4H(R)

Pearson Edexcel
International GCSE

EDEXCEL IGCSE

MATHEMATICS A

SOLUTIONS

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4MA0/4HR

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Within these solutions We have indicated where marks **might** be awarded for each question. We have used B marks, M marks and A marks in a similar, but **not identical**, way that the exam board uses these marks within their mark schemes. We have done this for simplicity and convenience. We have sometimes interchanged B marks, M marks and A marks and We have sometimes awarded the marks in different ways to the exam board.

B1 – This is an unconditional accuracy mark (the specific number, word or phrase must be seen. This type of mark cannot be given as a result of 'follow through').

M1 – This is a method mark. We have indicated where method marks might be awarded for the method that is shown. If You use a different method, then the same number of method marks would be awarded but We are not able to indicate for what the marks would be awarded for Your particular method. When appropriate, You should seek clarity and download the relevant examiner mark scheme from the exam board's web site

A1 – These are accuracy marks. Accuracy marks are typically awarded after method marks. If the correct answer is obtained, then You should normally (but not always) expect to be awarded all of the method marks (provided that You have shown Your method) and all of the accuracy marks.

$f = 5p - 4v$

Work out the value of p when $f = -22$ and $v = -5$

$(-22) = 5 \times p - 4 \times (-5)$ M1 [SUBSTITUTION]

$p = \dfrac{-22-20}{5}$ M1 [REARRANGEMENT]

A1

$p =$ -8.4

Here is part of a timetable for the Paris to Montpellier express train service.

Paris	06 07	10 07	12 07	18 07	20 07
Valence	08 22	12 24	14 24	20 24	22 24
Nimes	09 09	13 05	15 05	21 05	23 05
Montpellier	09 37	13 34	15 34	21 34	23 34

The average speed of the 20 07 train from Paris is 224 km/h.

Work out the distance this train travels from Paris to Montpellier.

TIME TAKEN = 23:34 − 20:07

= 3 HOURS 27 MINS (M1)

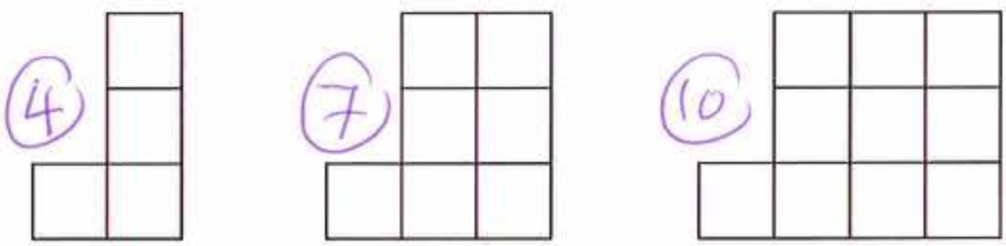
0.45 HOURS!

DISTANCE = SPEED × TIME

= 224 × 3.45 (M1) [MULTIPLYING]

= $\underline{\underline{772.8}}$ km (A1)

Here is a sequence of patterns made from centimetre squares.



Pattern number 1   Pattern number 2   Pattern number 3

(a) Find an expression, in terms of n, for the total number of centimetre squares in Pattern number n.

$4, 7, 10, \ldots$

$+3 \rightarrow 3n \pm ?$

A1 A1

$3n + 1$

(2)

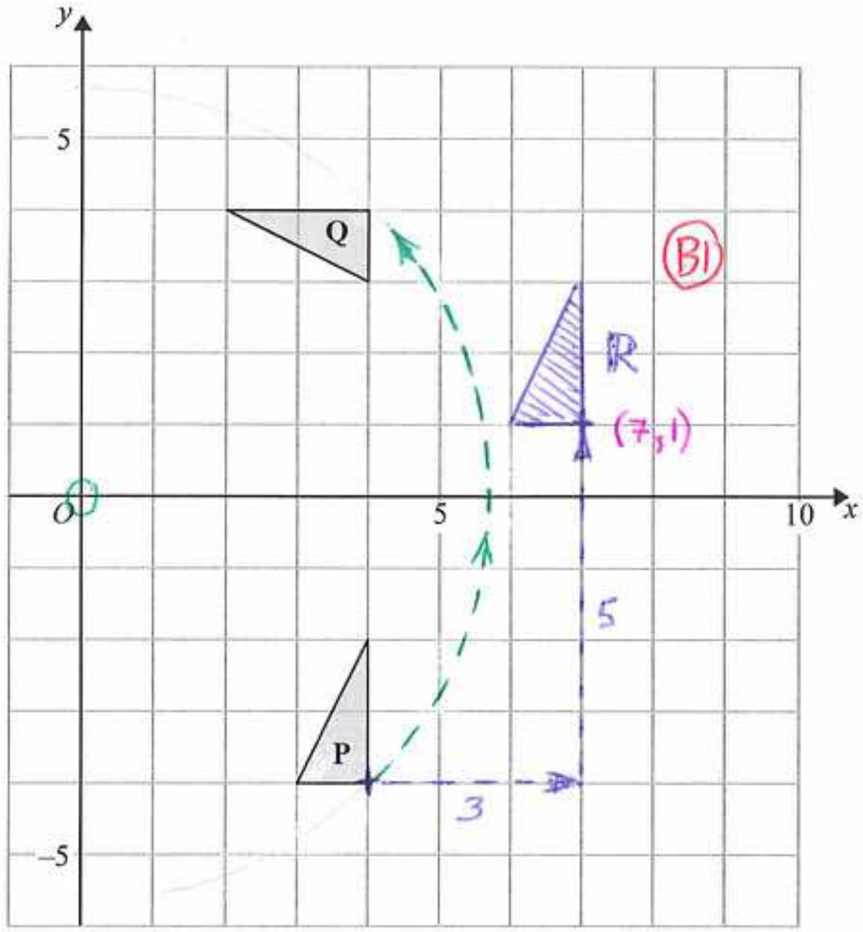
A pattern in this sequence has 88 centimetre squares.

