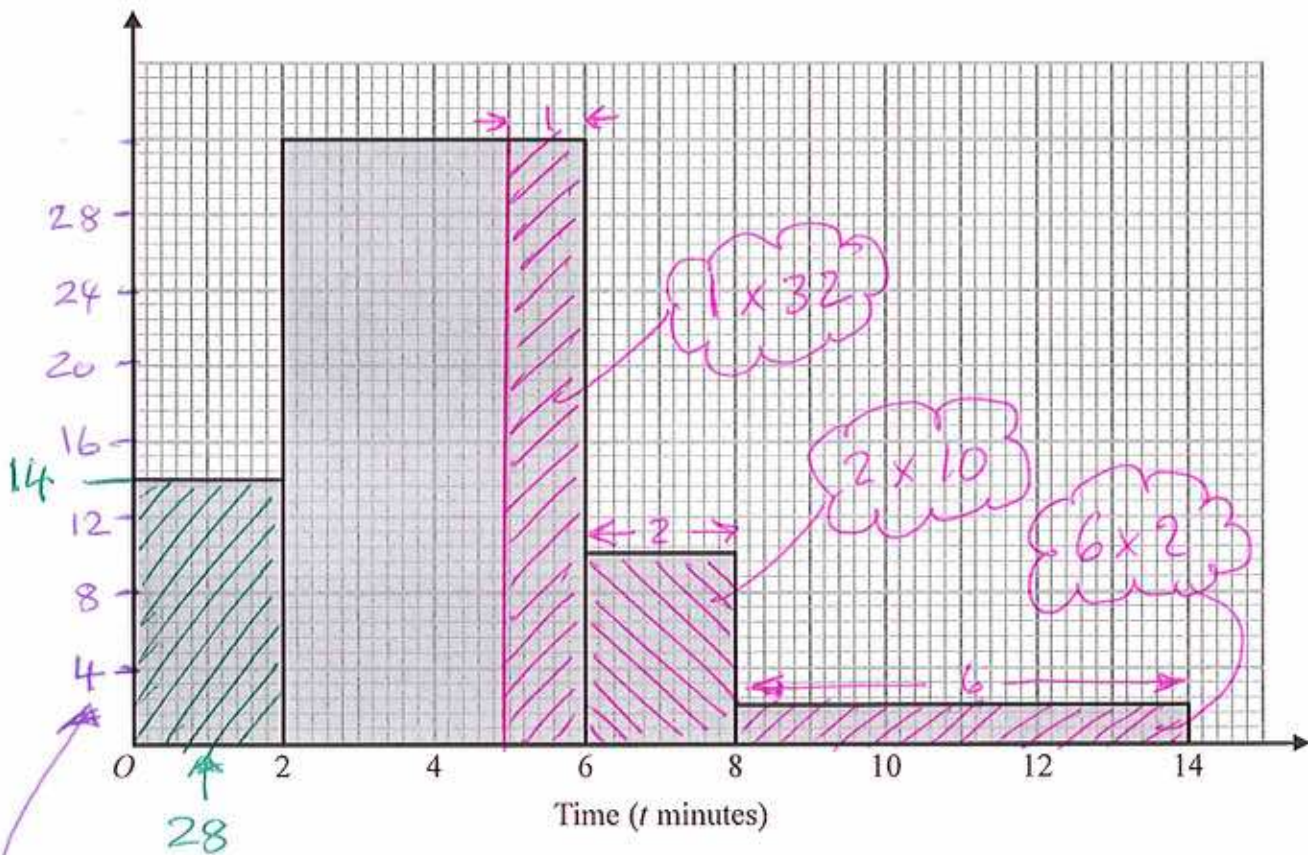
(M1) (B1)

$$A = 60 \times 20 - 1.5 \times 20^2$$

$$= \underline{\underline{600}} \text{ m}^2$$

(A1)

The histogram shows information about the times, t minutes, customers spent in a post office.



28 customers spent 2 minutes or less in the post office.

Calculate an estimate for the number of customers who spent between 5 and 14 minutes in the post office.

1ST

28 CUSTOMERS IN BAR OF WIDTH 2.

$$\begin{aligned} \therefore \text{HEIGHT} &= \frac{28}{2} \\ &= \underline{\underline{14}} \quad (\text{M1}) \end{aligned}$$

2ND

[WORK OUT FREQUENCY DENSITY SCALE]

3RD

$$1 \times 32 + 2 \times 10 + 6 \times 2 = \underline{\underline{64}} \quad (\text{M1}) \quad (\text{A1})$$

A circular clock face, centre O , has a minute hand OA and an hour hand OB .

$OA = 10$ cm.

$OB = 7$ cm.

Calculate the length of AB when the hands show 5 o'clock.

Give your answer correct to 3 significant figures.

