4H(R)

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A SOLUTIONS

JANUARY 2015 | 4MA0/4HR

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

Eric travels from the UK to India every year.

In 2010, the exchange rate was £1 = 67.1 rupees.
In 2012, the exchange rate was £1 = 82.5 rupees.

In 2010 Eric changed £600 into rupees.

How many pounds (£) did Eric have to change to rupees in 2012 to get the same number of rupees as he did in 2010?

2010

$600 \times 67.1 = 40260$ M1

2012

NEEDS 40260 RUPEES $= \dfrac{40260}{82.5}$ M1

$= £488$ A1

The wheel of the Singapore Flyer is a circle with a diameter of 150 metres.

(a) Calculate the circumference of the wheel.
Give your answer correct to the nearest metre.

$C = \pi D$

$= \pi \times 150$ (M1)

$= 471.238...$

$= \underline{471}$ (A1)

471 metres
(2)

$30 \times 60 = 1800$ SECONDS

The wheel takes 30 minutes to rotate once.

(b) Work out the average speed of a point on the circumference of the wheel as it rotates once.
Give your answer in metres per second correct to 3 significant figures.

$\text{SPEED} = \dfrac{\text{DISTANCE}}{\text{TIME}}$

$= \dfrac{471.238...}{1800}$ (M1)

(B1)

$= 0.261799...$

$= \underline{0.262}$ (A1)

The diagram shows a giant wheel above horizontal ground.

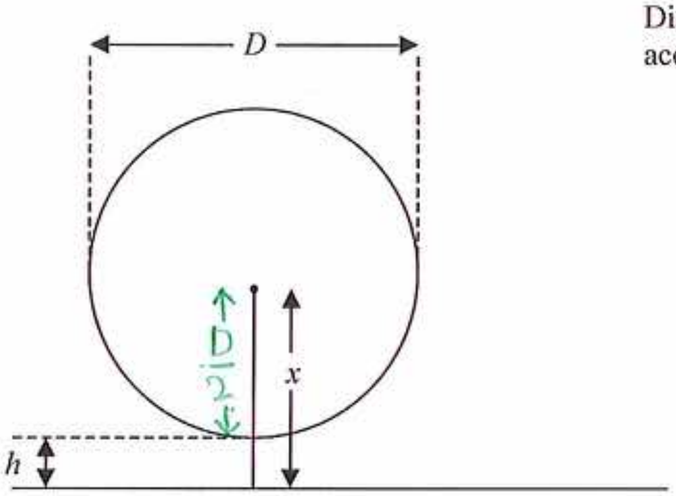


Diagram **NOT** accurately drawn

The wheel is a circle of diameter D metres.
The lowest point of the wheel is h metres above the ground.
The centre of the wheel is x metres above the ground.

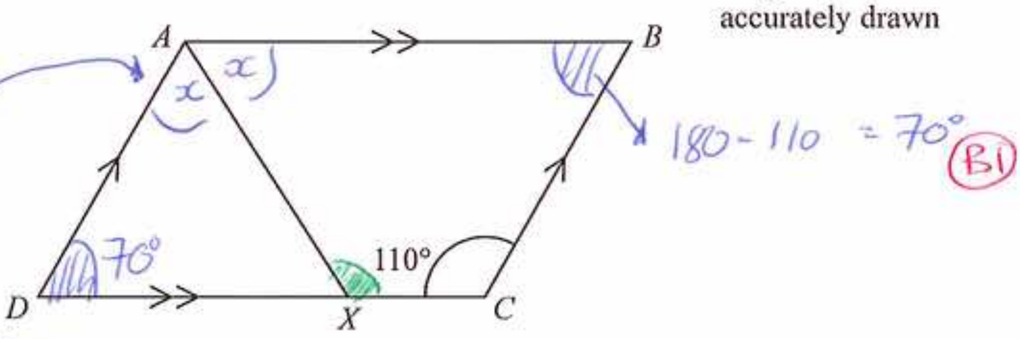
(c) Express h in terms of D and x

$$h + \frac{D}{2} = x \quad \text{M1}$$

$$\Rightarrow \quad h = x - \frac{D}{2} \quad \text{A1}$$

$$\left[\underline{\text{OR}} \quad h = \frac{2x - D}{2}\right]$$

Diagram **NOT** accurately drawn



ABCD is a parallelogram.
Angle *DCB* = 110°
X is the point on *DC* such that *AX* bisects the angle *DAB*. ⇐ KEY PART OF QUESTION!

