

4H(R)

Pearson Edexcel
International GCSE

EDEXCEL

IGCSE

MATHEMATICS A

SOLUTIONS

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4MA0/4HR

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or ‘best’ method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

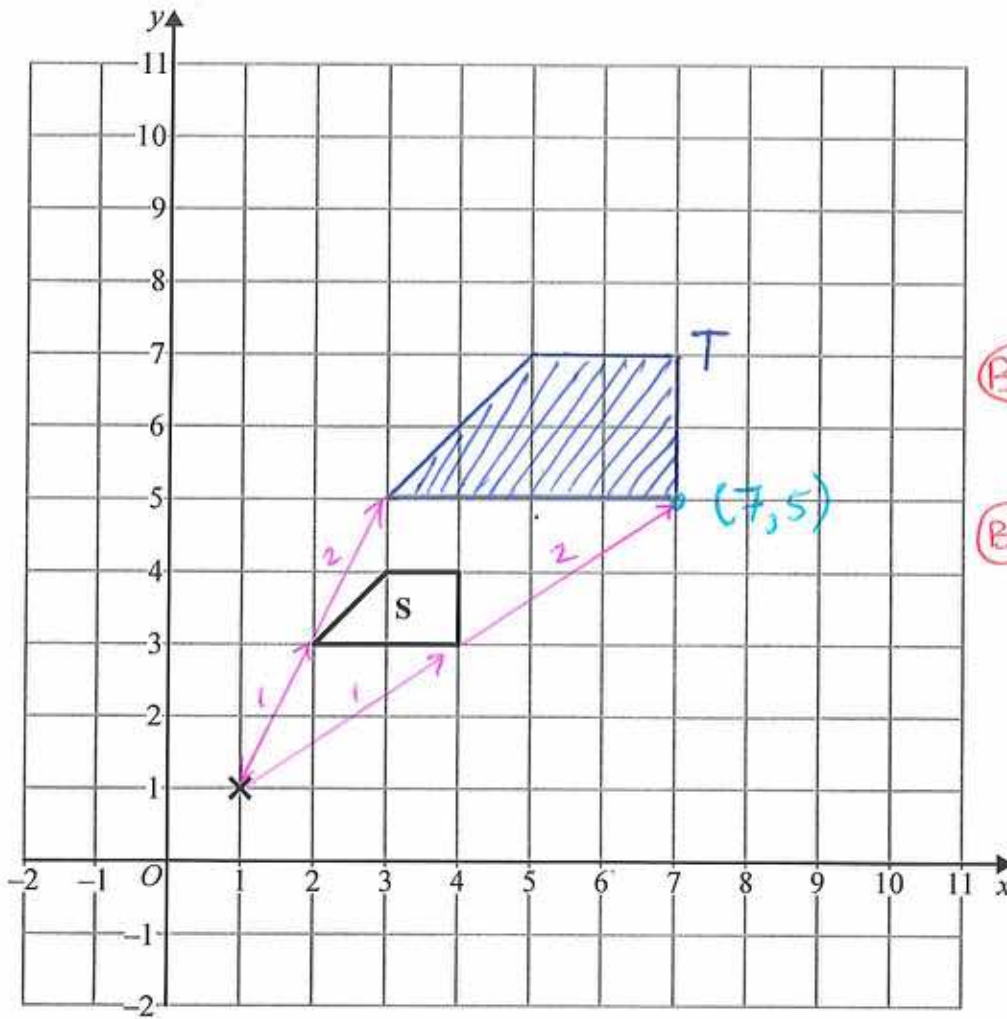
Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 - This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of ‘follow through’).

M1 - This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board’s web site

A1 - These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.



(B1) [CORRECT ENLARGEMENT]

(B1) [CORRECT POSITION]

- (a) Enlarge shape S, by scale factor 2, centre (1,1).
Label the new shape T.

(2)

- (b) Describe fully the single transformation that maps shape T onto shape S.

ENLARGEMENT, SCALE FACTOR $\frac{1}{2}$

(1)

CENTRE (1,1)

(B1) [ALL THREE STATEMENTS]

(a) Solve $6t - 5 = 2t + 9$

Show clear algebraic working.

$$6t - 2t = 9 + 5 \quad \text{(M1)} \quad [t \text{ ON SAME SIDE}]$$

$$4t = 14 \quad \text{(M1)}$$

$$t = \frac{14}{4}$$

$$t = \frac{3.5}{1} \quad \text{(A1)}$$

(b) Expand and simplify $3(2y + 2) + 2(y - 4)$

$$6y + 6$$

$$2y - 8 \quad \text{(M1)}$$

$$8y - 2 \quad \text{(A1)}$$

(c) Simplify fully $4wxy \div (8xy)$

$$\frac{4wxy}{8xy}$$

$$\frac{w}{2} \quad \text{(A1)}$$

There were 2.1 million people living in Dubai in 2013
1.75 million of these people were not born in Dubai.

