# 4H(R)

**Pearson Edexcel
International GCSE**

# EDEXCEL
# IGCSE

# MATHEMATICS A
# SOLUTIONS

# JANUARY 2016 | 4MA0/4HR

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Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

Here is a list of ingredients needed to make 16 cookies.

| Ingredients for 16 cookies |
| --- |
| 120 g butter |
| 150 g sugar |
| 240 g flour |
| 170 g chocolate |
| 1 egg |

Max wants to make 40 cookies.

(a) How much flour does Max need?          $\frac{240}{16} \times 40$ (M1)

                                                    600 (A1)
                                             _____ g
                                                    (2)

Abby made cookies to sell at a school fair.
She used 600 g of butter.

(b) How many cookies did she make?

        $\frac{600}{120} \times 16$ (M1)

                                                    (A1)
                                             80
                                             _____
                                                    (2)

(c) Find the ratio of the weight of sugar to the weight of flour in the list of ingredients.
    Give your answer in the form 1: $n$

                    S :  F
            $\div 150$ ( 150 : 240 ) $\div 150$      (M1)
                         1 : 1·6

                                                    (A1)
                                             1 :  1·6
                                             _____
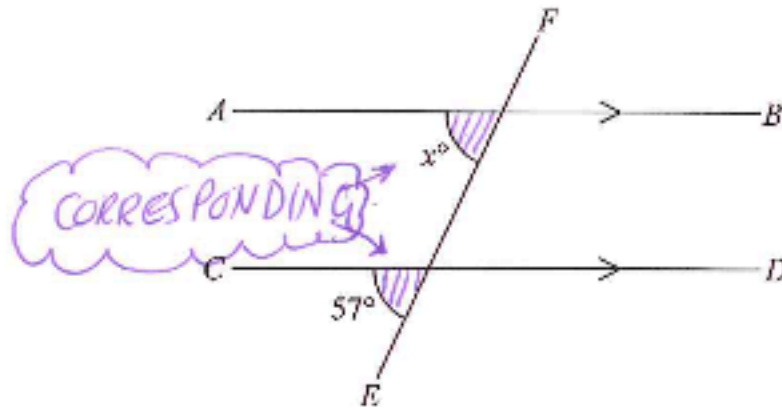                                                    (2)

Diagram **NOT** accurately drawn

CORRESPONDING

*AB* is parallel to *CD*
*EF* is a straight line.

(a) (i) Find the value of *x*

(B1)

$x =$ **57°**

(ii) Give a reason for your answer. (A1)

CORRESPONDING ANGLES ARE EQUAL

[OR A SIMILAR SENTENCE THAT CONTAINS THE WORD
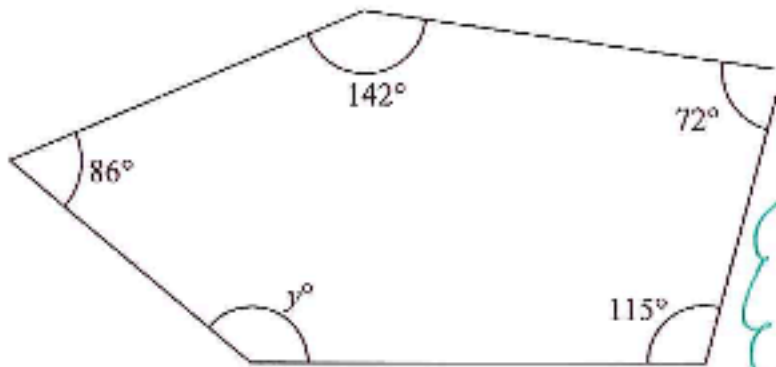'CORRESPONDING']

(2)

Here is a pentagon.

Diagram **NOT** accurately drawn

142°

72°

86°

INTERIOR ANGLES IN A PENTAGON ADD TO 540°

*y*°

115°

(b) Work out the value of *y*.

$540 - (86 + 142 + 72 + 115)$ (M1)

(B1)

$= 540 - 415$

(A1)

$y =$ **125°**

(3)

Show that $\frac{3}{4} + \frac{4}{5} = 1\frac{11}{20}$

ADD [COMMON DENOMINATORS]

$$\frac{3}{4} + \frac{4}{5} = \frac{15}{20} + \frac{16}{20}$$

(B1) [COMMON DENOMINATORS]

$$= \frac{31}{20}$$ (B1)

$$= 1\frac{11}{20} \quad \underline{QED!}$$

$\mathcal{E}$ = {whole numbers}
$A$ = {factors of 100}
$B$ = {multiples of 5} $\rightarrow$ 5, 10, 15, ECT

List the members of the set $A \cap B$

1, ⓪⓪
2, 50
4, ㉕
⑤, ㉜
⑩, ~~10~~

$\{5, 10, 20, 25, 50, 100\}$

Ⓑ② [-1 ee00]

Here are a rectangle and a square.