Calculate the size of angle *AXC*.

$2x = 110$

$\Rightarrow x = \underline{\underline{55}}$ B1

$AXC = 360 - (55 + 70 + 110)$ M1

$= \underline{\underline{125°}}$ A1

FACTS USED:

- OPPOSITE ANGLES IN A PARALLELOGRAM ARE EQUAL.
- ADJACENT ANGLES IN A PARALLELOGRAM ADD TO 180°
- ANGLES IN A QUADRILATERAL ADD TO 360°

Solve $x + 2y = 3$

$x - y = 6$

Show clear algebraic working.

$x + 2y = 3$ — 1 } SUBTRACT

$x - y = 6$ — 2

$3y = -3$ M1

$y = -\frac{3}{3}$

$= \underline{\underline{-1}}$ A1

SUBSTITUTE INTO 1

$x + 2 \times (-1) = 3$

$x - 2 = 3$

$x = \underline{\underline{5}}$ A1

Here are some rows of a number pattern.

Row number	Column 1	Column 2	Column 3
1	$1 \times 3 + 1$	4	2^2
2	$2 \times 4 + 1$	9	3^2
3	$3 \times 5 + 1$	16	4^2
⋮			
		676	
⋮			
$n \rightarrow$	$n(n+2)+1$	$(n+1)^2$	$(n+1)^2$

I DID THESE BEFORE I LOOKED AT QUESTIONS!

(a) Write down the Row number of the row that has 676 in Column 2

$\sqrt{676} = 26 \longrightarrow$

$n = 25$ ....

(1) A1

(b) For Row number n,

(i) write down an expression, in terms of n, that should go in Column 1

A1

$n(n+2)+1$ ....

(ii) write down an expression, in terms of n, that should go in Column 3

A1

$(n+1)^2$

The table gives information about the number of vehicles passing a point on a road in each of 70 intervals of equal length.

Number of vehicles	Frequency	MID VALUE	$x \times f$
1 to 5	8	3	24
6 to 10	10	8	80
11 to 15	18	13	234
16 to 20	20	18	360
21 to 25	10	23	230
26 to 30	4	28	112
		TOTAL	1040

M1

M1

(a) Write down the modal class interval.

HIGHEST FREQUENCY.

A1

16 to 20

(1)

(b) Calculate an estimate for the mean.

$$\text{MEAN} = \frac{\text{TOTAL NUMBER OF VEHICLES}}{\text{NUMBER OF INTERVALS}}$$

$$= \frac{1040}{70} \quad \text{M1}$$

$$= 14.8571\ldots$$

$$= \underline{\underline{14.9}} \quad \text{A1}$$

Here is a trapezium *ABCD*.

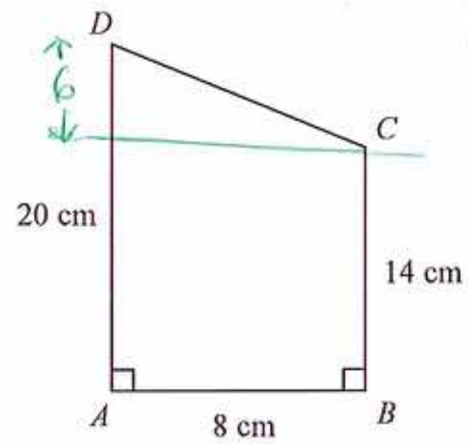


Diagram **NOT** accurately drawn

Angle DAB = angle $ABC = 90°$

$AD = 20$ cm
$AB = 8$ cm
$BC = 14$ cm

(a) Calculate the area of the trapezium *ABCD*.