- (a) Work out 1.75 as a percentage of 2.1
Give your answer correct to 1 decimal place.

$$\frac{1.75}{2.1} \times 100 = 83.333\dots$$

$$\underline{\hspace{1cm}} 83.3 \text{ \%}$$

(2)

The unit of currency in Dubai is the dirham.
The exchange rate is £1 = 5.52 dirham.

The cost of a pair of running shoes in Dubai is 343 dirham.
The cost of an identical pair of running shoes in the UK is £54.99

$$54.99 \times 5.52$$

The pair of running shoes is more expensive in Dubai than in the UK.

- (b) How much more expensive?
Give your answer to the nearest dirham.

UK: $54.99 \times 5.52 = 303.5448$ dirham

$$\text{DIFFERENCE} = 343 - 303.5\dots$$

$$= 39.4552$$

$$\underline{\hspace{1cm}} 39 \text{ dirham}$$

(3)

A plane flies a distance of 5522 km from London to Abu Dhabi in 7 hours 24 minutes.

- (c) Work out the average speed of the plane.
Give your answer in kilometres per hour, correct to 3 significant figures.

$$7.4 \text{ hours}$$

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$= \frac{5522}{7.4}$$

(M1) [DIVIDE]

$$\underline{\hspace{1cm}} 746 \text{ kilometres per hour}$$

(3)

$$= 746.216\dots$$

Here is a kite $ABCD$.

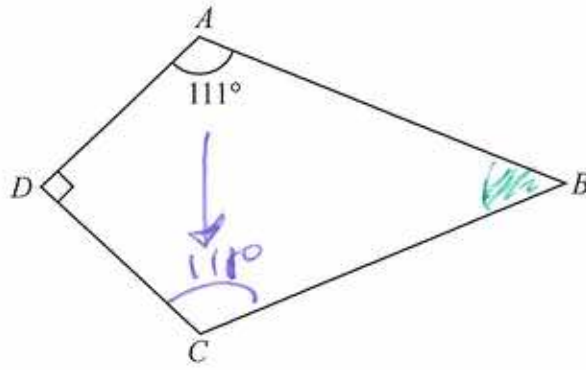



Diagram **NOT** accurately drawn

Angle $DAB = 111^\circ$

Angle $ADC = 90^\circ$

(a) Work out the size of angle ABC . 

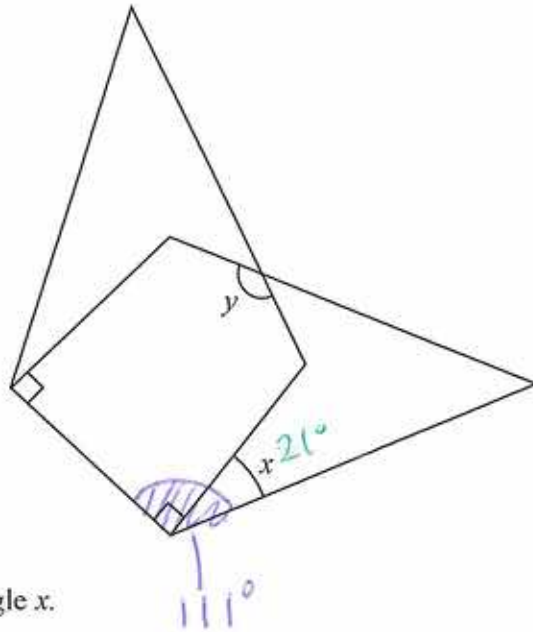
$$360 - (90 + 111 + 111) \text{ (ml)}$$

$$\underline{48^\circ} \text{ (AI)}$$

(2)

Two of these kites are arranged so that a shorter side of one of the kites is placed on top of a shorter side of the other kite, as shown in the diagram below.

Diagram **NOT** accurately drawn

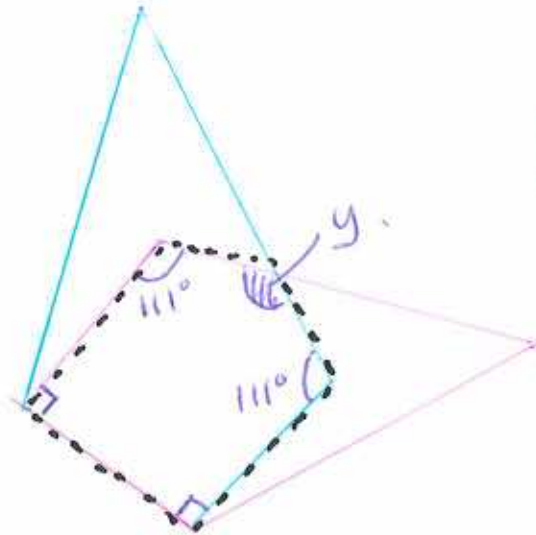


(b) Work out the size of angle x .

$$111 - 90 \quad \text{(mi)}$$

$$\frac{21 \quad \text{(AI)}}{(2)}$$

(c) Work out the size of angle y .