8 cm

Area = 48 cm²

$h = 6$

Diagram **NOT** accurately drawn

7

7
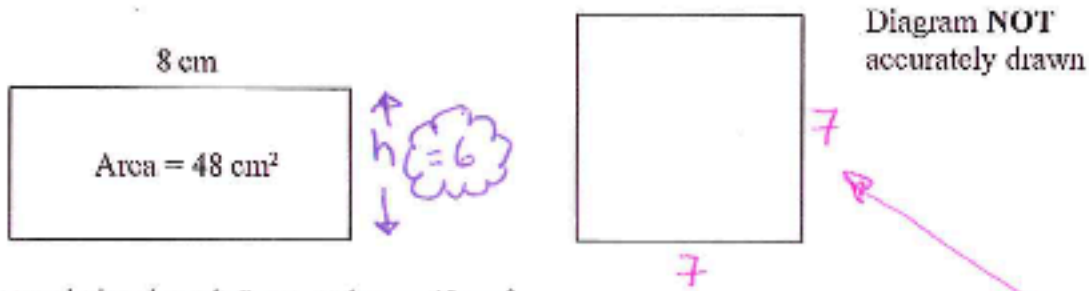
The rectangle has length 8 cm and area 48 cm²
The perimeter of the square is the same as the perimeter of the rectangle.

Calculate the area of the square.

RECTANGLE

$h = \dfrac{48}{8}$

$= 6$ (M1)

∴ PERIMETER IS

$8 + 8 + 6 + 6 = \underline{28}$ (M1)

∴ PERIMETER OF SQUARE

$= 28$

∴ SIDE $= \dfrac{28}{4}$

$= 7$ (M1)

AREA OF SQUARE IS

$7 \times 7 = \underline{49\,cm^2}$ (A1)

(a) Simplify $x^7 \times x^2$

$x^9$ (B1)

(1)

(b) Simplify $y^9 \div y^3$

$\dfrac{y^9}{y^3}$

$y^6$ (B1)

(1)

(c) Expand and simplify $4(2d + 3) - 2(3d - 5)$

*TWO MINUSES*

$8d + 12$      $-6d + 10$

(M1)

(A1)

$2d + 22$

(2)

(d) Solve $9y - 3 = 5y + 2$

$9y - 5y = 2 + 3$   (M1) [EITHER]

$4y = 5$

$y = \dfrac{5}{4}$

(A1)

$y = 1.25$

(2)

(e) Solve $\dfrac{7x - 1}{5} = x$

Show clear algebraic working.

$7x - 1 = 5x$   (M1)

$7x - 5x = 1$   (M1) [EITHER]

$2x = 1$

$x = \dfrac{1}{2}$

(A1)

$x = 0.5$

(3)

Mr Rowland has a class of 30 students.
He gave them 24 words to spell.
The table shows information about the number of correct spellings for each student.

| Number of correct spellings | Frequency | MID VALUE (x) | f×x |
|---|---|---|---|
| 0 – 4 | 1 | 2 | 2 |
| 5 – 9 | 5 | 7 | 35 |
| 10 – 14 | 6 | 12 (mi) | 72 |
| 15 – 19 | 10 | 17 | 170 |
| 20 – 24 | 8 | 22 | 176 |
| | | TOTAL | 455 |

(mi)

(a) Write down the modal class.

(B1)

15 – 19

(1)

(b) Work out an estimate for the mean number of correct spellings.
    Give your answer to 1 decimal place.

$$\text{MEAN} = \frac{\text{TOTAL NUMBER CORRECT}}{\text{NO. OF STUDENTS}}$$

$$= \frac{2 + 35 + 72 + 170 + 176}{30}$$ (mi)

$$= \frac{455}{30}$$

$$= 15.1666\ldots$$

(A1)

15·2

(4)

Ying eats some yoghurt.

The yoghurt contains 192 mg of calcium.
This is 16% of the total amount of calcium that Ying should have each day.

Work out the total amount of calcium that Ying should have each day.

$$16\% = 192$$

$$1\% = \frac{192}{16} \text{ M1}$$

$$100\% = \frac{192}{16} \times \frac{100}{\text{M1}}$$

1200 A1    mg

A plane flew from Bogotá to Quito.

