4H(R)

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

MAY 2013

4MA0/4HR

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Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

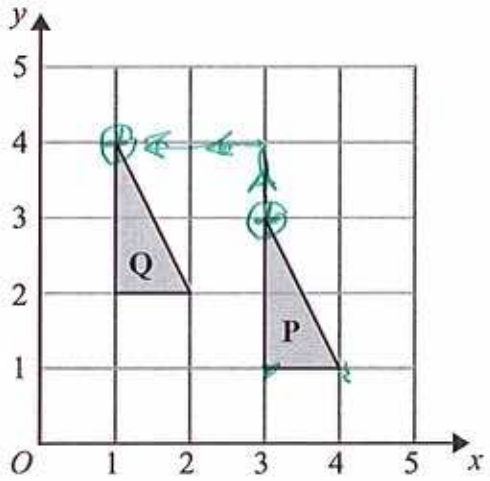
A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

There are 20 students in a class.
12 of the students are girls. ⟶ ∴ 8 BOYS

Find the ratio of the number of girls to the number of boys.
Give your ratio in the form $n : 1$

G : B
= 12 : 8 (B1) [÷ 8]

(A1) 1.5 : 1



Describe fully the single transformation which maps triangle **P** onto triangle **Q**.

TRANSCATION B1

BY VECTOR $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ B1

The table shows information about the number of letters in the first name of each of 50 people.

Number of letters	Frequency	$x \times f$
3	2	6
4	5	20
5	14	70
6	19	114
7	10	70

M1

(i) Work out the mean number of letters in the first names of the 50 people.

TOTAL = 280

MEAN = TOTAL NO. OF LETTERS / NO. OF PEOPLE

$= \frac{280}{50}$ M1 [DIVIDE BY 50]

A1

5.6

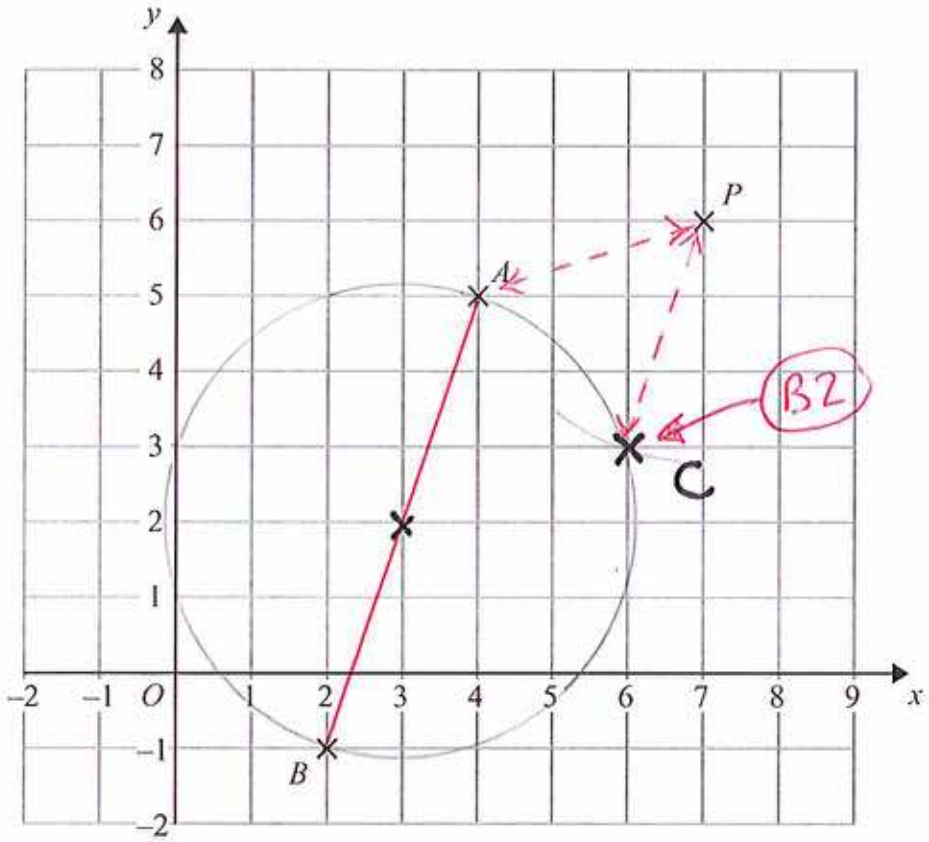
(ii) One more person joins the 50 people.
The mean number of letters in the first names of the 51 people is less than the mean number of letters in the first names of the 50 people.