(b) Work out the Pattern number of this pattern.

$3n + 1 = 88$ M1

$3n = 87 \rightarrow n = \frac{87}{3}$

$= \underline{\underline{29}}$ A1



(a) Describe fully the single transformation that maps triangle P onto triangle Q.

ROTATION, B1 90° ANTI-CLOCKWISE B1

CENTRE AT (0,0) B1

(3)

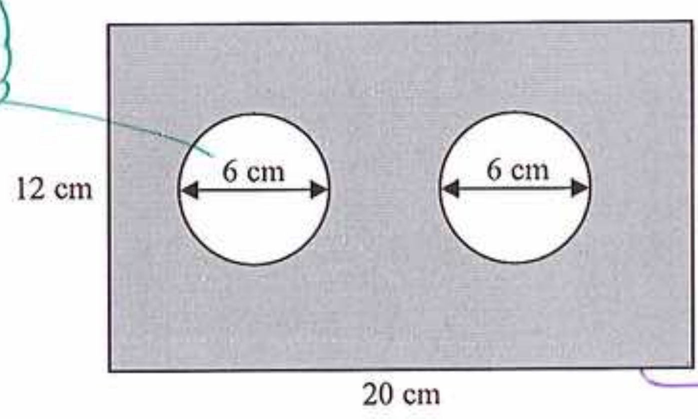
(b) On the grid, translate triangle P 3 squares to the right and 5 squares up. Label the new triangle R.

R

(1)

Diagram **NOT** accurately drawn

USE $A = \pi \times 3^2$



USE $A = 20 \times 12$

The diagram shows a metal plate in the shape of a rectangle.
The rectangle has length 20 cm and width 12 cm.
Two identical circles, each of diameter 6 cm, have been cut out of the plate.

Work out the area of the shaded region of the metal plate.
Give your answer correct to the nearest cm^2.

RECTANGLE

$20 \times 12 = 240$ M1

CIRCLES

$2 \times \pi \times 3^2 = 56.548\ldots$ M1

M1 [SUBTRACT]

SHADED REGION $= 183.451\ldots$

$= 183 \text{ cm}^2$ A1

Kim bought 12 boxes of drinks.
He paid \$15 for each box.
There were 12 drinks in each box.

Kim sold $\frac{3}{4}$ of the drinks for \$1.50 each.

He sold all of the other drinks at a reduced price.

He made an overall profit of 15%.

Work out how much Kim sold each reduced price drink for.

TOTAL COST = 12 × 15 = \$180 B1

NUMBER OF DRINKS = 12 × 12 = 144 B1

SOLD $\frac{3}{4} \times 144$ = 108 DRINKS

INCOME = 108 × 1.50 = \$162 B1

PROFIT WAS 0.15 × 180 = \$27 ← B1 [EITHER]

TOTAL INCOME = 180 + 27 = \$207

LAST DRINKS $\frac{1}{4} \times 144$ = 36 DRINKS

SOLD FOR 207 − 162 = \$45

PRICE = $\frac{45}{36}$ = \$1.25 A1

Reeta has a biased dice.

Each time Reeta rolls the dice, the probability that she will get a six is 0.1

(a) Write down the probability that she will not get a six.

A1

0.9
(1)

Reeta rolls the dice 50 times.