The distance the plane flew was 725 km.
The time taken by the plane was 1 hour 24 minutes.

$$\frac{24}{60} = 0.4 \text{ Hours}$$

Work out the average speed of the plane.
Give your answer correct to 3 significant figures.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$= \frac{725}{1.4} \quad \text{(M1)}$$

(B1)

$$= 517.857\ldots$$

518 (A1) km/h

The table gives the surface areas, in square kilometres, of five seas.

| Sea | Surface area in square kilometres |
|---|---|
| Mediterranean Sea | $2.97 \times 10^6$ |
| East China Sea | $1.25 \times 10^6$ |
| Baltic Sea | $4.22 \times 10^5$ |
| Red Sea | $4.38 \times 10^5$ |
| Okhotsk Sea | $1.59 \times 10^6$ |

(a) Write $1.59 \times 10^6$ as an ordinary number.

MILLION!

$\underline{1\ 590\ 000}$    (Bi)

(1)

(b) Work out the difference, in square kilometres, between the largest surface area and the smallest surface area for these five seas.
Give your answer in standard form.

$$2.97 \times 10^6 \ - \ 4.22 \times 10^5 \left.\right\} \text{(m1) [EITHER]}$$

$$= \ 2548\ 000$$

(A1)

$\underline{2.548 \times 10^6}$ km² 

(2)

The surface area of the East China Sea is $k$ times the surface area of the Baltic Sea.

(c) Work out the value of $k$.
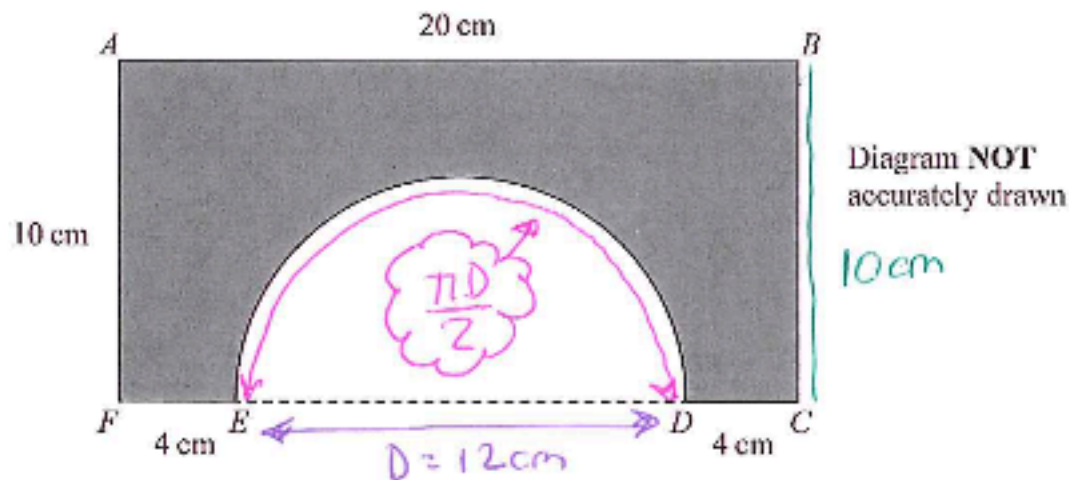Give your answer to the nearest whole number.

$$1.25 \times 10^6 = k \times (4.22 \times 10^5)$$

$$\Rightarrow \quad k = \frac{1.25 \times 10^6}{4.22 \times 10^5} \quad \text{(m1)}$$

$$= 2.962\ldots$$

$k = \underline{3}$   (A1)

(2)

20 cm

A                                                    B

Diagram **NOT**
accurately drawn

10 cm

$\frac{\pi D}{2}$

10 cm

F     4 cm     E                              D     4 cm     C

D = 12 cm

The shaded shape is made by cutting a semicircle from a rectangular piece of card, *ABCF*, as shown in the diagram.

*FEDC* is a straight line.
The centre of the semicircle lies on *ED*.
*AF* = *BC* = 10 cm,     *AB* = 20 cm,     *FE* = *DC* = 4 cm.

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

DIAMETER OF SEMI-CIRCLE IS

$20 - (4+4) = \underline{\underline{12 \text{ cm}}}$

LENGTH OF SEMI-CIRCLE ARC

$\frac{\pi \times 12}{2} = 18.849...$  (B1)

WHOLE PERIMETER

$10 + 20 + 10 + 4 + 4 + 18.849...$  (M1)

$= 66.849$

DON'T FORGET THIS ONE!