Write down the greatest number of letters in the first name of the person who joins the group.

NUMBER OF LETTERS < 5.6

$\therefore$ COULD BE 5 B1

The diagram shows three points, A, B and P, on a centimetre grid.



The point A has coordinates (4, 5) and the point B has coordinates (2, −1).

(a) Find the coordinates of the midpoint of AB.

$\left(\frac{4+2}{2}, \frac{5+-1}{2}\right) = (3, 2)$

M1

A1

(3, 2)

(2)

AB is a diameter of a circle.
P is the point (7, 6)
C is the point on the circle such that $PA = PC$.

(b) On the diagram, mark with a cross (×) the point C.
Label your point C.

(2)

A shop, *Furniture 4U*, had a sale.

(a) In the sale, normal prices were reduced by 15%. → 0·15 → 0·85

(i) The normal price of a table was \$280

Work out the sale price of the table.

280×0.85 M1 (MULTIPLY)

B1

\$ 238 A1

(ii) The normal price of a chair was reduced in the sale by \$24

Work out the normal price of the chair.

$\frac{24}{0.15}$ M1 (DIVIDE)

B1

\$ 160 A1

(6)

(b) Ruth, Suha and Yasmin went to the sale.
The amounts of money spent by Ruth, Suha and Yasmin were in the ratios 2 : 3 : 7
Ruth and Suha spent a total of \$320 in the sale.

Work out the amount of money Yasmin spent in the sale.

R : S : Y

2 : 3 : 7

320

1 PART = $\frac{320}{5}$

= 64 B1

YASMIN SPENT $64 \times 7 = 448$ M1 A1

SUM OF EXTERIOR ANGLES = 360°

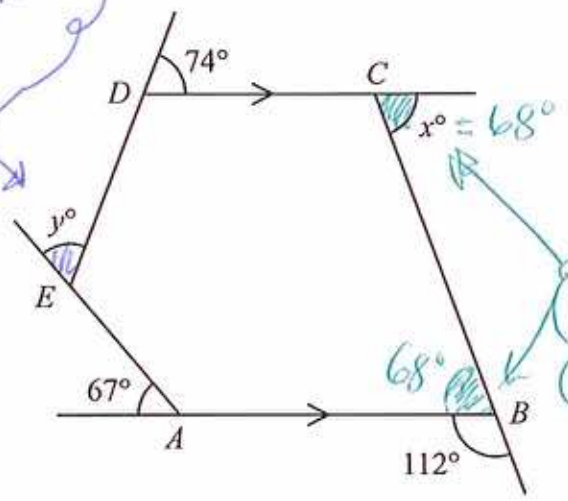


Diagram **NOT** accurately drawn

ALTERNATE ANGLES

The diagram shows a pentagon *ABCDE*.
DC is parallel to *AB*.

The size of an exterior angle at *A* is 67°
The size of an exterior angle at *B* is 112°
The size of an exterior angle at *C* is $x°$
The size of an exterior angle at *D* is 74°
The size of an exterior angle at *E* is $y°$

(a) (i) Work out the value of *x*.

$180 - 112$ (M1)

$x =$ 68° (A1)

(ii) Work out the value of *y*.

$y = 360 - (74 + 68 + 112 + 67)$ (M1)

$y =$ 39° (A1)

(4)

(b) Work out the sum of the interior angles of the pentagon *ABCDE*.

SUM = $(n-2) \times 180$ ⇐ 'MAGIC' FORMULA!

$= 3 \times 180$ (M1)

540° (A1)

ALTERNATIVE METHODS ARE FINE!

(i) Solve the inequalities $3 \leqslant x + 4 < 7$

$-1 \leqslant x < 3$ (-4 FROM EACH PART!)

