4H

EDEXCEL IGCSE

Pearson Edexcel International GCSE

MATHEMATICS A

SOLUTIONS

MAY 2013 | **4MA0/4H**

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The methods used in these solutions, where relevant, are methods which have been successfully used with students. The method shown for a particular question is not always the only method and We do not claim that the method we have used is necessarily the most efficient or 'best' method. We will, from time to time, update a solution to show a different method if We feel that it is a good idea to do so.

Sometimes a method used in these solutions might be unfamiliar to You. If You are able to use a different method to obtain the correct answer then We would usually recommend that You keep using your existing method and not change to the method that We have used here. However, the choice of method is always up to You and We believe that it is often useful if You know more than one method to solve a particular type of problem.

Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

A box contains four different kinds of chocolates.
Debbie takes at random a chocolate from the box.
The table shows the probability of Debbie taking an Orange or a Coffee or a Caramel chocolate.

Chocolate	Probability
Orange	0.15
Coffee	0.40
Caramel	0.35
Strawberry	

0.9 B1

(a) Work out the probability that Debbie takes a Strawberry chocolate.

$1 - 0.9 = 0.1$

A1

0.1

(2)

(b) Work out the probability that Debbie takes an Orange chocolate or a Coffee chocolate.

$0.15 + 0.40$

M1

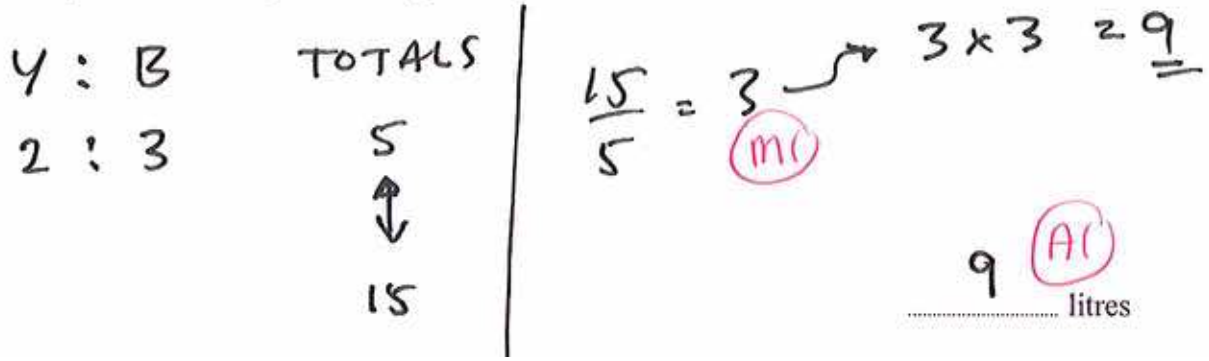
A1

0.55

(2)

Green paint can be made by mixing yellow paint and blue paint in the ratio 2 : 3
Wendy makes 15 litres of green paint.

Work out how many litres of blue paint Wendy uses.



Yoko flew on a plane from Tokyo to Sydney.
The plane flew a distance of 7800 km.
The flight time was 9 hours 45 minutes. — 9.75 HOURS

Work out the average speed of the plane in kilometres per hour.