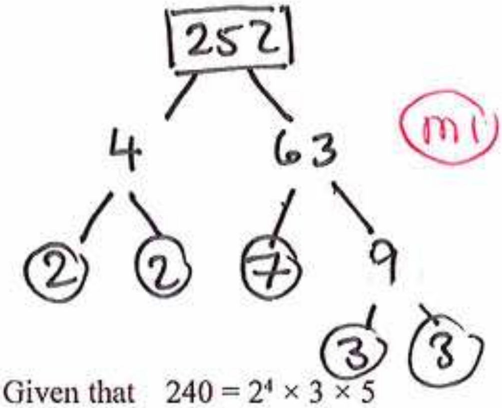
(b) Work out an estimate for the number of times that she will get a six.

50×0.1 M1

A1

5
(2)

(a) Write 252 as a product of its prime factors.



M1

A1

$2^2 \times 3^2 \times 7$

(2)

Given that $240 = 2^4 \times 3 \times 5$

and that $y = 240 \times 252$

(b) write y as a product of powers of its prime factors.

$2^4 \times 3 \times 5 \times 2^2 \times 3^2 \times 7$

A2

$2^6 \times 3^3 \times 5 \times 7$

(2)

The diagram shows a parallelogram $ABCD$.
In the diagram, all the angles are in degrees.

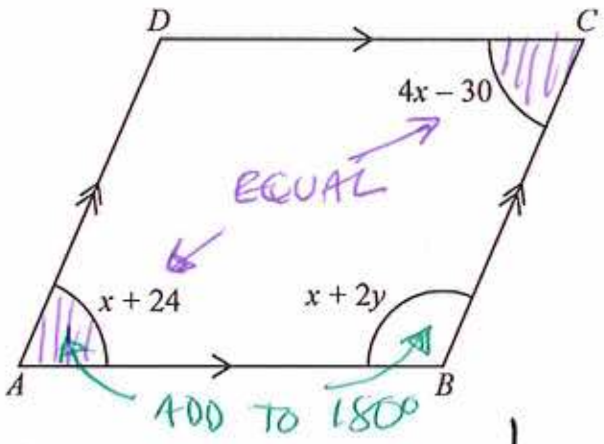


Diagram **NOT** accurately drawn

Work out the value of x and the value of y.

$4x - 30 = x + 24$ (M1)

$4x - x = 24 + 30$

$3x = 54$

$x = \underline{18°}$ (A1)

(M1)

$x + 24 + x + 2y = 180$

$2x + 24 + 2y = 180$

$2 \times 18 + 24 + 2y = 180$

$60 + 2y = 180$

$2y = 120$

$y = \underline{60°}$ (A1)

Mortar mix is made by mixing cement, sand and quicklime in the ratio 1 : 2 : 3

(a) Work out the volume of sand needed to make 2.1 m³ of mortar mix.

C : S : Q TOTAL

1 : 2 : 3 6

↓

2.1

$\frac{2.1}{6} = 0.35$ (M1)

$\therefore$ SAND $= 2 \times 0.35$

$= \underline{\underline{0.7\text{m}^3}}$ (A1)

Julie has 0.75 m³ of quicklime.

She has plenty of sand and cement.

(b) Work out the greatest volume of mortar mix she could make.

C : S : Q TOTAL

1 : 2 : 3 6

↓

0.75

$\frac{0.75}{3} = 0.25$ (M1)

$\therefore$ TOTAL $= 6 \times 0.25$

$= \underline{\underline{1.5\text{m}^3}}$ (A1)

a, b, c and d are four integers.
Their mean is 8 → TOTAL = 4 × 8 = $\underline{\underline{32}}$
Their mode is 7 $\boxed{\text{2ND}}$
Their median is $\boxed{7.5}$ $\boxed{\text{1ST}}$

(a) Find the value of the largest of the four integers.

$\boxed{7}$ $\boxed{7}$ $\boxed{8}$ $\boxed{10}$ — $\boxed{\text{4TH}}$ = 32 − (7 + 7 + 8)

↑ 7.5 $\boxed{\text{3RD}}$

10
(2)

(b) Find the mean value of the numbers $(2a - 3)$, $(2b - 3)$, $(2c - 3)$ and $(2d - 3)$.

ALL VALUES ARE DOUBLED
THEN 3 IS SUBTRACTED

[SAME HAPPENS TO THE MEAN]

SO

NEW MEAN = 2 × 8 − 3
= $\underline{\underline{13}}$

(a) Factorise $2t^2 - 7t + 3$

$(2t-1)(t-3)$

A1 A1

$(2t-1)(t-3)$

(2)

(b) Rearrange the formula $y = a - bx^2$ to make x the subject.

$bx^2 + y = a$

$bx^2 = a - y$ M1 EITHER

$x^2 = \dfrac{a-y}{b}$ M1 [DIVIDE]

$x = \pm\sqrt{\dfrac{a-y}{b}}$ M1 [SQUARE ROOT]

Here are the points that Carmelo scored in his last 11 basketball games.

23 20 14 23 17 24 24 18 16 22 21

(a) Find the interquartile range of these points.