$A = \frac{1}{2}(a+b)h$

$= \frac{1}{2} \times (14+20) \times 8$ (M1)

[OTHER METHODS ARE FINE]

(A1)

136 cm²
(2)

(b) Calculate the length of *CD*.

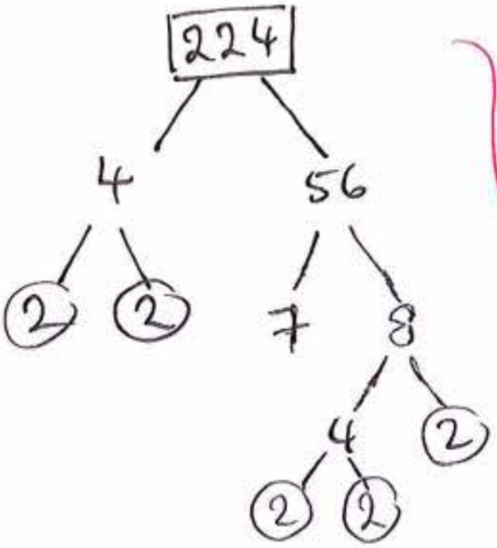
$CD^2 = 8^2 + 6^2$ (B1) (M1) [FOR USING PYTHAGORAS]

$= 100$

$CD = \sqrt{100}$ (M1)

$= \underline{\underline{10}}$ cm (A1)

(a) Write 224 as a product of powers of its prime factors.
Show your working clearly.



M1

M1

$2 \times 2 \times 2 \times 2 \times 2 \times 7$

A1

$2^5 \times 7$

(3)

(b) Write down 3 **different** factors of 224 with a sum between 99 and 110

FACTORS ARE:

2	7
$2^2 = 4$	$2 \times 7 = 14$
$2^3 = 8$	$2^2 \times 7 = 28$
$2^4 = 16$	$2^3 \times 7 = 56$
$2^5 = 32$	$2^4 \times 7 = \cancel{112}$
	$2^5 \times 7 = \cancel{224}$

TOO HIGH

M1 FOR ANY THREE FACTORS (< 99)

$56 + 28 + 16 = 100$

$56 + 32 + 16 = 104$

$56 + 32 + 14 = 102$

A1 [FOR ANY THREE FACTORS]

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{\text{even numbers}\}$ — {2, 4, 6, 8, 10}

$B = \{\text{multiples of } 3\}$ — {3, 6, 9}

(a) List the members of set B.

A1

{3, 6, 9}

(1)

(b) Find $A \cup B$

A1

{2, 3, 4, 6, 8, 9, 10}

(1)

(c) Find $A \cap B$

A1

{6}

(1)

x is a member of $\mathscr{E}$

$x \in B$

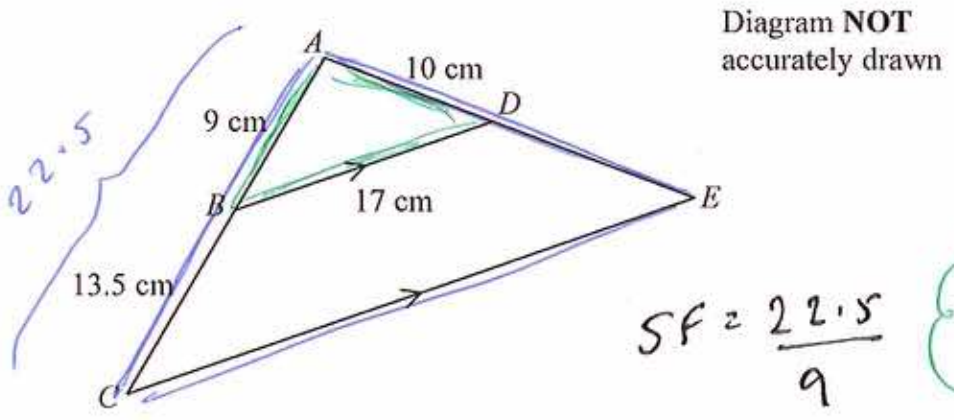
$x \notin A$

(d) What are the possible values of x?