A1 A1

$-1 \leqslant x < 3$

(ii) n is an integer. — WHOLE NUMBER!

Write down all the values of n which satisfy $3 \leqslant n + 4 < 7$

A2

$-1, 0, 1, 2$

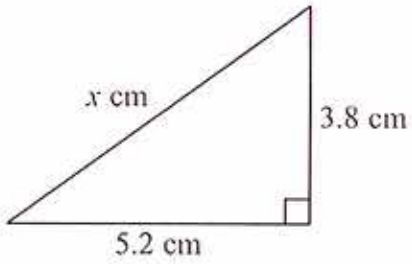


Diagram **NOT** accurately drawn

Calculate the value of x.
Give your answer correct to 3 significant figures.

$x^2 = 5.2^2 + 3.8^2$ (M1)

$= 41.48$ (A1)

$x = \sqrt{41.48}$

$= 6.44049...$

(A1)

$x =$ 6.44

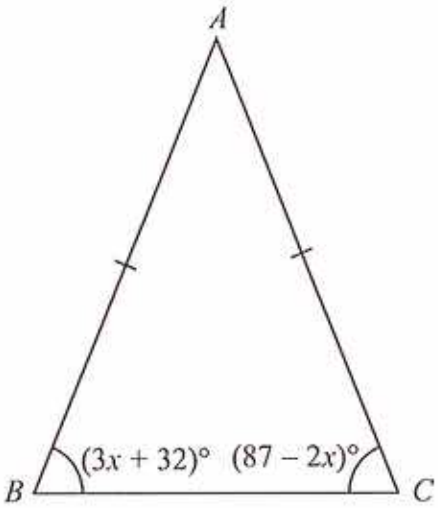


Diagram **NOT** accurately drawn

In the isosceles triangle ABC,
$AB = AC$
angle $B = (3x + 32)°$
angle $C = (87 - 2x)°$

Work out the value of x.
Show clear algebraic working.

ISOSCELES TRIANGLE SO...

$3x + 32 = 87 - 2x$ (M1)

$3x + 2x = 87 - 32$ (M1)

$5x = 55$ (M1)

$x = \frac{55}{5}$

$= 11$ (A1)

The grouped frequency table gives information about the weights of 180 airmail letters.