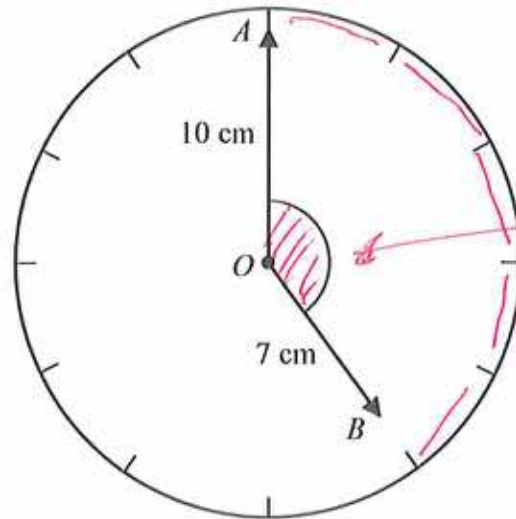
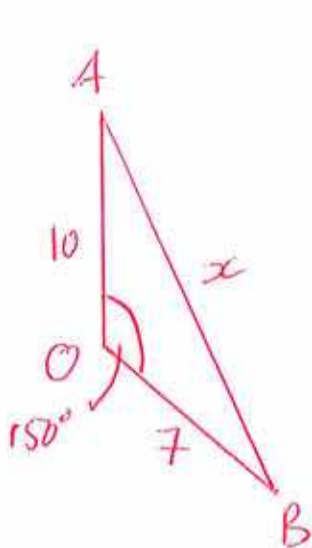


Diagram NOT accurately drawn

$$\frac{5}{12} \times 360 = 150^\circ$$



$$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 150^\circ$$

$$= 270.243 \dots$$

$$x = \sqrt{270.24 \dots}$$

$$= 16.439 \dots$$

$$= \underline{\underline{16.4 \text{ cm}}}$$

A rectangular lawn has a length of $3x$ metres and a width of $2x$ metres.
The lawn has a path of width 1 metre on three of its sides.

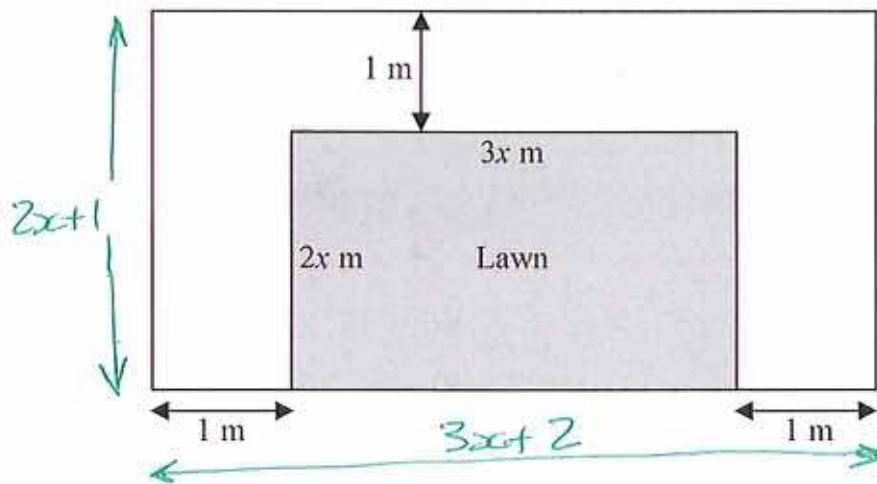


Diagram NOT
accurately drawn

The total area of the lawn and the path is 100 m^2

(a) Show that $6x^2 + 7x - 98 = 0$

$$(2x+1)(3x+2) = 100 \quad (M1)$$

$$6x^2 + 4x + 3x + 2 = 100 \quad (M1)$$

$$6x^2 + 7x - 98 = 0$$

(b) Calculate the area of the lawn.
Show clear algebraic working.

$$(3x+14)(2x-7) = 0 \quad (M1)$$

$$\begin{aligned} 3x+14 &= 0 \\ 3x &= -14 \\ x &= -\frac{14}{3} \end{aligned}$$

$$\begin{aligned} 2x-7 &= 0 \\ 2x &= 7 \\ x &= \underline{\underline{3.5}} \quad (A1) \end{aligned}$$

[NOT
POSSIBLE]

$$\begin{aligned} \text{AREA OF LAWN} &= 2x \times 3x \\ &= 6x^2 \quad (M1) \\ &= 6 \times 3.5^2 \quad (M1) \\ &= \underline{\underline{73.5 \text{ m}^2}} \quad (A1) \end{aligned}$$



Diagram NOT
accurately drawn

The diagram shows part of a regular polygon.

The interior angle and the exterior angle at a vertex are marked.

The size of the interior angle is 7 times the size of the exterior angle.

Work out the number of sides of the polygon.

$$7x + x = 180 \quad (m)$$

$$\Rightarrow 8x = 180$$

$$x = \frac{180}{8}$$

$$= \underline{\underline{22.5}} \quad (A)$$

$$\frac{360}{22.5} = \underline{\underline{16 \text{ SIDES}}} \quad (A)$$

Show that the recurring decimal $0.0\ddot{1}5 = \frac{1}{66}$

$$100x = 1.5151515 \dots \quad (m1)$$

$$x = 0.0151515 \dots$$

SUBTRACT

$$\begin{array}{r} 99x = 1.5 \\ x = 1.5 \\ \hline 99 \end{array} \quad (m1) \rightarrow x = \frac{15}{990}$$

$$= \frac{1}{66} \quad \text{Q.E.D.}$$

There are 1300 sheets of paper, correct to the nearest 100 sheets, in a pile. $\rightarrow 1300 \pm 50$
Each sheet is of equal thickness.