A1 A1

3 OR 9

(2)



$SF = \frac{22.5}{9} = 2.5$

In the diagram *ABC* and *ADE* are straight lines.
BD is parallel to *CE*.

AB = 9 cm, *BC* = 13.5 cm, *AD* = 10 cm, *BD* = 17 cm

(a) Calculate the length of *CE*.

$CE = 17 \times 2.5$ M1

A1
42.5 cm
(2)

(b) Calculate the length of *DE*.

$AE = 10 \times 2.5$
$= 25$

M1
$DE = 25 - 10$
$= 15$

A1
15 cm
(2)

The area of triangle *ABD* is 36 cm²

(c) Calculate the area of quadrilateral *BDEC*.

TRIANGLE ACE $= 36 \times 2.5^2$
$= 225$ M1

$\therefore$ TRAPEZIUM $= 225 - 36$ M1
$= 189$

A1
189 cm²

$t^n = \frac{1}{t^3}$

(a) Write down the value of n.

A1

$n =$ -3

(1)

(b) Simplify $\frac{6xy^5}{3xy^2}$

A1 A1

$2y^3$

(2)

(c) Expand and simplify $(3x - 2y)(x + 2y)$

F O I L M1

$3x^2 + 6xy - 2xy - 4y^2$

A1

$3x^2 + 4xy - 4y^2$

(2)

(d) Factorise $4x^2 - 7x - 2$ 1×2

$(4x + 1)(x - 2)$

$+x$

$-8x$

$-7x$

A1 A1

$(4x+1)(x-2)$

(2)

$I = kT^4$

$k = 5.67 \times 10^{-8}$

$T = 5800$

(a) Work out the value of I.
Give your answer in standard form correct to 3 significant figures.

$I = (5.67 \times 10^{-8}) \times 5800^4$

$= 64164532.32$ M1

$I =$ 6.42×10^7 A1

(2)

(b) Rearrange the formula $I = kT^4$ to make T the subject.