$$V = \frac{d}{t}$$

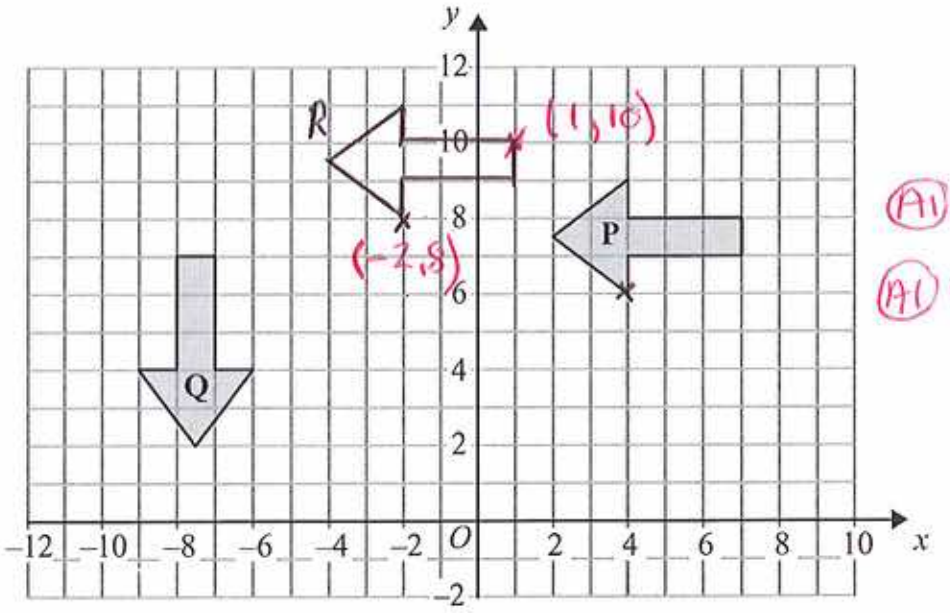
$$= \frac{7800}{9.75}$$ M1

B1 →

$$= 800$$

A1

800 km/h



A1 ANY TRANSLATION

A1 CORRECT POSITION

(a) Describe fully the single transformation that maps shape **P** onto shape **Q**.

A1 ROTATION, 90° ANTICLOCKWISE A1

CENTRE AT (0,0) A1

(3)

(b) On the grid, translate shape **P** by the vector $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix}$
Label the new shape **R**.

(2)

(a) Show that $\frac{7}{8} - \frac{5}{6} = \frac{1}{24}$

$\frac{7}{8} - \frac{5}{6} = \frac{42}{48} - \frac{40}{48}$ B1 [COMMON DENOMINATOR]

$= \frac{2}{48}$ B1 $= \frac{1}{24}$

(b) Show that $\frac{5}{8} \div \frac{7}{12} = 1\frac{1}{14}$

$\frac{5}{8} \div \frac{7}{12} = \frac{5}{\cancel{8}_2} \times \frac{\cancel{12}^3}{7}$ M1 [FOR $\times \frac{12}{7}$]

$= \frac{5}{2} \times \frac{3}{7}$

$= \frac{15}{14}$ B1

$= 1\frac{1}{14}$

Solve $7y - 6 = 2y + 8$

Show clear algebraic working.

$7y - 2y = 8 + 6$ (M1)

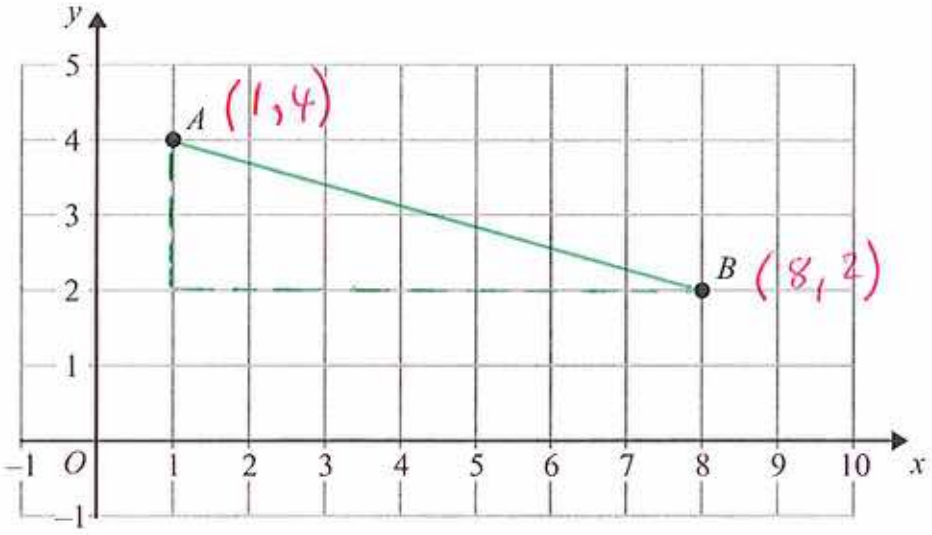
$5y = 14$ (M1)

$y = \frac{14}{5}$

$= 2.8$

$y =$ 2.8 (A1)

Two points, A and B, are plotted on a centimetre grid.
A has coordinates (1, 4) and B has coordinates (8, 2).



(a) Work out the coordinates of the midpoint of AB.

$\left(\frac{8+1}{2}, \frac{2+4}{2}\right) = (4.5, 3)$

A1 A1

(4.5, 3)

(2)

(b) Use Pythagoras' Theorem to work out the length of AB.
Give your answer correct to 3 significant figures.