$\underline{66.8}$  (A1)          cm

Simplify fully $(2x + 3)^2 - (2x - 3)^2$

$(2x+3)(2x+3) - (2x-3)(2x-3)$

$4x^2 + 6x + 6x + 9 - \left[ 4x^2 - 6x - 6x + 9 \right]$ (m1) [EXPANSION]

$= 4x^2 + 6x + 6x + 9 - 4x^2 + 6x + 6x - 9$

(m1) [SIGNS CORRECT]

$= \underline{24x}$ (A1)

**METHOD 2**

$(2x+3)^2 - (2x-3)^2$ ← {DOTS!}

$= \left[(2x+3) + (2x-3)\right] \times \left[(2x+3) - (2x-3)\right]$ (m1)

$= \left[ 4x \right] \times \left[ 6 \right]$ (m1)

$= \underline{24x}$ (A1)

The diagram shows triangle *ABC*.

*SIMILAR TRIANGLES*

Diagram **NOT**
accurately drawn
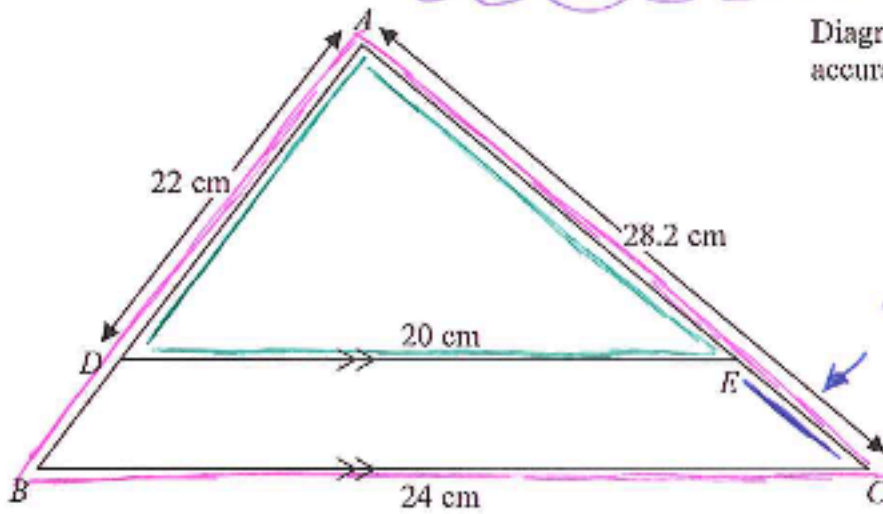


22 cm

28.2 cm

*PART (b)*

20 cm

24 cm

*ADB* and *AEC* are straight lines.
*DE* is parallel to *BC*.
*DE* = 20 cm,   *BC* = 24 cm,   *AD* = 22 cm,   *AC* = 28.2 cm

$$SCALE\ FACTOR = \frac{24}{20} = 1\cdot2$$

(a) Work out the length of *AB*.

$$AB = 22 \times 1\cdot2$$

B1

26·4   A1   cm

(2)

(b) Work out the length of *EC.*

$$AE = \frac{28\cdot2}{1\cdot2}$$

$$= 23\cdot5$$   M1

$$EC = 28\cdot2 - 23\cdot5$$

$$= 4\cdot7$$   A1

_____ cm

(2)

(a) Complete the table of values for $y = x^3 - 5x + 2$

$$y = (-3)^3 - 5(-3) + 2$$

*TYPE INTO CALCULATOR LIKE THIS*

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-10$ | $4$ | $6$ | $2$ | $-2$ | $0$ | $14$ |

(B2) [-1 eeo] (2)

(b) On the grid, draw the graph of $y = x^3 - 5x + 2$ for $-3 \leqslant x \leqslant 3$

(B1)[POINTS]  (B1)[CURVE]