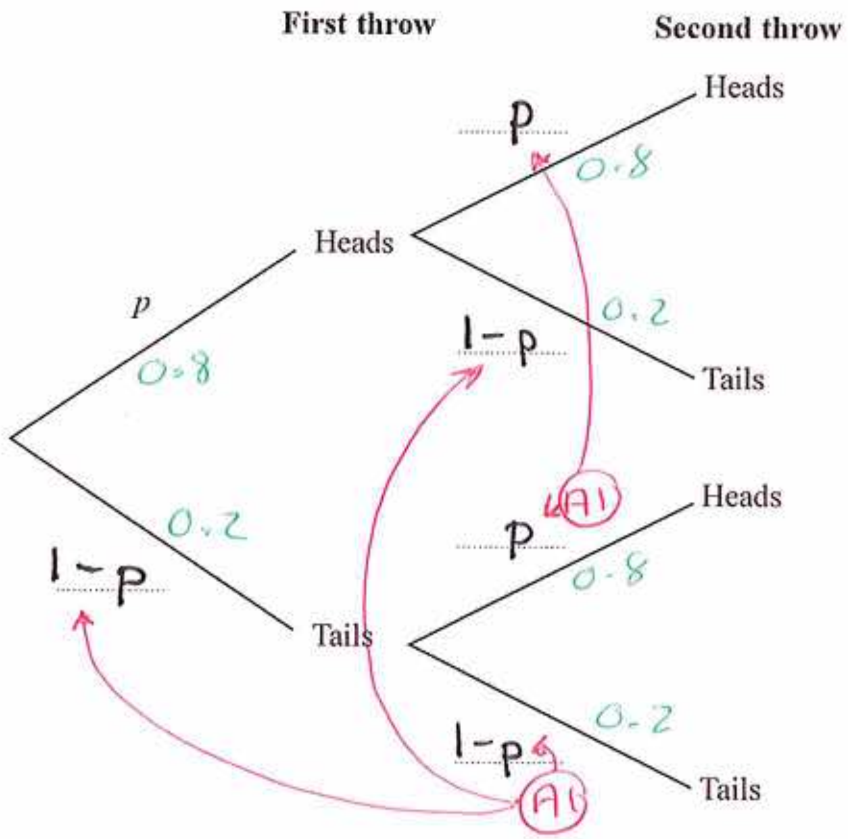
$I = kT^4$

$T^4 = \frac{I}{k}$ M1

$T = \sqrt[4]{\frac{I}{k}}$ A1

Jim has a biased coin.
The probability that Jim will throw Heads on any throw is p.
Jim throws the coin twice.

(a) Complete the probability tree diagram.
Give your probabilities in terms of p.



(2)

(b) Find an expression, in terms of p, for the probability that Jim will throw two Heads.

$P(H,H) = p \times p$

p^2 (A1)

(1)

Given that $p = 0.8$,

(c) work out the probability that Jim will throw exactly one Head.

$P(HT) = 0.8 \times 0.2 = 0.16$

$P(TH) = 0.2 \times 0.8 = 0.16$

TOTAL $= 0.32$ (A1)

(M1) [MULTIPLY] (M1) [TWO ANSWERS]

(a) Solve $x^2 - 4x - 1 = 0$ $a = 1,\ b = -4,\ c = -1$

Show your working clearly.
Give your solutions correct to 3 significant figures.

M1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-4) \pm \sqrt{16 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{20}}{2}$$ M1

$x = 4.24$, $x = -0.236$

(3)

A1 [Both]

Hence, or otherwise,

(b) solve $(x + 3)^2 - 4(x + 3) - 1 = 0$

giving your solutions correct to 3 significant figures.

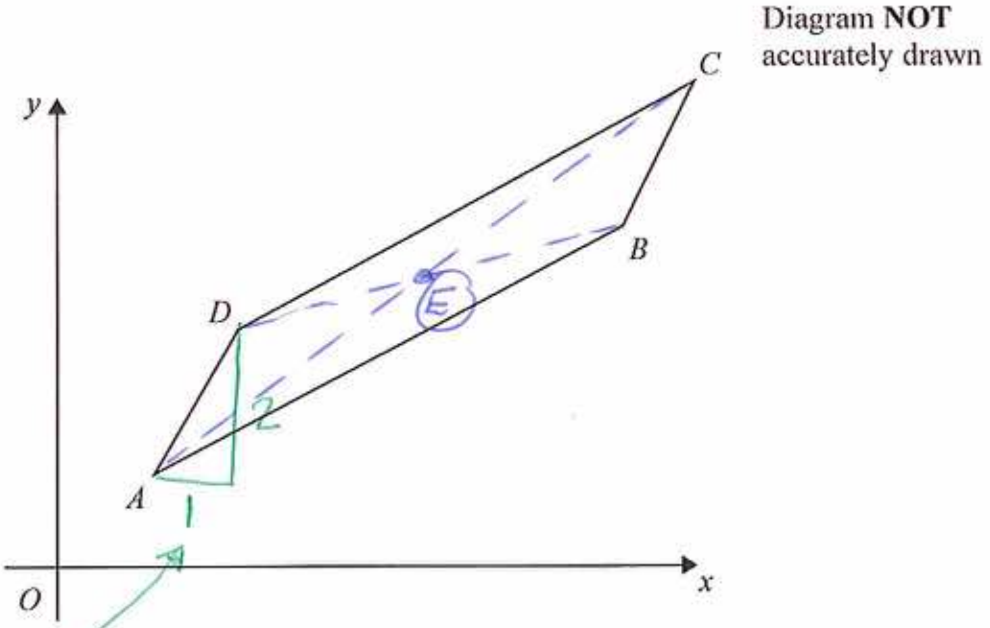
$x + 3 = 4.24$ $x + 3 = -0.236$

$\Rightarrow x = \underline{1.24}$ $\Rightarrow x = -3.236$

$= -\underline{3.24}$

A1 [Both].