M1 PYTHAGORAS

$7^2 + 2^2 = 49 + 4$

$= 53$ A1

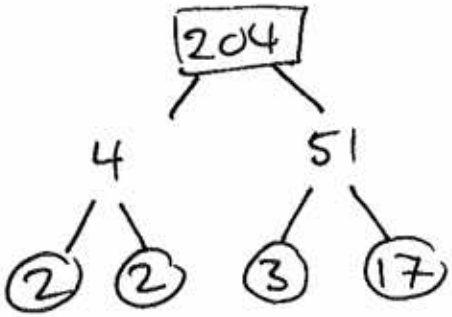
B1 (EITHER)

$\sqrt{53} = 7.280109...$

7.28 cm A1

(4)

Express 204 as a product of its prime factors.



$2^2 \times 3 \times 17$

(a) Solve the inequalities $-6 \leqslant 3x < 9$

DIVIDE BY 3

$-\frac{6}{3} \leq \frac{3x}{3} < \frac{9}{3}$ (M1)

(A1)

$-2 \leq x < 3$

(2)

(b) n is an integer.

Write down all the values of n which satisfy $-6 \leqslant 3n < 9$

SAME INEQUALITY

$-2 \leq n < 3$

(B2) [-1 eeo]

$n = -2, -1, 0, 1, 2$

(2)

The scale of a map is 1 : 25 000
On the map, the distance between two railway stations is 22 cm.

Work out the real distance between the two railway stations.
Give your answer in kilometres.

$22 \times 25\,000 = 550\,000$ cm (A1)

$= \frac{550\,000}{100}$ m

$= \frac{5500}{1000}$ km (M1)

$= 5.5$ km

5.5 km (A1)

For $y = x^3 - 6x^2 + 20$

(a) (i) show that $y = 4$ when $x = 2$

$y = (2)^3 - 6 \times (2)^2 + 20$ M1

$= 8 - 6 \times 4 + 20 = 4$

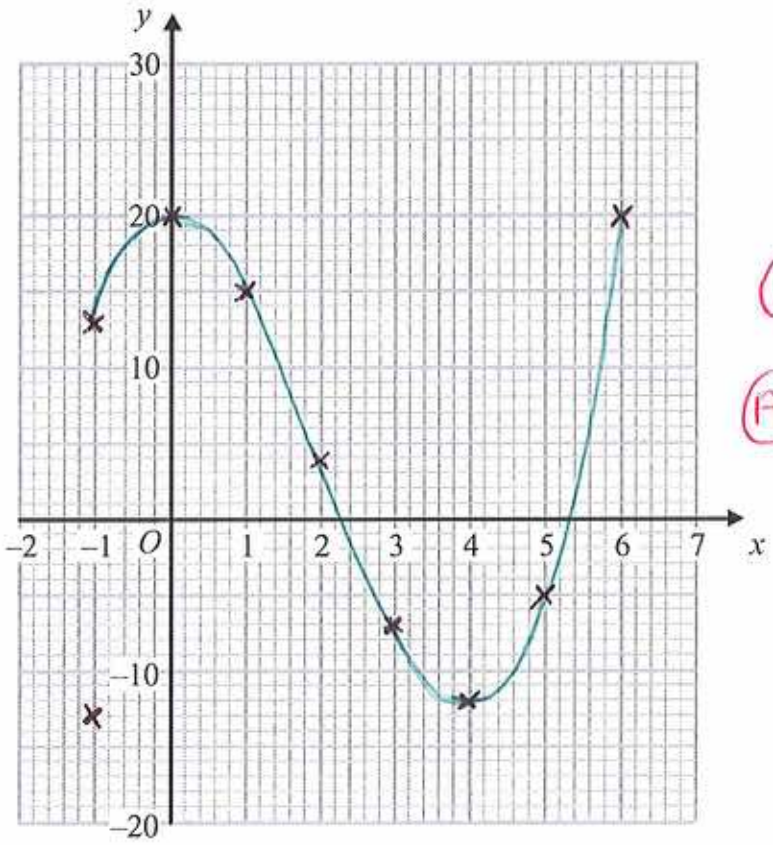
(ii) complete the table of values

x	−1	0	1	2	3	4	5	6
y	13	20	15	4	−7	−12	−5	20

A1

(2)

(b) On the grid, draw the graph of $y = x^3 - 6x^2 + 20$ for values of x from −1 to 6



A1 POINTS

A1 SMOOTH CURVE

(c) For the curve with equation $y = x^3 - 6x^2 - 20$

(i) find $\frac{dy}{dx}$

M1 A1

$3x^2 - 12x$

(ii) find the gradient of the curve at $x = -3$

$3(-3)^2 - 12(-3)$ M1

A1

63

(4)

The table shows information about the amount of money, in dollars, spent in a shop in one day by 80 people.