The height of the pile is 160 mm, correct to the nearest 10 mm. $\rightarrow 160 \pm 5$

Calculate the upper bound, in millimetres, for the thickness of one sheet of paper.

$$\text{THICKNESS} = \frac{\text{HEIGHT}}{\text{NO. OF SHEETS}} \quad (\text{M1}) \quad [\text{CORRECT DIVISION}]$$

$$= \frac{160 + 5}{1300 - 50} \quad (\text{UPPER})$$
$$\quad \quad \quad (\text{LOWER})$$

$$= \frac{165}{1250} \quad (\text{M1}) \quad [\text{CORRECT VALUES}]$$

$$= \underline{\underline{0.132}} \text{ mm} \quad (\text{A1})$$

Solve the simultaneous equations

$$2x - y = 7 \Rightarrow y = 2x - 7 \quad \text{--- (1)}$$

$$x^2 + y^2 = 34 \quad \text{--- (2)}$$

Show clear algebraic working.

SUBSTITUTE (1) INTO (2)

$$x^2 + (2x - 7)^2 = 34 \quad \text{(M1) [FOR EQUATION]}$$

$$x^2 + (2x - 7)(2x - 7) = 34$$

$$x^2 + 4x^2 - 14x - 14x + 49 = 34 \quad \text{(M1) [EXPANDING BRACKETS]}$$

$$5x^2 - 28x + 49 - 34 = 0$$

$$5x^2 - 28x + 15 = 0 \quad \text{(B1) [QUADRATIC]}$$

$$(5x - 3)(x - 5) = 0 \quad \text{(M1) [FACTORISING]}$$

$$\begin{aligned} \swarrow \\ 5x - 3 &= 0 \\ x &= \underline{\underline{0.6}} \end{aligned}$$

$$\begin{aligned} \searrow \\ x &= \underline{\underline{5}} \quad \text{(A1) [BOTH x-VALUES]} \end{aligned}$$

$$\begin{aligned} y &= 2 \times 0.6 - 7 \\ &= \underline{\underline{-5.8}} \end{aligned}$$

$$\begin{aligned} y &= 2 \times 5 - 7 \\ &= \underline{\underline{3}} \end{aligned}$$

(A1) [BOTH y-VALUES]

A pyramid has a horizontal square base $ABCD$ with sides of length 230 metres.

M is the midpoint of AC .

The vertex, T , is vertically above M .

The slant edges of the pyramid are of length 218 metres.



Calculate the height, MT , of the pyramid.

Give your answer correct to 3 significant figures.

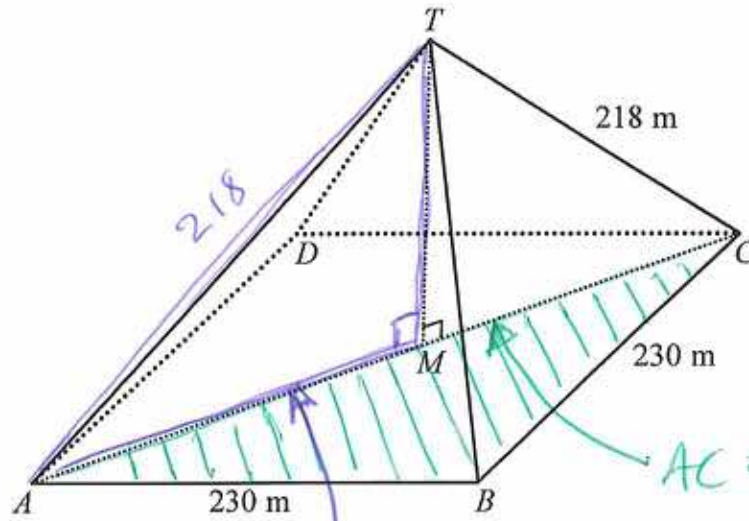
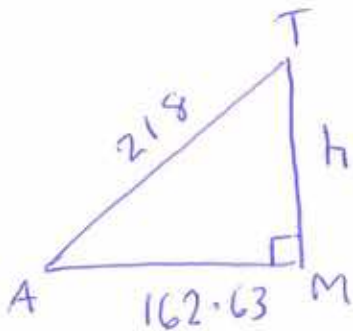


Diagram NOT
accurately drawn

[2ND]

$$AM = \frac{325.269}{2}$$

$$= 162.63\dots$$



[3RD]

$$h = \sqrt{218^2 - 162.63^2}$$

$$= 145.168\dots$$

$$= \underline{145 \text{ m}}$$

[1ST]

$$AC = \sqrt{230^2 + 230^2}$$

$$= 325.269\dots$$