-2·8

LINE FOR PART (d)
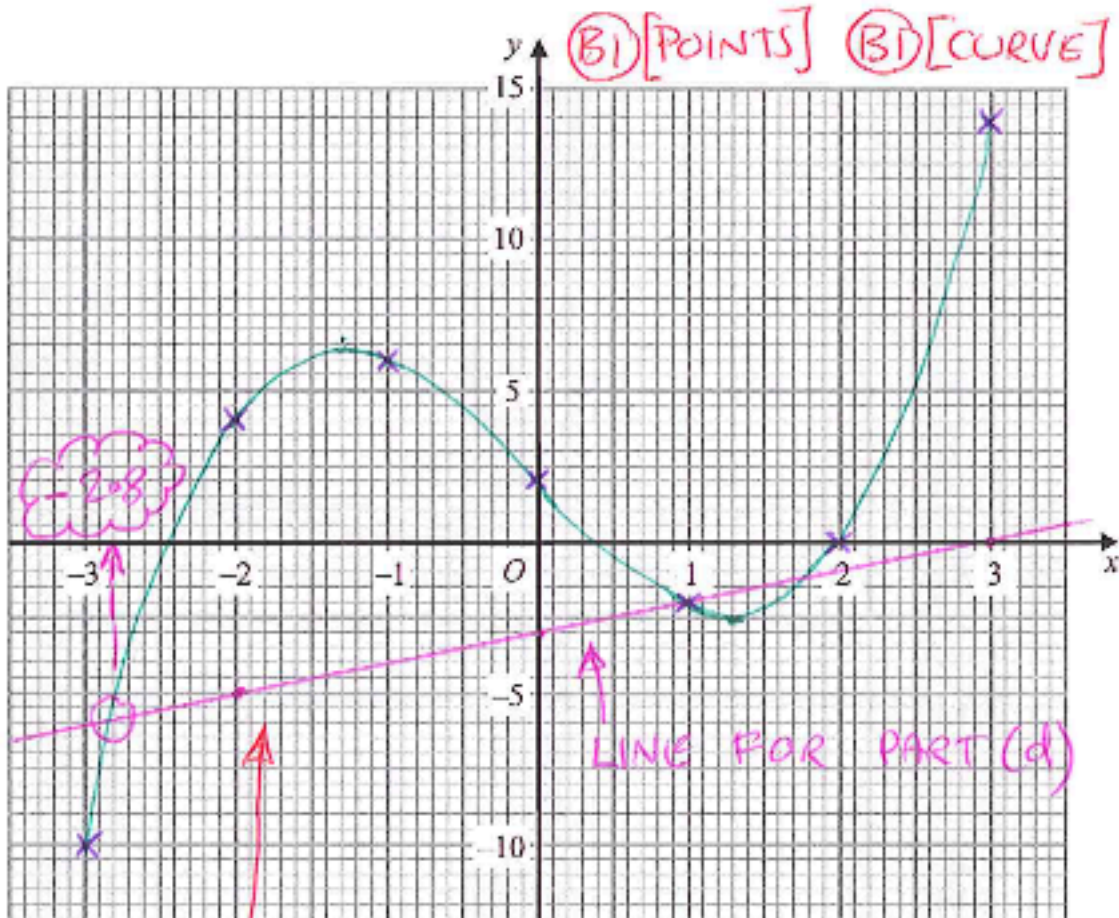
(B1) [STRAIGHT LINE - PART (d)]

(2)

The equation $x^3 - 6x + m = 0$, where $m$ is an integer, has one negative solution and two positive solutions.

(c) Given that $x = 1$ is one of the positive solutions, show that $m = 5$

$$x^3 - 6x + m = 0$$

$$(1)^3 - 6(1) + m = 0$$

$$\Rightarrow -5 + m = 0$$

(B1) [EITHER]

$$m = 5$$

(1)

(d) By drawing a suitable straight line on the grid, find an estimate for the negative solution of $x^3 - 6x + 5 = 0$

Give your estimate to 1 decimal place.

$$x^3 - 6x + 5 = 0$$

$$x^3 - 5x + 2 = x - 3$$

THIS IS THE GRAPH

DRAW THE LINE $y = x - 3$

$-2.8$ (A1)

(2)

Use algebra to show that the recurring decimal $0.2\dot{6} = \frac{4}{15}$

(B1)

$$x = 0.2666\ldots$$
$$10x = 2.6666\ldots$$ } SUBTRACT

$$9x = 2.4$$
$$x = \frac{2.4}{9}$$ (B1)
$$= \frac{24}{90} \longrightarrow = \frac{4}{15}$$

$Q$ is inversely proportional to $t^2$
$Q = 320$ when $t = 0.5$

Find a formula for $Q$ in terms of $t$

$$Q = \frac{k}{t^2} \quad \text{(M1)} \quad (Q = 320, \ t = 0.5)$$

$$320 = \frac{k}{0.5^2}$$

$$\Rightarrow k = 320 \times 0.5^2 \qquad \text{M1 [EITHER]}$$

$$= 80 \longrightarrow \qquad Q = \frac{80}{t^2} \quad \text{(A1)}$$

Chaiwat either cycles to work or goes by bus.