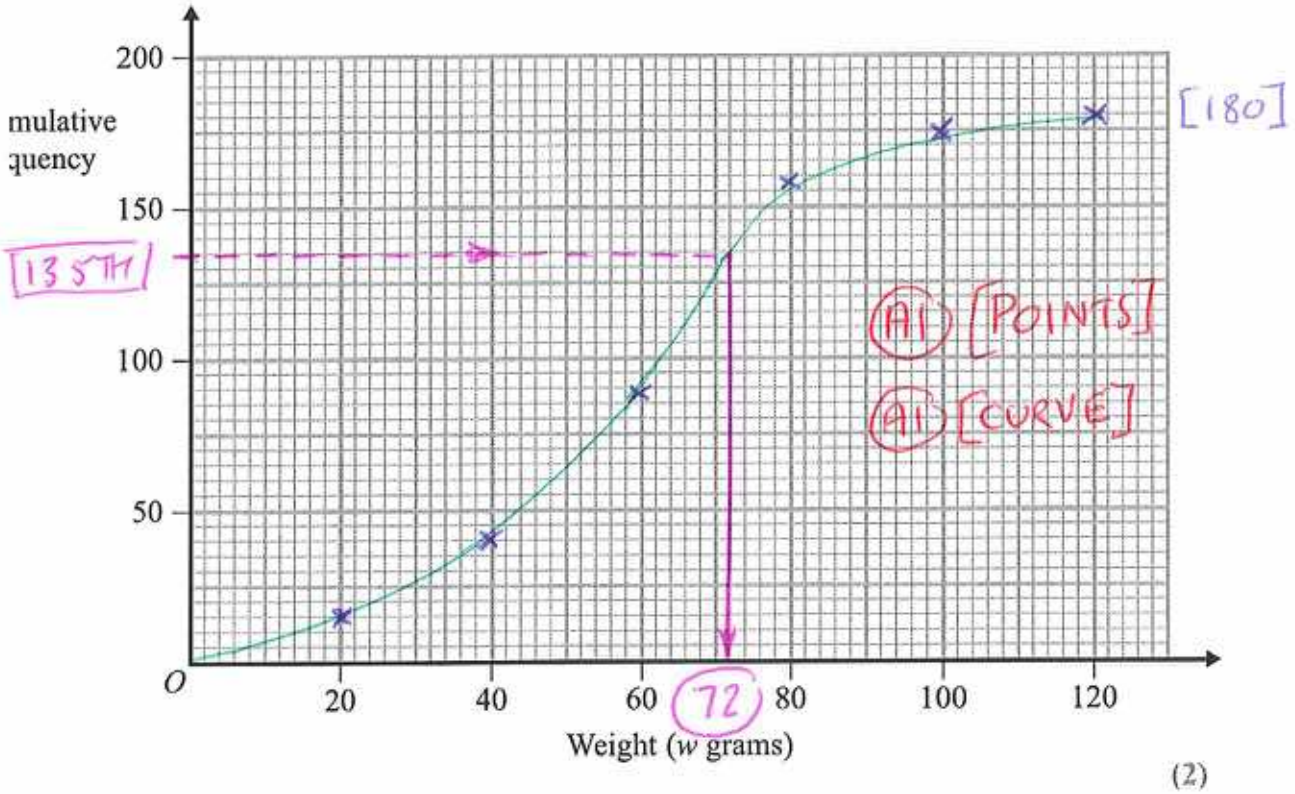
Weight (w grams)	Frequency
$0 < w \leqslant 20$	15
$20 < w \leqslant 40$	25
$40 < w \leqslant 60$	47
$60 < w \leqslant 80$	70
$80 < w \leqslant 100$	18
$100 < w \leqslant 120$	5

(a) Complete the cumulative frequency table.

Weight (w grams)	Cumulative frequency
$0 < w \leqslant 20$	15
$0 < w \leqslant 40$	40
$0 < w \leqslant 60$	87
$0 < w \leqslant 80$	157
$0 < w \leqslant 100$	175
$0 < w \leqslant 120$	180

B1

(b) On the grid, draw a cumulative frequency graph for your table.



(2)

(c) Find an estimate for the upper quartile of the weights of the 180 letters.

$Q_3 = 3 \times \frac{180}{4}$

= 135 TH VALUE (B1)

= 72 (A1)

$A = 2^3 \times 3^2 \times 5^4$ $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

$B = 3^5 \times 5 \times 7^3$ $= 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 7 \times 7 \times 7$

Find the Highest Common Factor (HCF) of A and B.

COMMON FACTORS ARE (MATCHING)

3, 3, 5 (M1)

$\therefore$ HCF $= 3 \times 3 \times 5$

45 (A1)

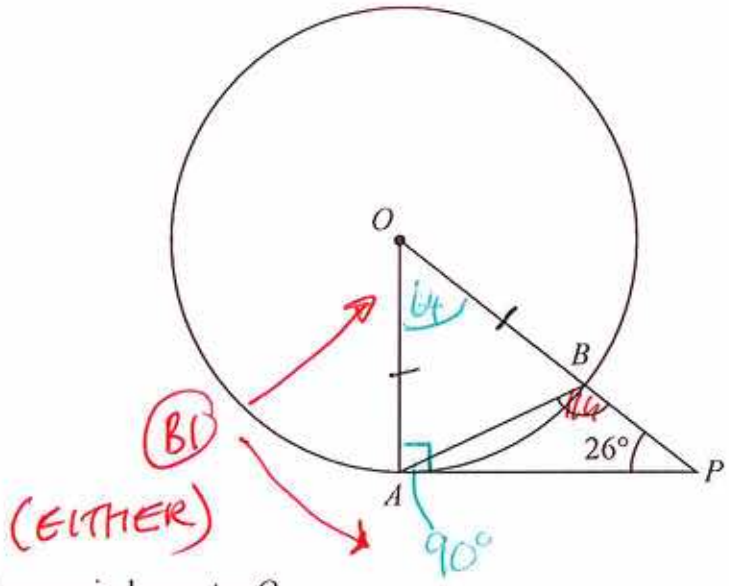


Diagram **NOT** accurately drawn

A and *B* are points on a circle, centre *O*.
PA is the tangent to the circle at *A*.
OBP is a straight line.
Angle *APO* = 26°