[MANY OTHER METHODS ARE POSSIBLE]

$$y = 540 - (2 \times 111 + 2 \times 90) \quad \frac{138 \quad \text{(AI)}}{(3)}$$

(a) Complete the table of values for $y = x^2 - 4x + 2$

$$\rightarrow y = (-1)^2 - 4(-1) + 2$$

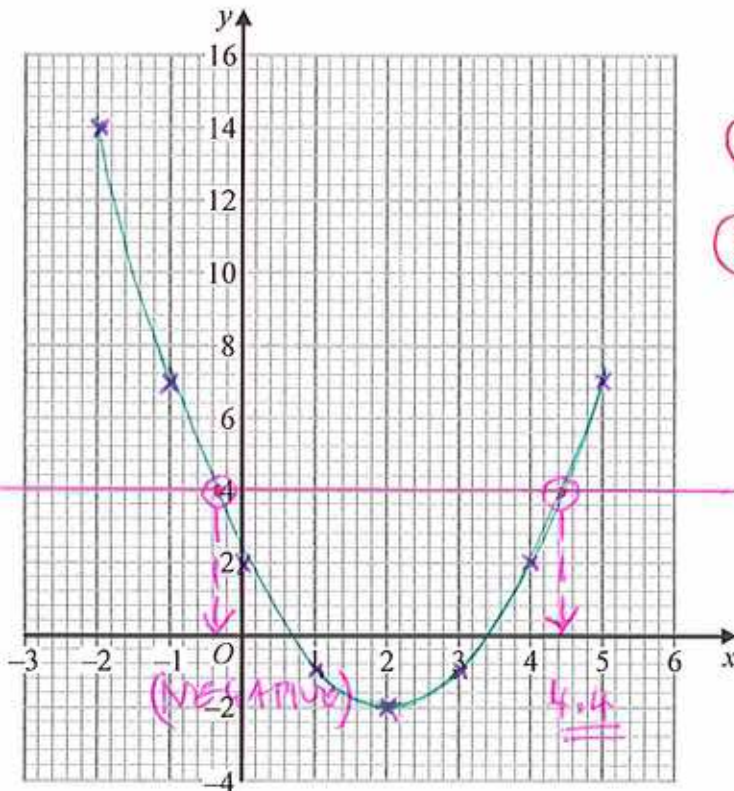
x	-2	-1	0	1	2	3	4	5
y	14	7	2	-1	-2	-1	2	7

(A1)

(2)

(A1)

(b) On the grid, draw the graph of $y = x^2 - 4x + 2$ for values of x from -2 to 5



(B1) [POINTS]

(B1) [CURVE]

y = 4!

(2)

The point $P(k, 4)$ where $k > 0$ lies on the graph of $y = x^2 - 4x + 2$

(c) Use your graph to find an estimate for the value of k .

4.4

(A1)

(1)

Here is a list of numbers written in order of size.

3 6 x y

The numbers

have a median of 8

have a mean of 11 \rightarrow TOTAL = 11×4

Find the value of x and the value of y .

$$= \underline{\underline{44}} \quad \text{BI}$$

3 6 10

1ST

MEDIAN = 8

2ND

$$y = 44 - (3 + 6 + 10)$$

$$= \underline{\underline{25}}$$

$$x = \dots 10 \quad \text{AD}$$

$$y = \dots 25 \quad \text{AD}$$

Here are two circles.

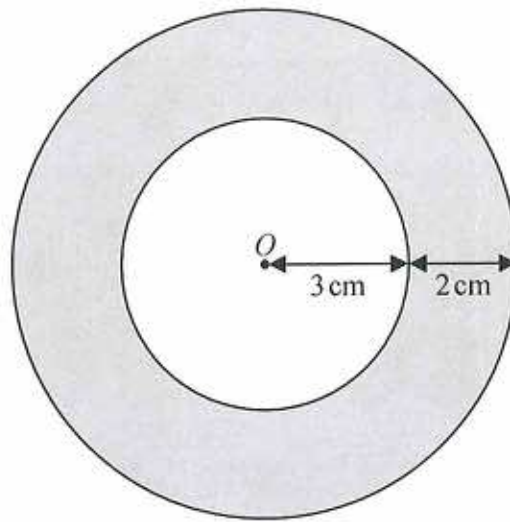


Diagram NOT
accurately drawn

The circles have the same centre O .

The radius of the inner circle is 3 cm .

The width of the shaded region between the inner circle and outer circle is 2 cm .