Here is the parallelogram *ABCD*.



$\overrightarrow{AD} = \begin{pmatrix}1\\2\end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix}5\\3\end{pmatrix}$

(a) Find the magnitude of $\overrightarrow{AD}$.

Give your answer correct to 3 significant figures.

$AD^2 = 1^2 + 2^2$ M1

$= 5$

$AD = \sqrt{5} = 2.2360\ldots$ A1

2.24

(2)

The point *A* has coordinates (4, 2)

(b) Work out the coordinates of the point *C*.

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}5\\3\end{pmatrix}$

$= \begin{pmatrix}6\\5\end{pmatrix}$ M1

∴ COORDINATES OF

$C = (4+6, 2+5)$ M1

(10, 7) A1

The diagonals of the parallelogram $ABCD$ cross at the point E.

(c) Find as a column vector, $\overrightarrow{OE}$.

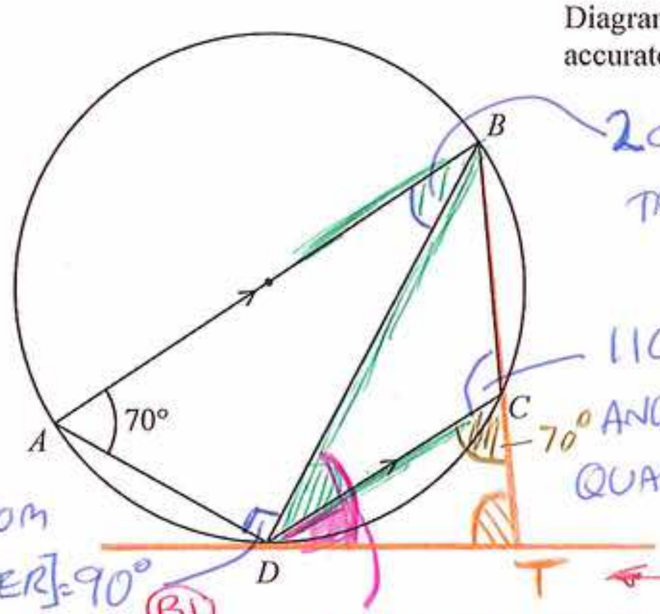
E IS THE MIDPOINT OF AC

$\therefore$ ITS COORDINATES ARE $\left(\frac{4+10}{2}, \frac{2+7}{2}\right)$ M1

$= (7, 4 \cdot 5)$ A1

$\therefore$ VECTOR FROM 'O' IS $\begin{pmatrix} 7 \\ 4 \cdot 5 \end{pmatrix}$ A1

Diagram **NOT** accurately drawn



20° [ANGLES IN A TRIANGLE ADD TO 180°]

110° [OPPOSITE ANGLES IN CYCLIC QUAD ADD TO 180°]

[ANGLE FROM DIAMETER] = 90° (B1)

70° − 20 = 50 (B1)

(B1) [FOR SHOWING 'T']

A, *B*, *C* and *D* are points on a circle.
AB is a diameter of the circle.
DC is parallel to *AB*.
Angle *BAD* = 70°

(a) Calculate the size of angle *BDC*.

BDC = 20° BECAUSE IT IS THE ALTERNATE ANGLE TO ABD

(A1)

20° °

(2)

The tangent to the circle at *D* meets the line *BC* extended at *T*.

(b) Calculate the size of angle *BTD*.

180 − (70 + 50) = 60° (A1)

(a) Show that $(3 + 2\sqrt{2})(4 - \sqrt{2}) = 8 + 5\sqrt{2}$

Show your working clearly.