Calculate the size of angle *ABP*. (4)

OAP = 90° (ANGLE BETWEEN A TANGENT AND RADIUS)

AOP = 64 (180 − 90 − 26)

SINCE AOB IS AN ISOSCELES TRIANGLE:

$$ABO = \frac{180 - 64}{2}$$

$$= 58^\circ \quad (B1)$$

$$\therefore ABP = 180 - 58$$

$$= 122^\circ$$

(a) Solve the simultaneous equations

$5x + 3y = 9$ — ① ×2
$7x - 2y = 25$ — ② ×3

Show clear algebraic working.

M1 $\left\{\begin{array}{l} 10x + 6y = 18 \quad -③ \\ 21x - 6y = 75 \quad -④ \end{array}\right\}$ ADD

$31x = 93$

$x = 3$ A1

SUBSTITUTE $x = 3$ INTO EQ ①

$5 \times 3 + 3y = 9$ M1

$3y = 9 - 15 \Rightarrow y = -2$ A1

$x =$ 3

$y =$ -2

(4)

(b) P is the point of intersection of the lines with equations $5x + 3y = 9$ and $7x - 2y = 25$

Write down the coordinates of P.

A1

(3 , -2)

(1)

POINTS OF INTERSECTION ARE THE SOLUTIONS TO SIMULTANEOUS EQUATIONS!

Jomo invested an amount of money at 4% per annum **compound interest**.
At the end of 2 years, the value of his investment was £3380

How much of the £3380 was interest?

1·04

INITIAL INVESTMENT $= \frac{3380}{1.04^2}$ M1

$= 3125$ A1

INTEREST EARNT $= 3380 - 3125$ M1

$= 255$

£ 255 A1

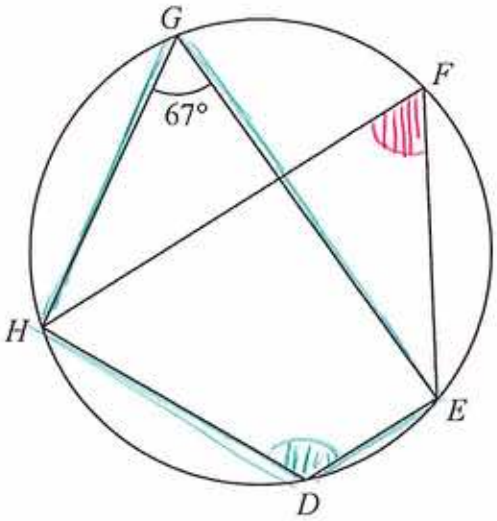


Diagram **NOT** accurately drawn

CYCLIC QUADRILATERAL

D, *E*, *F*, *G* and *H* are points on a circle.
Angle *EGH* = 67°

(a) Find the size of angle *EFH*.

F

[ANGLES IN THE SAME SEGMENT ARE EQUAL!] 67 ° A1

(1)

(b) (i) Find the size of angle *EDH*.

D

180 - 67

113 ° A1

(ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL ADD UP TO 180°. A1

(2)

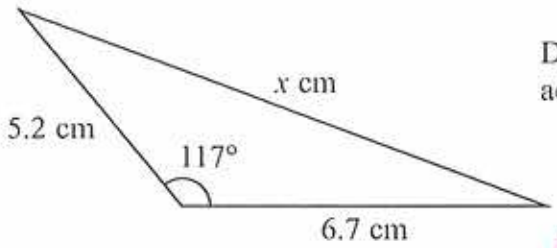


Diagram **NOT** accurately drawn

COSINE RULE

Calculate the value of x.
Give your answer correct to 3 significant figures.