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

USE $A = \pi r^2$

LARGE CIRCLE

$$A = \pi \times 5^2$$

$$= 25\pi \quad (\text{M1})$$

SMALL CIRCLE

$$A = \pi \times 3^2$$

$$= 9\pi$$

SHADED PART

$$= 25\pi - 9\pi \quad (\text{M1})$$

$$= 16\pi$$

$$= 50.265\dots$$

$$\underline{50.3} \quad (\text{A1}) \text{ cm}^2$$

Louis makes a model of a plane.

The wingspan of the model is 50 centimetres.

The wingspan of the real plane is 80 metres.

- (a) Work out the scale of the model.
Give your answer in the form 1 : n

MODEL: REAL

$$50\text{cm} : 80\text{m}$$

$$50\text{cm} : 8000\text{cm} \quad (\text{M1})$$

$$50 : 8000$$

$$\rightarrow 1 : 160 \quad (\text{A1})$$

(2)

The length of the real plane is 72 metres.

- (b) Work out the length of the model.
Give your answer in centimetres.

MODEL: REAL

$$1 : 160$$

$$\text{MODEL} = \frac{1}{160} \times 72 \quad (\text{M1})$$

$$= 0.45\text{m}$$

$$\rightarrow 45 \quad (\text{A1})$$

centimetres
(2)

There are 30 apples in a box.

The mean weight of these 30 apples is 120 grams. \rightarrow TOTAL WEIGHT = 3600 g

There are 10 apples in a bag.

The mean weight of these 10 apples is 95 grams. \rightarrow TOTAL WEIGHT = 950 g

Work out the mean weight of the 40 apples.

$$\text{MEAN} = \frac{3600 + 950}{40}$$

(mi)

(mi)
[EITHER]

\rightarrow 113.75 (A1) grams

Solve $4x + 3y = 6$ \longrightarrow ① $\times 5$

$3x + 5y = -1$ \longrightarrow ② $\times 3$

Show clear algebraic working.

$$\begin{array}{r}
 20x + 15y = 30 \quad \xrightarrow{\text{mi}} \text{③} \\
 9x + 15y = -3 \quad \xrightarrow{\text{mi}} \text{④} \\
 \hline
 \end{array}
 \left. \begin{array}{l} \text{③} \\ \text{④} \end{array} \right\} \text{SUBTRACT}$$

$$11x = 33$$

$$x = \underline{\underline{3}} \quad \text{①}$$

SUBSTITUTE $x = 3$ INTO ①:

$$4(3) + 3y = 6 \quad \text{mi}$$

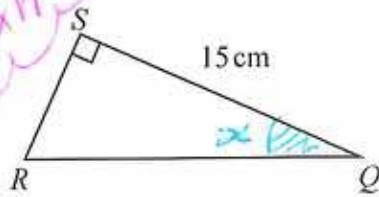
$$y = \frac{6 - 4(3)}{3}$$

$$= \underline{\underline{-2}} \quad \text{②}$$

$$x = \underline{\quad 3 \quad}$$

$$y = \underline{\quad -2 \quad}$$

Here is a triangle QRS .



$$SQ = 15 \text{ cm}$$

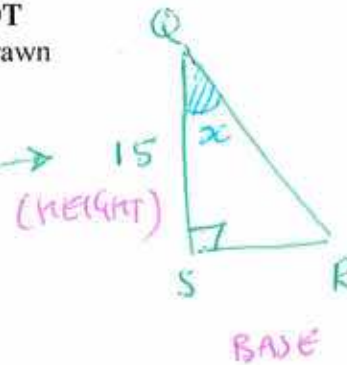
$$\text{Angle } RSQ = 90^\circ$$

$$\text{Area of triangle } QRS = 60 \text{ cm}^2$$

Work out the size of angle SQR .

Give your answer correct to 1 decimal place.

Diagram NOT
accurately drawn



1ST FIND SR

$$\frac{1}{2} b \times h = 60$$

$$\frac{1}{2} SR \times 15 = 60$$

$$SR \times 15 = 120$$

$$SR = \frac{120}{15}$$

$$= \underline{\underline{8}} \text{ (m)}$$

2ND

$$\text{TAN } \alpha = \frac{\text{OPP}}{\text{ADJ}}$$

$$\text{TAN } \alpha = \frac{8}{15} \text{ (m)}$$

$$\Rightarrow \alpha = \text{TAN}^{-1} \left(\frac{8}{15} \right) \text{ (m)}$$

$$= 28.0724\dots$$

$$= \underline{\underline{28.1}}^\circ \text{ (A)}$$

The table gives some information about the incomes, $\pounds I$, of 100 people in the UK.