F O (M1) I L

$(3+2\sqrt{2})(4-\sqrt{2}) = 12 - 3\sqrt{2} + 8\sqrt{2} - 2\times 2$ (M1)

$= 12 - 4 - 3\sqrt{2} + 8\sqrt{2}$ [$\sqrt{2}\times\sqrt{2} = 2$]

$= \underline{8 + 5\sqrt{2}}$

(2)

(b) Rationalise the denominator and simplify fully $\dfrac{10 + 3\sqrt{2}}{\sqrt{2}}$

Show your working clearly.

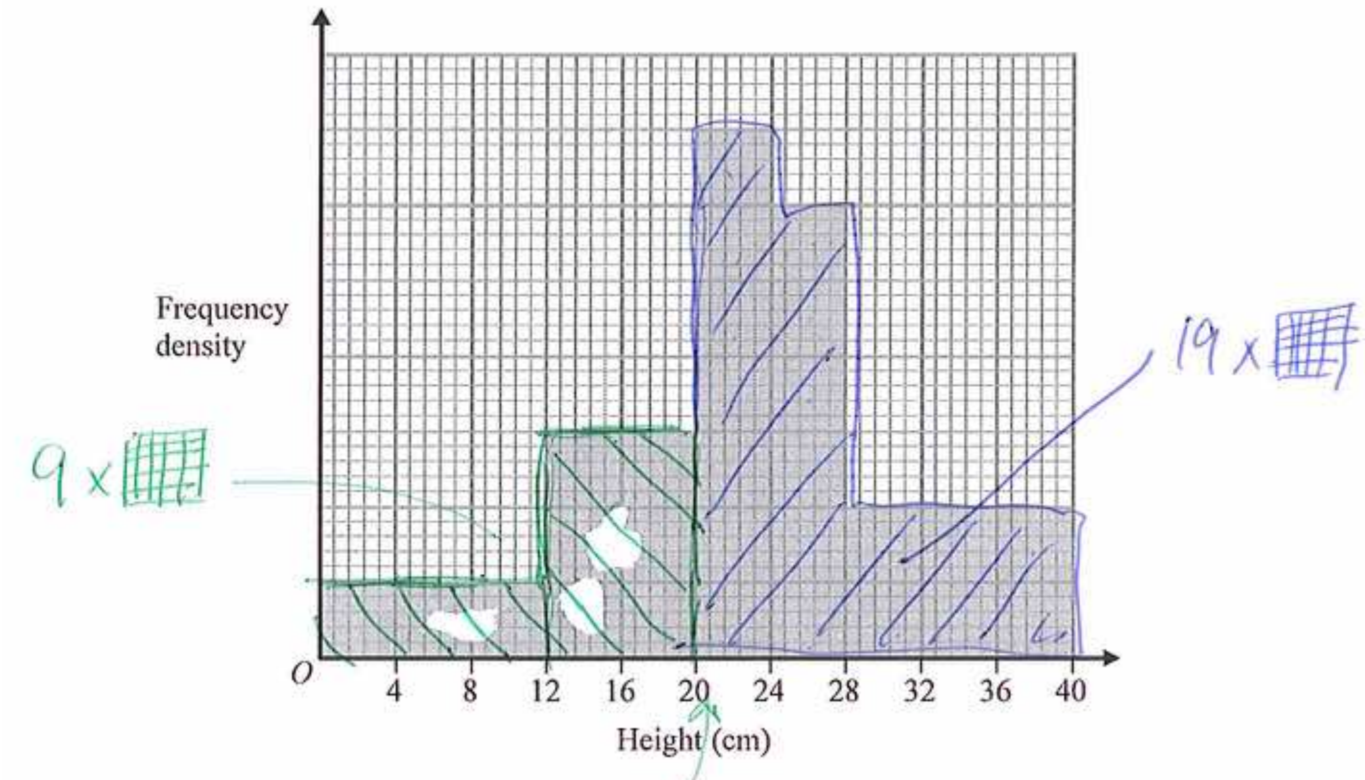
$\dfrac{10+3\sqrt{2}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{10\sqrt{2} + 3\times 2}{2}$

(M1)

$= \dfrac{10\sqrt{2}}{2} + \dfrac{6}{2}$

$= \underline{5\sqrt{2} + 3}$ (A1)

[I.E $3 + 5\sqrt{2}$]



The histogram gives information about the heights of some plants.