$x^2 = 5.2^2 + 6.7^2 - 2 \times 5.2 \times 6.7 \times \cos 117$

$= 103.564...$

$x =$ 104 cm

y is directly proportional to x^3
When $x = 10$, $y = 250$

(a) Find a formula for y in terms of x.

$y = kx^3 \quad (x = 10, y = 250)$

$250 = k \times 10^3$ M1

$k = \frac{250}{10^3}$

$= 0.25$ A1

$y = 0.25x^3$ A1

(3)

(b) Calculate the value of x when $y = 54$

$y = 0.25x^3 \quad (y = 54)$

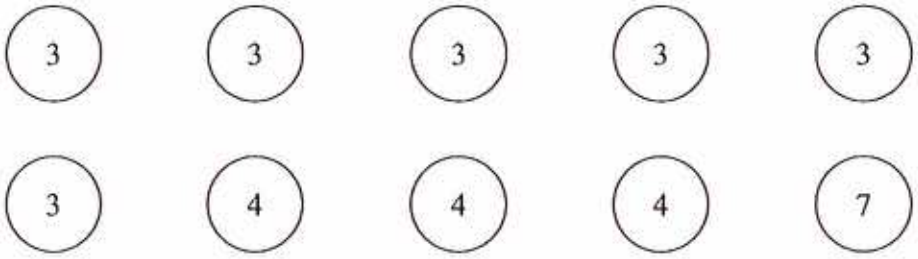
$54 = 0.25x^3$ M1

$x^3 = \frac{54}{0.25} \Rightarrow x = \sqrt[3]{\frac{54}{0.25}}$

$x =$ 6 A1

(2)

Here are ten counters.
Each counter has a number on it.



Fern puts the ten counters in a bag.
She takes at random a counter from the bag.

(a) Find the probability that the number on the counter is 3 or 4

$\frac{6}{10} + \frac{3}{10}$ M1

$\frac{9}{10}$ A1

(2)

Fern puts the counter back into the bag.
Then Rajan takes at random one of the ten counters from the bag.
He does not put the counter back into the bag.
He then takes at random a second counter from the bag.

PROBABILITIES CHANGE!

(b) Calculate the probability that 3 is the number on each of the two counters he takes.

$P(3,3) = \frac{6}{10} \times \frac{5}{9}$

M1

$= \frac{30}{90}$

A1

$\frac{1}{3}$

(2)

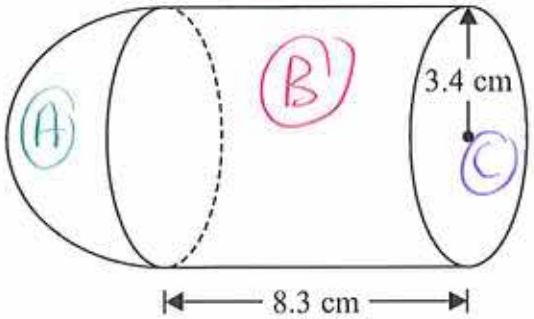


Diagram **NOT** accurately drawn

The diagram shows a shape made from a solid cylinder and a solid hemisphere.
The cylinder has a radius of 3.4 cm and a length of 8.3 cm.
The hemisphere has a radius of 3.4 cm.

Calculate the total surface area of the solid shape.
Give your answer correct to 3 significant figures.

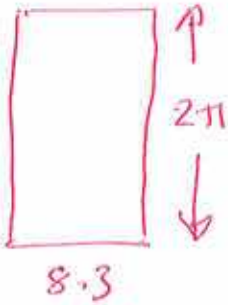
3 SURFACES

(A) [HEMISPHERE]

$\frac{4\pi r^2}{2}$ $(r = 3.4)$

$= \frac{4\pi \times 3.4^2}{2}$ $= 72.6336...$ (B1)

(B) [RECTANGLE]



$2 \times \pi \times 3.4 \times 8.3$

$= 177.3114...$ (B1)

(C) [CIRCLE]

πr^2 $(r = 3.4)$

$= \pi \times 3.4^2$ $= 36.3168...$ (B1)

TOTAL $= 286.26...$

286 cm² (A1)