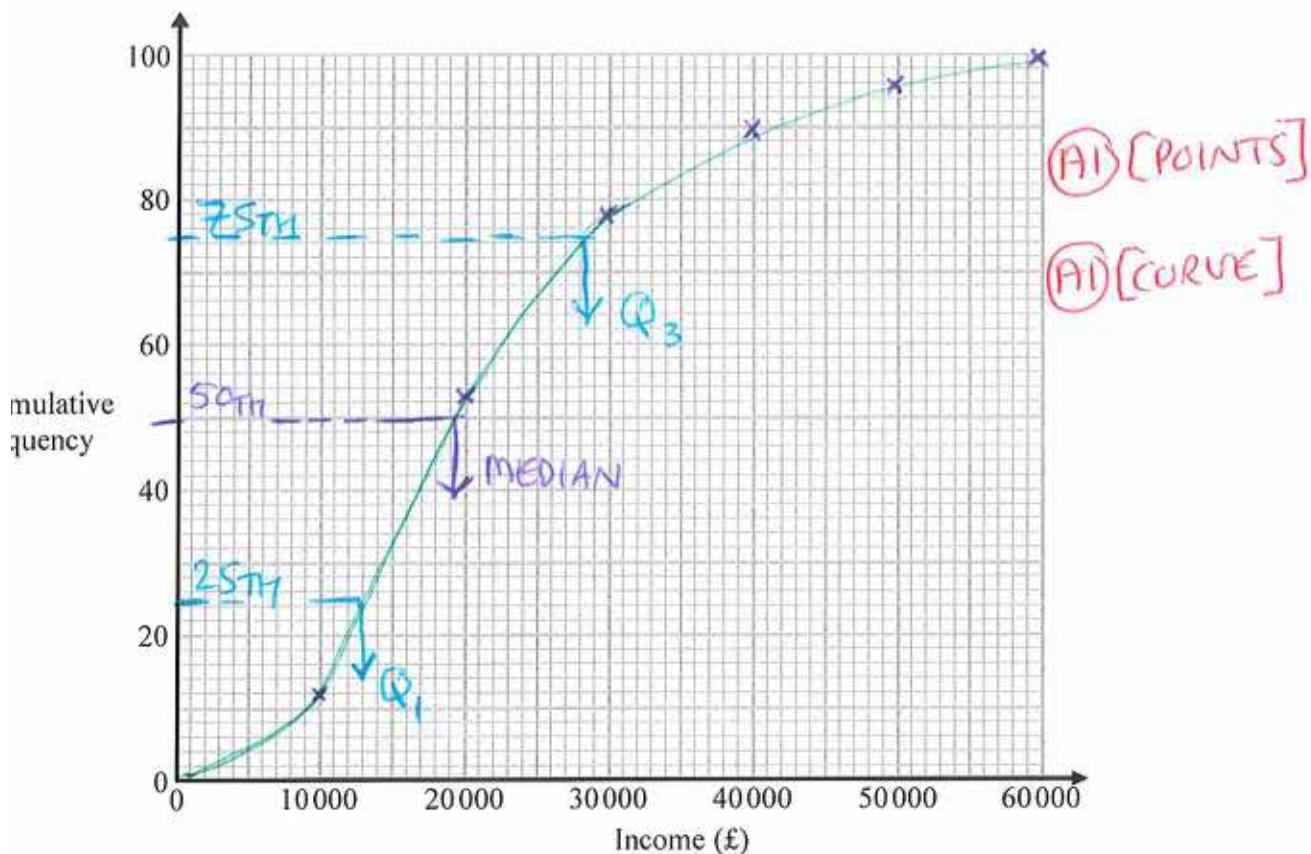
Income ($\pounds I$)	Frequency
$0 < I \leq 10000$	12
$10000 < I \leq 20000$	41
$20000 < I \leq 30000$	25
$30000 < I \leq 40000$	12
$40000 < I \leq 50000$	6
$50000 < I \leq 60000$	4

(a) Complete the cumulative frequency table.

Income ($\pounds I$)	Cumulative frequency
$0 < I \leq 10000$	12
$0 < I \leq 20000$	53
$0 < I \leq 30000$	78
$0 < I \leq 40000$	90
$0 < I \leq 50000$	96
$0 < I \leq 60000$	100

(10000, 12)
 (20000, 53)
 (30000, 78)
 ⋮

(A1) (1)



(2)

(c) Use your graph to find an estimate for

(i) the median,

$$\frac{100}{2} = \boxed{50\text{th}} \text{ VALUE}$$

£ 19 000 (AI)

(ii) the interquartile range.

$$Q_1 = \frac{100}{4} = \boxed{25\text{th}} \text{ VALUE}$$

[18 000 - 20 000]

$$Q_3 = 3 \times \frac{100}{4} = \boxed{75\text{th}} \text{ VALUE}$$

£ 15 000 (AI)
(3)

$$IQR = Q_3 - Q_1$$

$$= 28 000 - 13 000$$

[14 000 - 16 000]

(mi)

(mi)

(a) Write 250 000 in standard form.

$$\underline{2.5 \times 10^5} \quad \text{(B1)}$$

(1)

The radius of the planet Jupiter is 6.99×10^7 metres.