MID POINT	Money spent (x dollars)	Frequency	$x \times f$
10	$0 < x \leqslant 20$	24	$10 \times 24 = 240$
30	$20 < x \leqslant 40$	20	600
50	$40 < x \leqslant 60$	9	450
70	$60 < x \leqslant 80$	12	840
90	$80 < x \leqslant 100$	15	1350

B1

Work out an estimate for the total amount of money spent in the shop that day.

B1

TOTAL = 240 + 600 + ... + 1350

= 3480

A1

3480 dollars

The diagram shows an incomplete regular polygon.

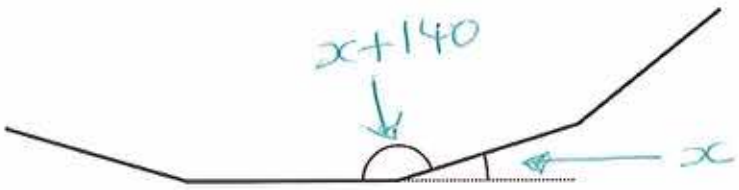


Diagram **NOT** accurately drawn

The size of each interior angle is 140 degrees greater than the size of each exterior angle.

Work out the number of sides the regular polygon has.

$(x+140) + x = 180$ (B1) (M1)

$\Rightarrow 2x + 140 = 180$

$2x = 40$

$x = 20^\circ$ (A1)

EXTERIOR,

$\text{SIDES} = \frac{360}{20}$

$= 18$ (A1)

The table shows the surface areas, in km², of five oceans.

Ocean	Surface area (km²)
Atlantic	7.68×10^7
Indian	6.86×10^7
Pacific	1.56×10^8
Southern	2.03×10^7
Arctic	1.41×10^7

(a) Which of these oceans has the largest surface area?

A1

PACIFIC

(1)

(b) Work out the total surface area, in km², of all five oceans.
Give your answer in standard form.

USE CALCULATOR TO GET

335 800 000 M1

A1

3.358×10^8 km²

(2)

The total surface area of the Earth is 5.10×10^8 km².

(c) Express the total surface area of the five oceans as a percentage of the total surface area of the Earth.
Give your answer correct to 1 decimal place.

$\frac{3.358 \times 10^8}{5.10 \times 10^8} \times 100 = 65.8431...$

M1

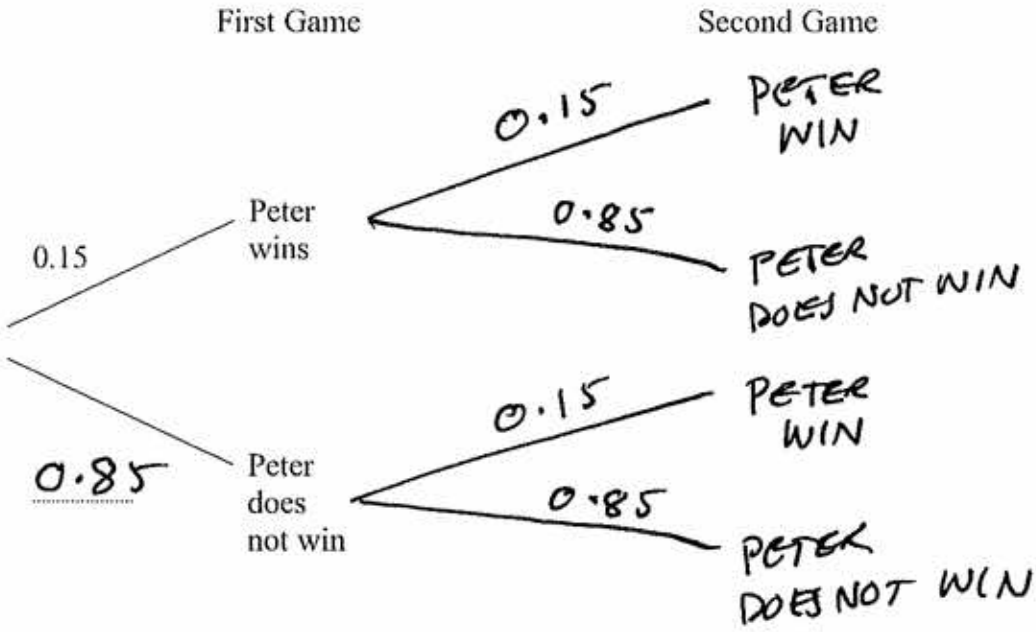
A1

65.8 %

(2)

Peter and John play two games of badminton against each other.
For each game, the probability that Peter wins is 0.15

(a) Complete the probability tree diagram.