There are 360 plants with a height of 20 cm or less.

Work out the number of plants with a height of more than 20 cm.

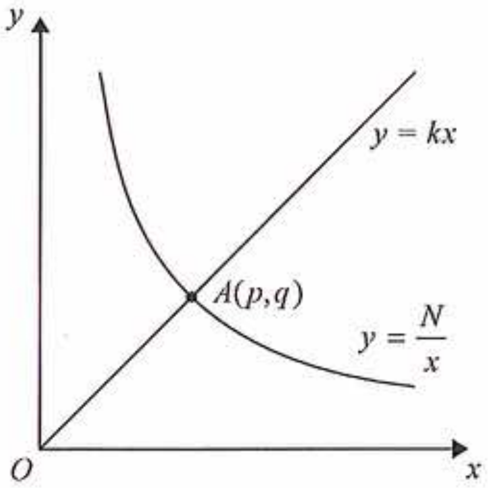
$9 \times \boxplus = 360 \quad \therefore \quad 1 \times \boxplus = \frac{360}{9}$ B1

$= 40$ PLANTS

[NO NEED TO WORK OUT ACTUAL FREQ DENSITIES BECAUSE ALL BARS CONTAIN A WHOLE NUMBER OF SQUARES!]

PLANTS WITH HEIGHT $> 20 = 19 \times 40$ M1

$= 760$ A1



The diagram shows the straight line with equation $y = kx$ intersecting the curve with equation $y = \frac{N}{x}$ at the point $A(p, q)$.

$y = kx$ — (1)

$y = \frac{N}{x}$ — (2)

(a) Find p and find q.

Give each answer in its simplest form, in terms of k and N.

$kx = \frac{N}{x}$ M1

$\Rightarrow kx^2 = N$

$x^2 = \frac{N}{k}$

$x = \sqrt{\frac{N}{k}}$

$p = \sqrt{\frac{N}{k}}$ A1

$y = k\sqrt{\frac{N}{k}}$

$= \sqrt{\frac{k^2N}{k}}$

$= \sqrt{kN}$

$q = \sqrt{kN}$ A1

Given that $p = 2q$

(b) find the value of k.

$\sqrt{\frac{N}{k}} = 2\sqrt{kN}$ M1

$\Rightarrow \frac{N}{k} = 4kN$

$\Rightarrow 4k^2N = N$

$4k^2 = 1$

$k^2 = \frac{1}{4}$

$k = \frac{1}{2}$ A1

(a) Factorise $4x^2 - 1$

A1 A1

$(2x+1)(2x-1)$

(2)

(b) Solve $\frac{4}{2x+1} + \frac{1}{4x^2-1} = 3$

Show clear algebraic working.

$$\frac{4}{2x+1} + \frac{1}{(2x+1)(2x-1)} = 3$$

$$4(2x-1) + 1 = 3(2x-1)(2x+1) \quad \text{M1}$$

$$\Rightarrow 8x - 4 + 1 = 3 \times [4x^2 - 1]$$

$$\Rightarrow 8x - 4 + 1 = 12x^2 - 3$$

M1 EITHER

$$\Rightarrow 12x^2 = 8x \quad \text{M1}$$

[NOTE THAT x COULD BE ZERO!]

$$\Rightarrow 12x = 8$$

[OR EQUIVALENT]

$$x = \frac{8}{12}$$

$$= \frac{2}{3} \quad \text{A1}$$