The radius of the Earth is 6.37×10^6 metres.

The volume of Jupiter is k times the volume of the Earth.

(b) Assuming that both planets are spheres, calculate the value of k .
Give your answer correct to 3 significant figures.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{V_J}{V_E} = k \quad \Rightarrow \quad k = \frac{\frac{4}{3}\pi r_J^3}{\frac{4}{3}\pi r_E^3} \quad \text{(M1)}$$

[DIVISION ATTEMPTED]

$$= \frac{r_J^3}{r_E^3}$$

$$= \frac{(6.99 \times 10^7)^3}{(6.37 \times 10^6)^3} \quad \text{(M1)}$$

$$= 1321.335\dots$$

$$= \underline{\underline{1320}} \quad \text{(A1)}$$

(a) Solve $2y + \frac{2-3y}{4} = \frac{1}{4}$

[MULTIPLY EACH TERM BY 4]

Show clear algebraic working.

$$8y + 2 - 3y = 1 \quad \text{(M1)}$$

$$\Rightarrow \left. \begin{array}{l} 5y + 2 = 1 \\ 5y = -1 \end{array} \right\} \text{(M1) [EITHER]}$$

$$y = \frac{-0.2}{(3)} \quad \text{(A1)}$$

(b) Factorise $3x^2 - 8x - 3$

$$(3x+1)(x-3) \quad \text{(B2)}$$

(c) Expand and simplify $4x(x+3) - (2x-3)^2$

$$4x^2 + 12x - [(2x-3)(2x-3)]$$

$$= 4x^2 + 12x - [4x^2 - 12x + 9] \quad \text{(M1)}$$

$$= 4x^2 + 12x - 4x^2 + 12x - 9$$

(M1)
[SIGNS CORRECT]

$$\underline{24x - 9} \quad \text{(A1)}$$

Naveed has two bags of tiles, bag A and bag B.

There are 10 tiles in bag A.
7 of these tiles are red.
The other 3 tiles are white.

There are 8 tiles in bag B.
5 of these tiles are red.
The other 3 tiles are white.

Naveed takes at random one tile from each bag.

(a) Work out the probability that the tiles are the same colour.

$$\begin{aligned}
 P(R,R) &= \frac{7}{10} \times \frac{5}{8} = \frac{35}{80} \quad \text{(BI)} \\
 P(W,W) &= \frac{3}{10} \times \frac{3}{8} = \frac{9}{80} \quad \text{(BI)}
 \end{aligned}
 \left. \vphantom{\begin{aligned} P(R,R) \\ P(W,W) \end{aligned}} \right\} \text{TOTAL} = \frac{44}{80} \quad \text{(AI)}$$

$$\frac{11}{20}$$

(3)

All 18 tiles are put in a box.

Naveed takes at random one tile from the box.

He does not replace the tile.

Naveed then takes at random a second tile from the box.

PROBABILITIES CHANGE!

(b) Work out the probability that both tiles are red.

$$\begin{aligned}
 P(R,R) &= \frac{12}{18} \times \frac{11}{17} \quad \text{(MI)} \\
 &= \frac{22}{51} \quad \text{(AI)} \quad \text{[ACCEPT EQUIVALENTS!]}
 \end{aligned}$$

Solve $2x^2 - 6x + 3 = 0$

Give your solutions correct to 3 significant figures.
Show your working clearly.

USE $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2, b = -6, c = 3$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)}$ (M1) [SUBSTITUTION]

$= \frac{6 \pm \sqrt{36 - 24}}{4}$ (M2) [SIMPLIFICATION]

2.37 0.634 (A1) [BOTH]

The diagram shows a prism.

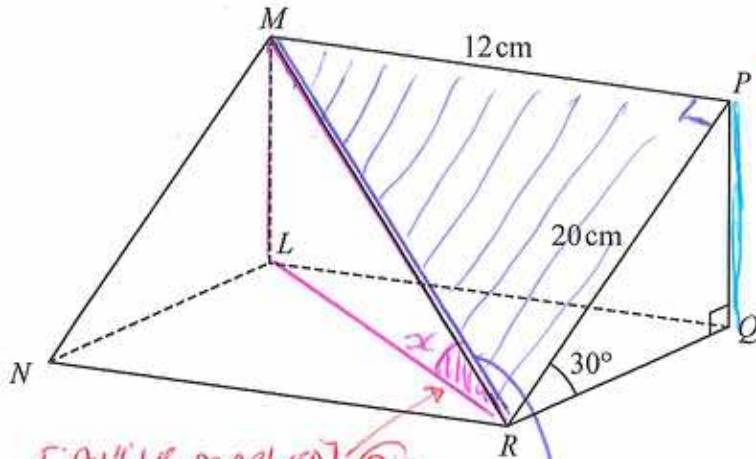


Diagram NOT accurately drawn

2ND
 $ML = PQ = 20 \sin 30$
 $= 10 \text{ cm}$ (BI)

[ANGLE MARKED] (BI)

Triangle PQR is a cross section of the prism.