On any day that he goes to work, the probability that he cycles is 0.6
When he cycles, the probability that he is late is 0.1
When he goes by bus, the probability that he is late is 0.3

(a) Complete the probability tree diagram.

late

0.1

cycle

0.6

0.9    not late

0.3    late

0.4
(B1)

bus

(B1)
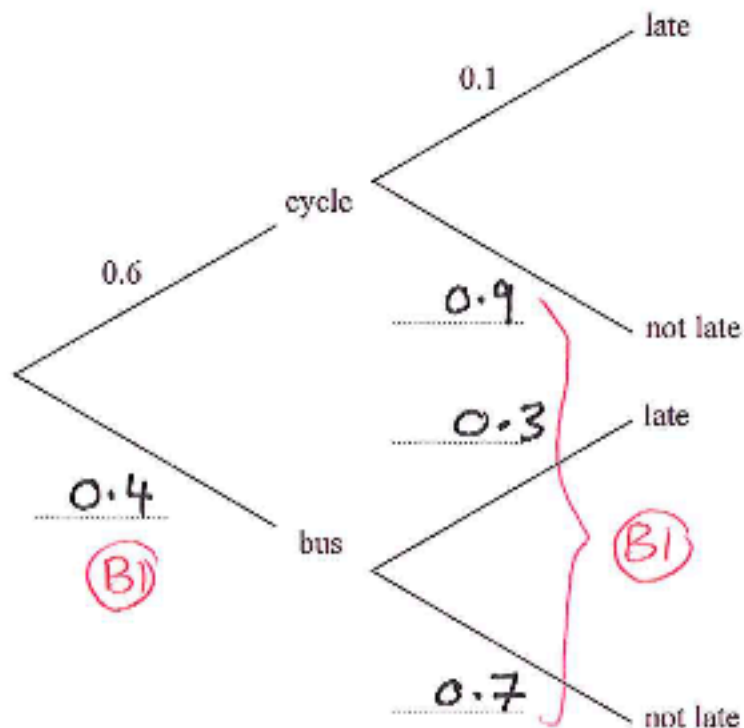
0.7    not late

(2)

(b) Calculate the probability that on a day Chaiwat goes to work, he cycles and is late for work.

$$P(C, L) = 0.6 \times 0.1 \quad \text{(m1)}$$

(A1)

0.06

(2)

(c) Calculate the probability that on a day Chaiwat goes to work, he is not late for work.

$$P(C, \bar{L}) = 0.6 \times 0.9 = 0.54 \quad \text{(m1)}$$
$$P(B, \bar{L}) = 0.4 \times 0.7 = 0.28 \quad \text{(m1)}$$

(A1)

TOTAL = 0.82

The incomplete histogram and the incomplete table show information about the ages of people watching a film in a cinema.