(3)

(b) Calculate the probability that Peter wins both games.

$P(WW) = 0.15 \times 0.15$

$= 0.0225$

0.0225

(2)

The pressure P, of water leaving a cylindrical pipe, is inversely proportional to the square of the radius, r, of the pipe.

$P = 22.5$ when $r = 2$

(a) Find a formula for P in terms of r.

$P = \frac{k}{r^2}$ $(P = 22.5,\ r = 2)$

$\Rightarrow 22.5 = \frac{k}{2^2}$ M1

$\Rightarrow k = 22.5 \times 2^2$

$= 90$ B1

$P = \frac{90}{r^2}$ A1

(3)

(b) Calculate the value of P when $r = 1.5$

$P = \frac{90}{r^2}$ $(r = 1.5)$

$P = \frac{90}{1.5^2}$

A1

$P = 40$

(1)

(c) Calculate the value of r when $P = 10$

$P = \frac{90}{r^2}$ $(P = 10)$

$10 = \frac{90}{r^2} \Rightarrow r^2 = \frac{90}{10}$ M1

$r = \sqrt{\frac{90}{10}}$

A1

$r = 3$

(2)

The function f is defined as

$$f(x) = \frac{x-6}{2}$$

(a) Find f(8)

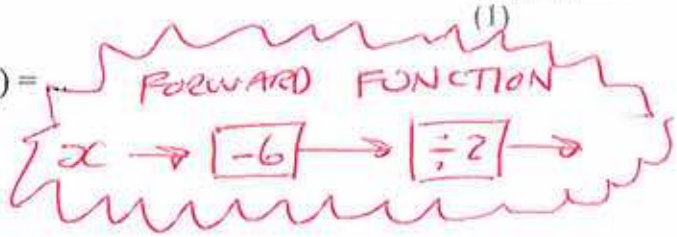
$f(8) = \frac{8-6}{2} = 1$

1

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f^{-1}(x) = x \times 2 + 6$



$f^{-1}(x) = 2x+6$

(2)

The function g is defined as

$$g(x) = \sqrt{x-4}$$

(c) Which values of x cannot be included in a domain of g?

$x - 4 < 0 \Rightarrow x < 4$

$x < 4$

(2)

(d) Express the function gf in the form $gf(x) = ...$
Give your answer as simply as possible.