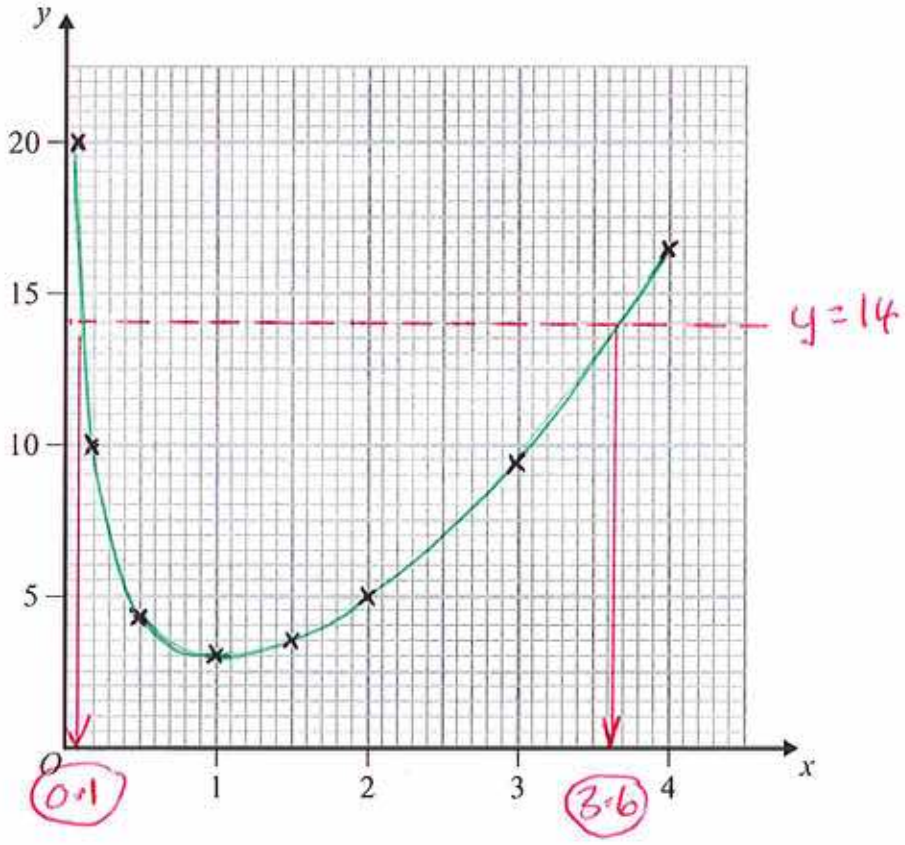
(a) Complete the table of values for $y = x^2 + \frac{2}{x}$

$y = (0.1)^2 + \frac{2}{(0.1)}$ ETC

x	0.1	0.2	0.5	1	1.5	2	3	4
y	20.01	10.04	4.25	3	3.58	5	9.67	16.5

(1)

(b) On the grid, draw the graph of $y = x^2 + \frac{2}{x}$ for $0.1 \leqslant x \leqslant 4$



(2)

(c) Use your graph to find estimates for the solutions of $x^2 + \frac{2}{x} = 14$ in the interval $0.1 \leqslant x \leqslant 4$

Give your estimates correct to 1 decimal place.

$x = 0.1$ AND $x = 3.6$

A garage tests cars for faults.
There are three types of fault – braking, steering and lighting.
A car fails the test if it has one or more of these three types of fault.