Age in years
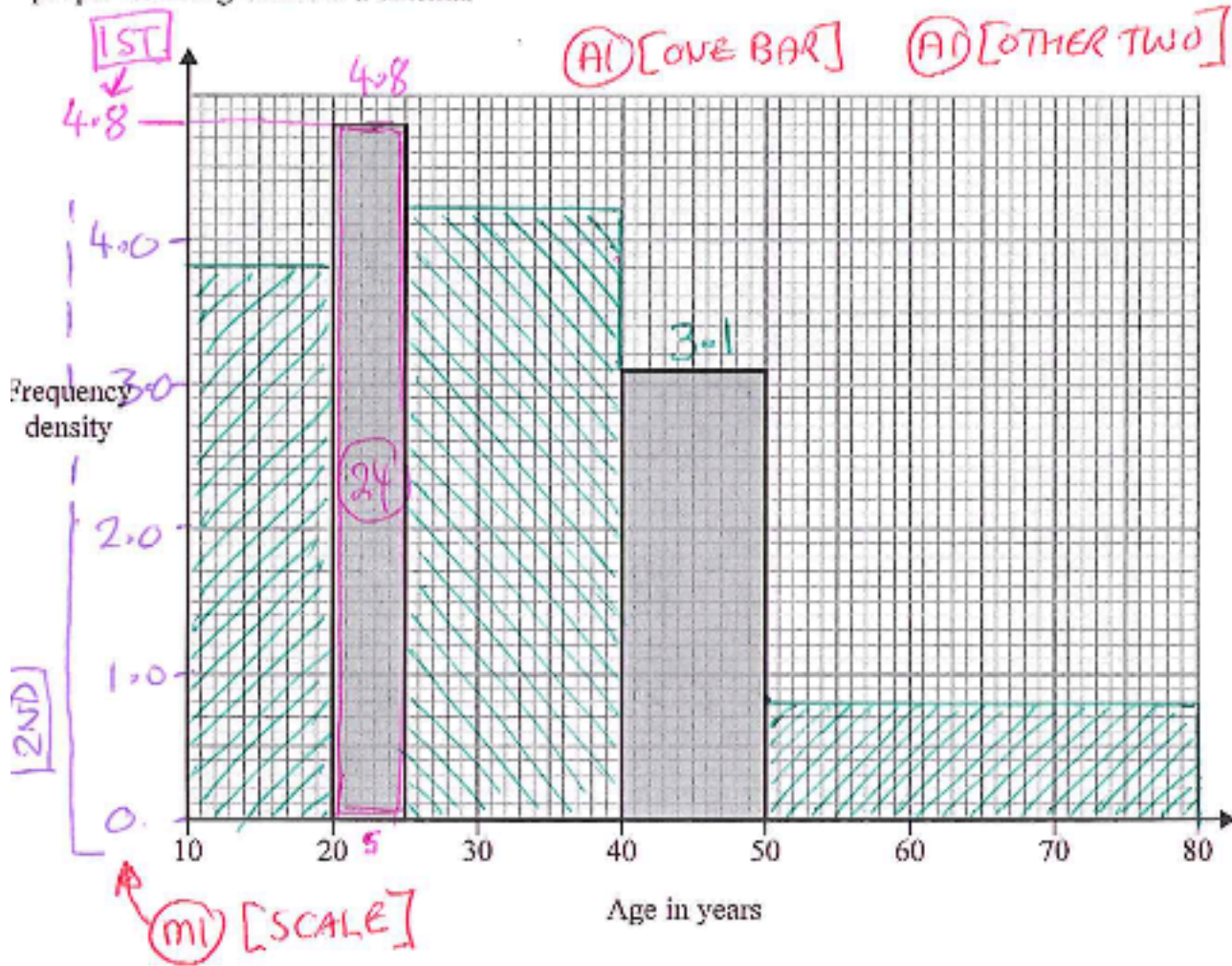
| WIDTH | Age ($a$ years) | Number of people | HEIGHT |
|---|---|---|---|
| 10 | $10 \leqslant a < 20$ | 38 | 3·8 |
| 5 | $20 \leqslant a < 25$ | 24 | 4·8 |
| 15 | $25 \leqslant a < 40$ | 63 | 4·2 |
| 10 | $40 \leqslant a < 50$ | 31 (B1) | 3·1 |
| 30 | $50 \leqslant a < 80$ | 24 | 0·8 |

(a) Use the histogram to complete the table.

(2)

(b) Use the table to complete the histogram.

(2)

Make g the subject of $3e + 4g = 7 + 9eg$

$$4g - 9eg = 7 - 3e \quad \text{(M1)} [\text{'g's ON LHS}]$$

$$g(4 - 9e) = 7 - 3e \quad \text{(m)} [\text{FACTORISING}]$$

$$g = \frac{7 - 3e}{4 - 9e} \quad \text{(A1)}$$

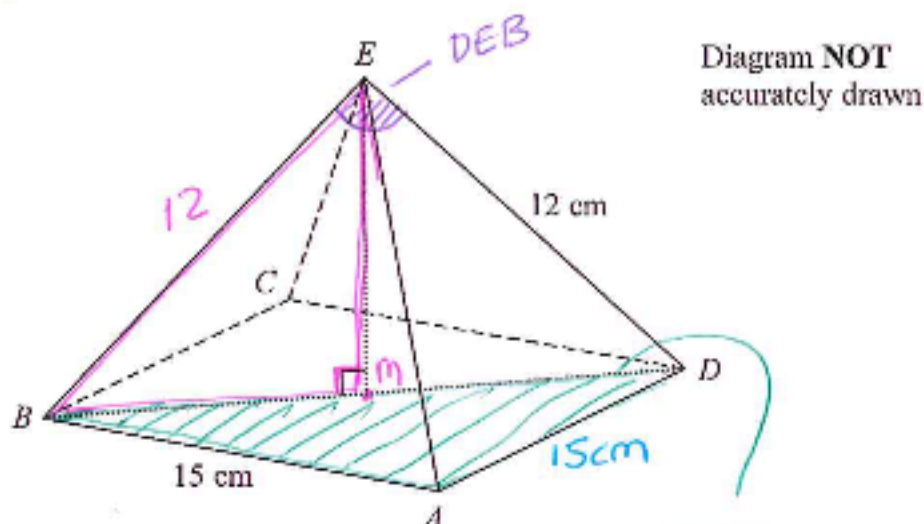Express $\dfrac{3}{x+2} - \dfrac{6}{2x+5}$ as a single fraction.