$g[f(x)] = \sqrt{f(x) - 4}$

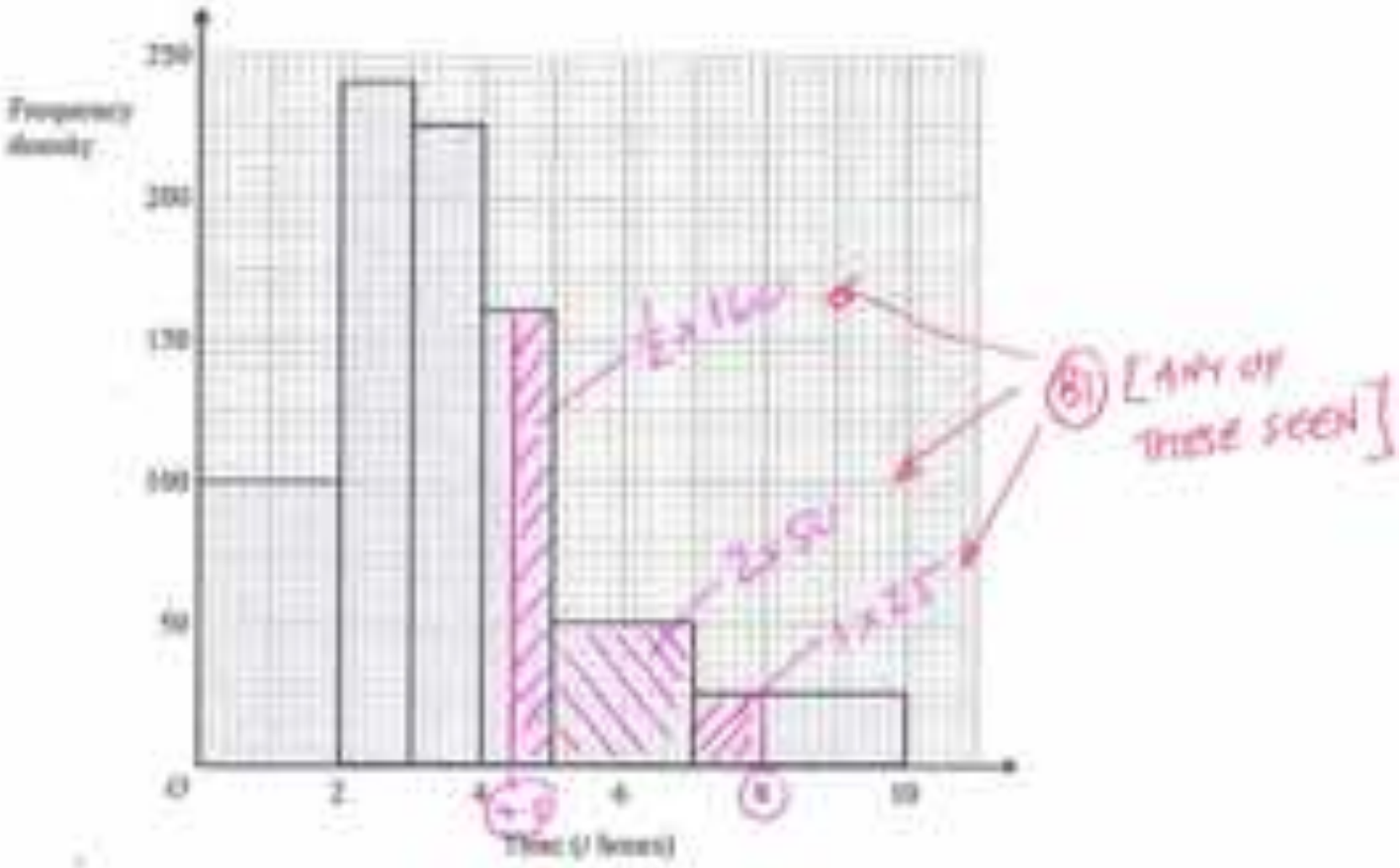
$= \sqrt{\frac{x-6}{2} - 4}$

$= \sqrt{\frac{x-6-8}{2}}$

$= \sqrt{\frac{x-14}{2}}$

$gf(x) = \sqrt{\frac{x-14}{2}}$

The histogram shows information about the times, t hours, for which some cars were left in a car park.



Calculate an estimate for the number of cars which were left in the car park for between 4.5 hours and 8 hours.

NUMBER OF CARS = AREA OF BARS

$= \frac{1}{2} \times 160 + 2 \times 50 + 1 \times 25$ (M1)

$= \underline{205}$ (A1)

The sides of triangle PQR are tangents to a circle.
The tangents touch the circle at the points S, T and U.
$QS = 6$ cm. $PS = 7$ cm.

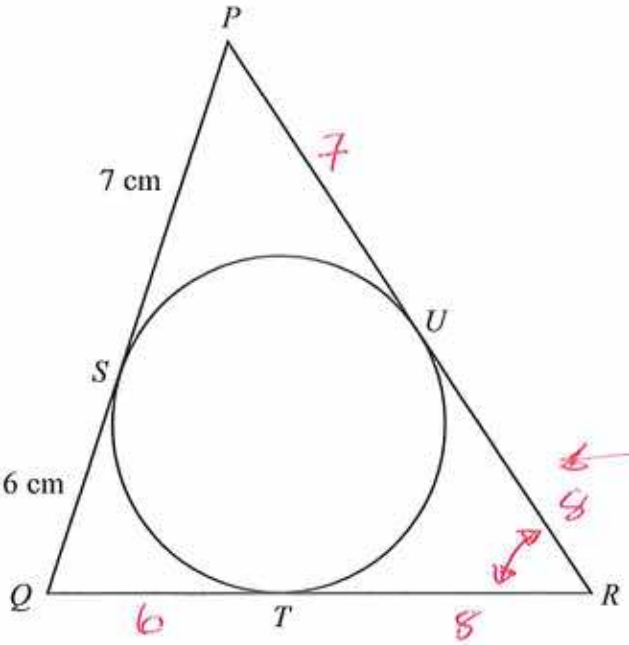


Diagram **NOT** accurately drawn

(a) (i) Write down the length of QT.

6 cm (A1)

(ii) Give a reason for your answer.