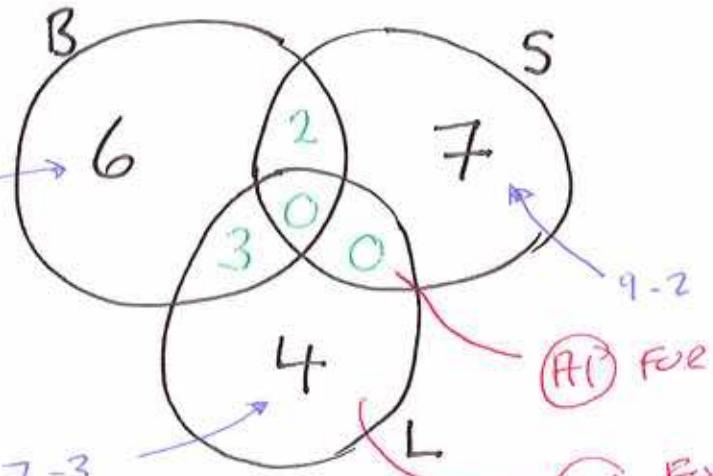
3 SETS

Last week, 11 cars had braking faults — $n(B) = 11$
9 cars had steering faults — $n(S) = 9$
7 cars had lighting faults — $n(L) = 7$
no car had both steering faults and lighting faults — $n(S \cap L) = 0$
2 cars had both braking faults and steering faults — $n(B \cap S) = 2$
3 cars had both braking faults and lighting faults. — $n(B \cap L) = 3$

By drawing a Venn Diagram, or otherwise, find the number of cars which failed the test last week.

WORKING

FILL IN INTERSECTIONS FIRST!



11 − (2+3)

9 − 2

7 − 3

A1 FOR 2, 3, 0, 0

A1 FOR 4, 6, 7

NOTE THE ASSUMPTION THAT NO CAR HAD ALL THREE FAULTS

FROM THE DIAGRAM,

NUMBER OF CARS WITH A FAULT (OR MORE)

AND SO FAILED THE TEST = 6+7+4+2+3

= 22 A1

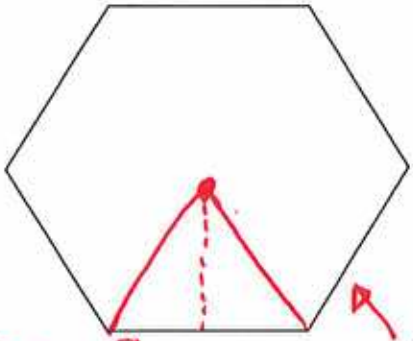


Diagram **NOT** accurately drawn

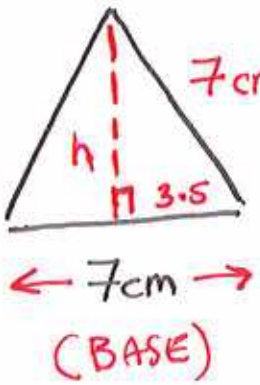
EQUILATERAL TRIANGLE

LENGTH OF ONE SIDE = $\frac{42}{6}$ = 7cm (M1)

The diagram shows a regular hexagon.
The perimeter of the hexagon is 42 cm.

Calculate the area of the hexagon.
Give your answer correct to 3 significant figures.

HEXAGON CAN BE MADE FROM 6 EQUILATERAL TRIANGLES



USING PYTHAGORAS

$h^2 = 7^2 - 3.5^2$ (M1)

$= 36.75$

$h = \sqrt{36.75}$

$= 6.06217...$ (A1)

AREA OF SINGLE TRIANGLE

$A = \frac{\text{BASE} \times \text{HEIGHT}}{2}$

$= \frac{7 \times 6.062...}{2}$ (M1)

$= 21.217...$

$\therefore$ WHOLE HEXAGON $= 21.217 \times 6$ (M1) $= 127.3$

$= 127 \text{ cm}^2$ (A1)

(a) Show that $\frac{x^2 + 3x}{2x^2 + 5x - 3}$ can be written as $\frac{x}{kx - 1}$

State the value of k.

$$\frac{x(x+3)}{(2x-1)(x+3)} = \frac{x}{2x-1}$$

M1 [FACTORISING]

A1

$k =$ 2

(2)

(b) $f(x) = \frac{x}{2x - 1}$

Find the inverse function f^{-1} in the form $f^{-1}(x) =$ .....
Show your working clearly.

$$y = \frac{x}{2x-1}$$

$$x = \frac{y}{2y-1}$$ M1

$$(2y-1)x = y$$

$$2xy - x = y$$ M1 [EITHER]

$$2xy - y = x$$

$$y(2x-1) = x$$

$$y = \frac{x}{2x-1}$$

$$\Rightarrow f^{-1}(x) = \frac{x}{2x-1}$$ A1