Simplify your answer.

$$\frac{3}{x+2} - \frac{6}{2x+5} = \frac{6x+15 - 6x - 12}{(x+2)(2x+5)} \quad \text{M1} \quad \text{B1}$$

$$= \frac{3}{(x+2)(2x+5)} \quad \text{A1}$$

ABCDE is a square-based pyramid.



Diagram **NOT** accurately drawn

$AE = BE = CE = DE = 12$ cm
$AB = 15$ cm

Calculate the size of angle *DEB*.
Give your answer to the nearest degree.

$$BD = \sqrt{15^2 + 15^2}$$
$$= 21 \cdot 213\ldots \quad \text{M1}$$

IST
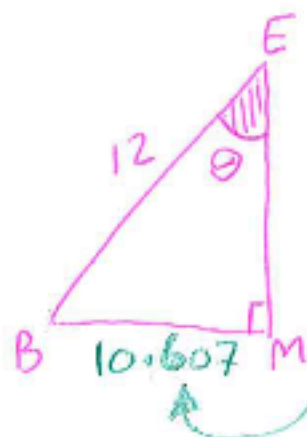
$\div 2$

$$\sin \theta = \frac{10 \cdot 607}{12} \quad \text{M1}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{10 \cdot 607}{12}\right)$$

$$= 62 \cdot 113\ldots \quad \text{M1}$$

$$\therefore DEB = 2 \times 62 \cdot 113\ldots$$

$$= 124 \cdot 226\ldots$$

$$= 124° \quad \text{A1}$$

$(a + \sqrt{b})^2 = 49 + 12\sqrt{b}$     where $a$ and $b$ are integers, and $b$ is prime.

Find the value of $a$ and the value of $b$

$$(a + \sqrt{b})(a + \sqrt{b}) = a^2 + a\sqrt{b} + a\sqrt{b} + \sqrt{b}\sqrt{b}$$

$$= a^2 + 2a\sqrt{b} + b \quad \text{(M1)}$$

$$= (a^2 + b) + 2a\sqrt{b}$$

$$2a\sqrt{b} = 12\sqrt{b} \qquad a^2 + b = 49$$

$$\Rightarrow 2a = 12 \qquad\qquad b = 49 - a^2$$

$$\Rightarrow a = \underline{6} \quad \text{(A1)} \qquad = 49 - 36$$

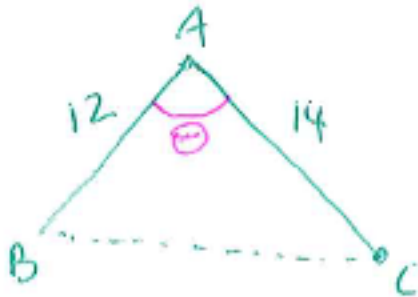$$\qquad\qquad\qquad\qquad\qquad = \underline{\underline{13}} \quad \text{(A1)}$$

*ABC* is a triangle.

*AB* = 12 cm
*AC* = 14 cm
The area of triangle *ABC* is 72 cm²

Find, in degrees, the two possible sizes of angle *BAC*.
Give your answers correct to the nearest degree.

USE $A = \frac{1}{2}ab\sin C$

$\frac{1}{2} \times 12 \times 14 \sin\theta = 72$ (M1)

$\Rightarrow \sin\theta = \frac{72 \times 2}{12 \times 14}$

$\Rightarrow \sin\theta = \frac{6}{7}$ (M1)

$\therefore \theta = \sin^{-1}\left(\frac{6}{7}\right)$

$= 58.997\ldots$

$= 59°$ (A1)

BUT $\theta$ COULD ALSO BE OBTUSE $\Rightarrow 180 - 59 = 121°$ (A1)

Solve the simultaneous equations

$$y = 3x + 2$$
$$x^2 + y^2 = 20$$

Show clear algebraic working.

$x^2 + (3x+2)^2 = 20$   (m1) [SUBSTITUTION]

$x^2 + (3x+2)(3x+2) = 20$

$x^2 + 9x^2 + 6x + 6x + 4 = 20$ } (m1) [EITHER]

$10x^2 + 12x + 4 = 20$

$10x^2 + 12x - 16 = 0$ } (m1) [EITHER]
$5x^2 + 6x - 8 = 0$

$(5x - 4)(x + 2) = 0$   (m1) [FACTORISING]

[OR USE OF FORMULA]

$x = \frac{4}{5}$      $x = -2$   (A1) [BOTH]

FIND y

$y = 3 \times \left(\frac{4}{5}\right) + 2$      $y = 3(-2) + 2$

$= 4.4$      $= -4$   (A1) [BOTH]