TANGENTS FROM THE SAME POINT ARE EQUAL (A1)

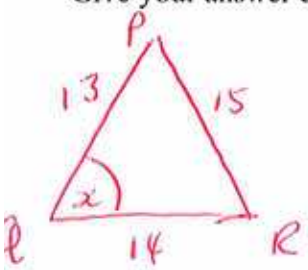
(2)

The perimeter of triangle PQR is 42 cm.

$42 - (2\times7 + 2\times6) = 16$

SO $TR = UR = 8$ (B1)

(b) Calculate the size of angle PQR.
Give your answer correct to 1 decimal place.



$\cos x = \dfrac{13^2 + 14^2 - 15^2}{2\times13\times14}$ (M1)

$= 0.3846...$

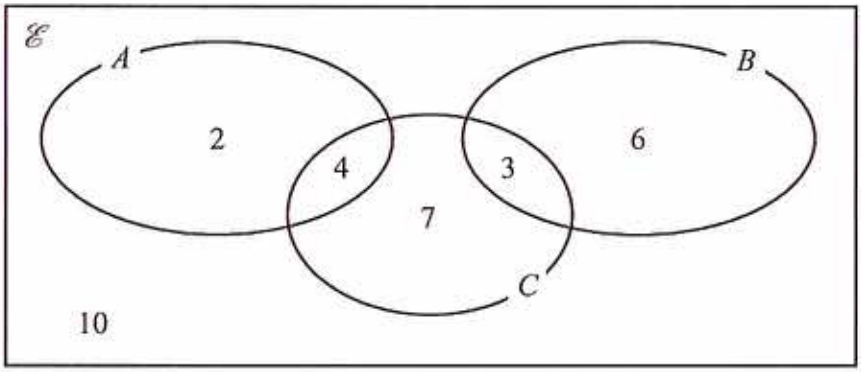
$x = \cos^{-1}(0.3846...)$ (M1)

$= 67.380...$

67.4° (A1)

(4)

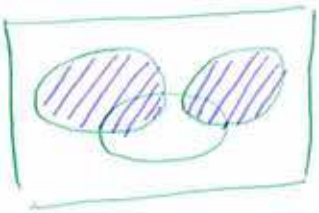
The Venn diagram shows a universal set $\mathscr{E}$ and 3 sets A, B and C.



2, 4, 7, 3, 6 and 10 represent **numbers** of elements.

Find

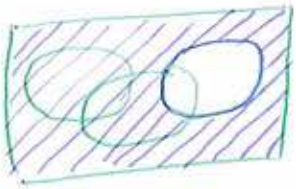
(i) n $(A \cup B)$



2 + 4 + 3 + 6

15 B1

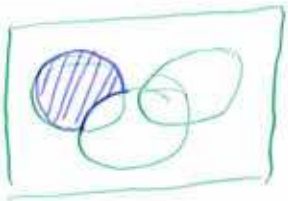
(ii) n (B')



10 + 2 + 4 + 7

23 B1

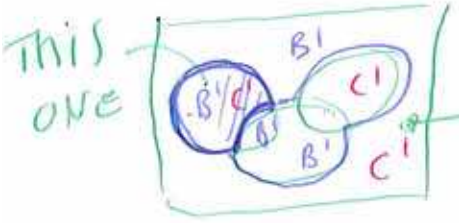
(iii) n $(A \cap C')$



2

2 B1

(iv) n $(B' \cap C')$



10 + 2

12 B1

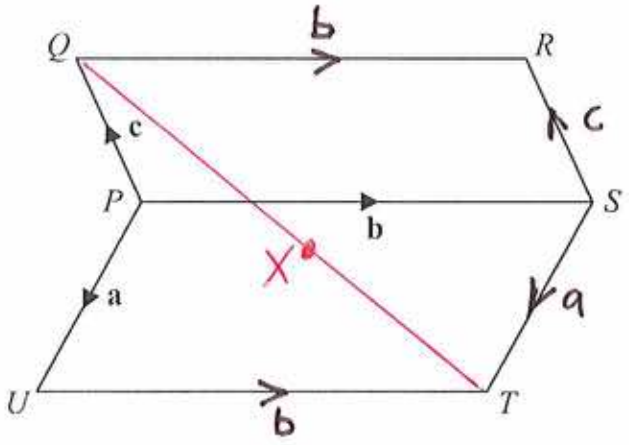


Diagram **NOT** accurately drawn

PQRS and *PSTU* are parallelograms.