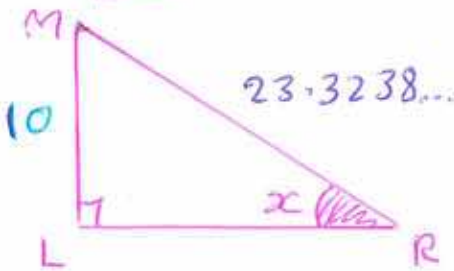
- PR = 20 cm
- MP = 12 cm
- Angle PRQ = 30°
- Angle PQR = 90°

1ST

$MR = \sqrt{20^2 + 12^2}$
 $= 23.3238...$ (BI)

Calculate the size of the angle that the line MR makes with the plane RQLN.
 Give your answer correct to 1 decimal place.

3RD



$\sin \alpha = \frac{\text{OPP}}{\text{HYP}}$

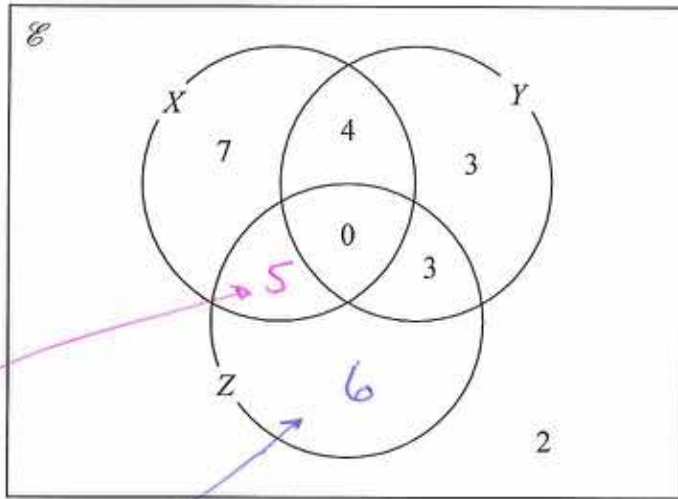
$\sin \alpha = \frac{10}{23.3238}$ (MI)

$\Rightarrow \alpha = \sin^{-1} \left(\frac{10}{23.3238} \right)$

$= 25.388...$

→ 25.4 (AI)

The Venn diagram shows a universal set \mathcal{E} and three sets X , Y and Z .



(B2)

The numbers shown represent **numbers** of elements.

$n(X') = 14$
 $n(Z) = 14$

1ST

(a) Complete the Venn diagram.

(2)

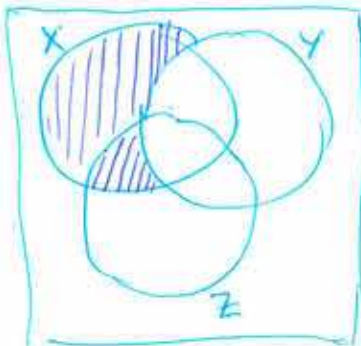
(b) Find the value of

(i) $n(X \cup Z)$

$7 + 4 + 5 + 0 + 3 + 6$

25 (A1)

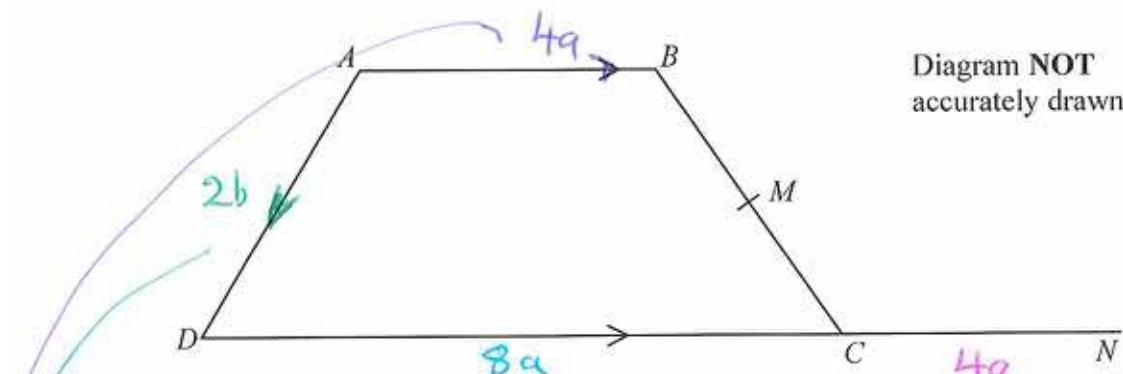
(ii) $n(X \cap Y)$



$7 + 5$

12 (A1)

(2)



AB is parallel to DC
 DC = 2AB
 M is the midpoint of BC

$\vec{AD} = 2b$
 $\vec{AB} = 4a$

$\vec{BM} = \frac{1}{2} \vec{BC}$

(a) Find \vec{BM} in terms of a and b .
 Give your answer in its simplest form.

$\vec{BC} = -4a + 2b + 8a$ (m) [FOR \vec{BC}]
 $= 4a + 2b$

$\Rightarrow \vec{BM} = \frac{1}{2} (4a + 2b)$

$2a + b$ (A)
 (2)

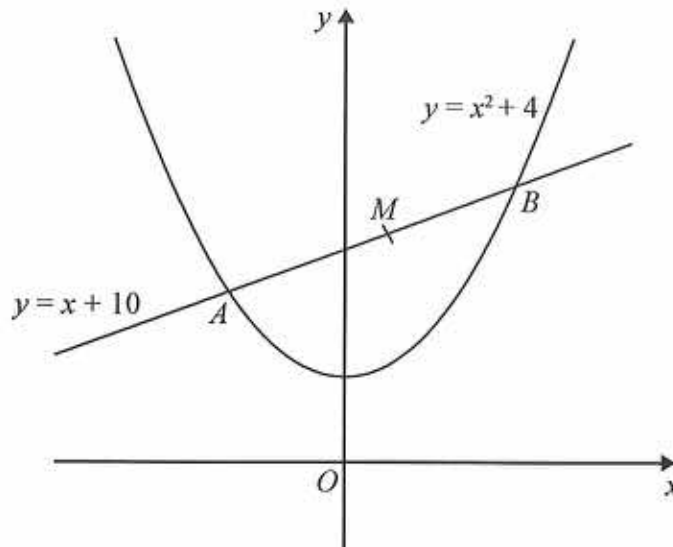
N is the point such that DCN is a straight line and $DC : CN = 2 : 1 \rightarrow \vec{CN} = 4a$
 (b) Show that AMN is a straight line.

$\vec{AM} = \vec{AB} + \vec{BM}$
 $= 4a + (2a + b) = 6a + b$

$\vec{AN} = \vec{AD} + \vec{DC} + \vec{CN}$
 $= 2b + 8a + 4a$ (m) [BOTH]
 $= 12a + 2b$
 $= 2(6a + b)$
 $= 2\vec{AM}$

EXPLANATION
 SINCE $\vec{AM} = k \times \vec{AN}$
 (m) [EXPLANATION]
 AM AND AN MUST BE PARALLEL AND SINCE THEY BOTH HAVE POINT A IN COMMON (NEEDED!)
 FORM A STRAIGHT LINE

The sketch shows the curve with equation $y = x^2 + 4$ and the line with equation $y = x + 10$



The line cuts the curve at the points A and B .

M is the midpoint of AB .

Find the coordinates of M .

Show clear algebraic working.

1ST [FIND INTERSECTIONS A AND B]

$$x^2 + 4 = x + 10 \quad (m1)$$

$$\Rightarrow x^2 - x - 6 = 0 \quad (m2)$$

$$(x-3)(x+2) = 0 \quad (m3)$$

$$\begin{array}{l} \downarrow \qquad \downarrow \\ x = 3 \qquad x = -2 \quad (A1) \end{array}$$

$$\Rightarrow \begin{array}{l} y = 3 + 10 \\ \qquad = 13 \end{array} \quad \begin{array}{l} y = -2 + 10 \\ \qquad = 8 \quad (A1) \end{array}$$

2ND [MIDPOINT]

$$\left(\frac{3 + (-2)}{2}, \frac{13 + 8}{2} \right)$$

$$= \underline{\underline{(0.5, 10.5)}} \quad (A1)$$

COORDINATES ARE $(3, 13)$ AND $(-2, 8)$

$$y = at^2 - 2at$$

$$x = 2a\sqrt{t}$$

Express y in terms of x and a .

Give your answer in the form

$$y = \frac{x^p}{ma^3} - \frac{x^q}{na}$$

where p , q , m and n are integers.

$$x = 2a\sqrt{t} \Rightarrow t = \left(\frac{x}{2a}\right)^2 = \frac{x^2}{4a^2} \quad (B1)$$

$$\Rightarrow t^2 = \frac{x^4}{16a^4} \quad (B1)$$

$$\therefore y = at^2 - 2at$$

$$\Rightarrow y = a \times \frac{x^4}{16a^4} - 2a \times \frac{x^2}{4a^2} \quad (M1)$$

$$= \frac{x^4}{16a^3} - \frac{x^2}{2a} \quad (A1)$$