$\overrightarrow{PU} = \mathbf{a}$ $\overrightarrow{PS} = \mathbf{b}$ $\overrightarrow{PQ} = \mathbf{c}$

Find, in terms of **a**, **b** and **c**

(i) $\overrightarrow{TQ}$

$= \overrightarrow{TU} + \overrightarrow{UP} + \overrightarrow{PQ}$

$= -b - a + c$

$c - a - b$

(ii) $\overrightarrow{PX}$ where X is the midpoint of TQ.

Simplify your answer as much as possible.

$\overrightarrow{PX} = \overrightarrow{PU} + \overrightarrow{UT} + \overrightarrow{TX}$

$= \overrightarrow{PU} + \overrightarrow{UT} + \frac{1}{2}\overrightarrow{TQ}$

$= a + b + \frac{1}{2}(c - a - b)$

$= a + b + \frac{1}{2}c - \frac{1}{2}a - \frac{1}{2}b$

$= \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c$

$\frac{1}{2}(a + b + c)$

The diagram shows a triangular prism with a horizontal rectangular base $ABCD$.
$AB = 10$ cm. $BC = 7$ cm.
M is the midpoint of AD.
The vertex T is vertically above M.
$MT = 6$ cm.

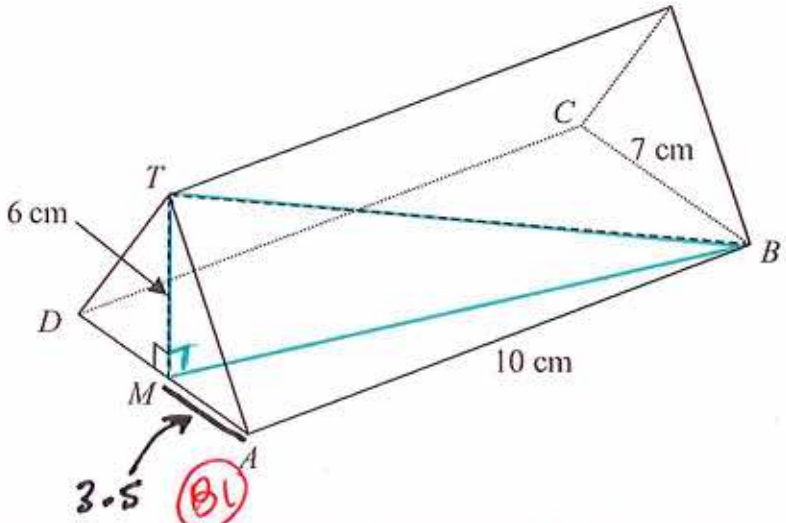
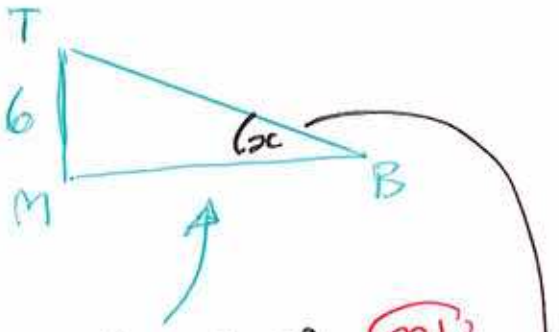


Diagram **NOT** accurately drawn

Calculate the size of the angle between TB and the base $ABCD$.

Give your answer correct to 1 decimal place.



$MB^2 = 10^2 + 3.5^2$ M1

$= 112.25$

$MB = \sqrt{112.25}$

$= 10.5948...$ A1

$\tan x = \frac{6}{10.5948}$ M1

$x = \tan^{-1}\left(\frac{6}{10.5948}\right)$

$= 29.5°$ A1

Solve $\frac{3}{(x+1)} + \frac{2}{(2x-3)} = 1$

Show clear algebraic working.

$$\frac{3(2x-3) + 2(x+1)}{(x+1)(2x-3)} = 1$$ M1 [COMMON DENOMINATOR]

$3(2x-3) + 2(x+1) = (x+1)(2x-3)$ M1 [NO DENOMINATOR]

$6x - 9 + 2x + 2 = 2x^2 - 3x + 2x - 3$

$8x - 7 = 2x^2 - x - 3$

$2x^2 - 9x + 4 = 0$ M1 [QUADRATIC]

$(2x - 1)(x - 4) = 0$ M1 [FACTORISING]

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$x - 4 = 0$

$x = 4$

A1 [BOTH SOLUTIONS]