14 16 17 18 20 21 22 23 23 24 24

Q_1 MEDIAN Q_3

$Q_1 = \frac{11+1}{4}$ (M1)

$= 3^{RD}$ VALUE [$Q_3 = 9^{TH}$ VALUE]

$IQR = Q_3 - Q_1$

$= 23 - 17$ (M1)

$= \underline{\underline{6}}$ (A1)

Kobe also plays basketball.
The median number of points Kobe has scored in his games is 18.5
The interquartile range of these points is 10

(b) Which of Carmelo or Kobe is the more consistent points scorer?
Give a reason for your answer.

LOWEST INTERQUARTILE RANGE INDICATES THE MOST CONSISTENT SCORES. THEREFORE CARMELO WAS MORE CONSISTENT (A1)

(1)

Rob is making a scale model of the Solar System on the school field.
He wants the distance from the Sun to Jupiter to be 8 metres on his scale model.

The real distance from the Sun to Jupiter is 7.8×10^8 kilometres.

(a) Find the scale of the model.
Give your answer in the form $1 : n$, where n is written in standard form.

m : R

8m : 7.8×10^8 km

8 : 7.8×10^{11} M1 [BOTH IN METRES]

1 : 0.975×10^{11} M1

[ANY 1:n RATIO]

A1

1: 9.75×10^{10}

(3)

Rob wants to put the position of a space probe on the scale model.
The real distance of the space probe from the Sun is 1.9×10^{10} kilometres, correct to 2 significant figures.

(b) Work out the maximum distance of the space probe from the Sun on the scale model.
Give your answer in metres.

UPPER BOUND FOR DISTANCE $= 1.95 \times 10^{10}$ km

$= 1.95 \times 10^{13}$ m

M1

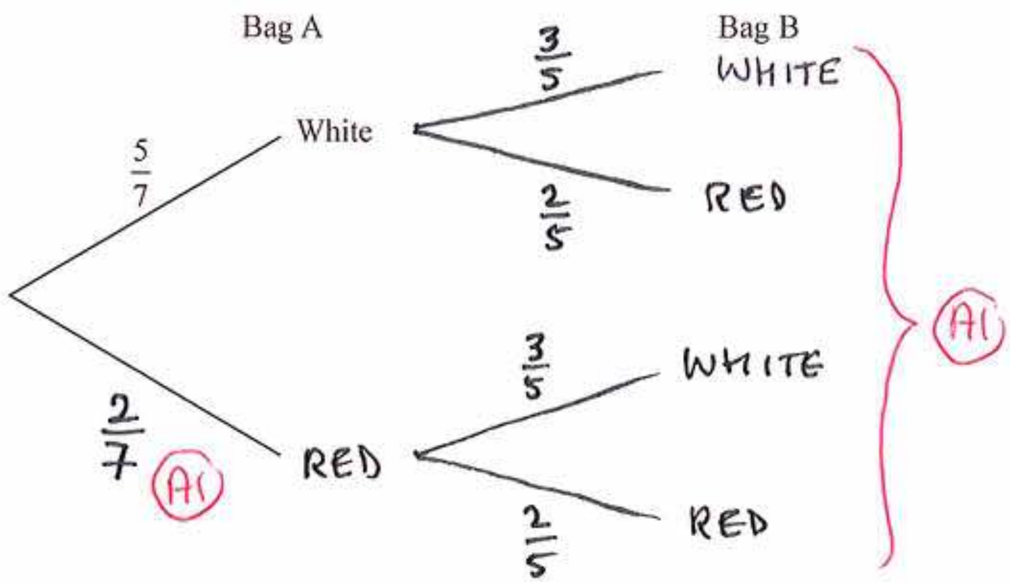
∴ SCALE MODEL DISTANCE $= \dfrac{1.95 \times 10^{13}}{9.75 \times 10^{10}}$ M1 [DIVIDING]

$=$ 200 m A1

Maria has two bags.
In bag A, there are 5 white counters and 2 red counters. → $\frac{5}{7}$ AND $\frac{2}{7}$
In bag B, there are 3 white counters and 2 red counters. → $\frac{3}{5}$ AND $\frac{2}{5}$

Maria is going to take at random one counter from bag A and one counter from bag B.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both counters will be white.

$$P(WW) = \frac{5}{7} \times \frac{3}{5} \quad \text{M1}$$

$$= \frac{15}{35}$$

A1

$\frac{3}{7}$

(2)

(c) Work out the probability that exactly one of the counters will be white.

$$P(WR) = \frac{5}{7} \times \frac{2}{5} = \frac{10}{35}$$

M1 [TWO POSSIBLITIES]

$$P(RW) = \frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$$

TOTAL = $\frac{16}{35}$ A1

M1 [ADDING]

Here is a hexagon.

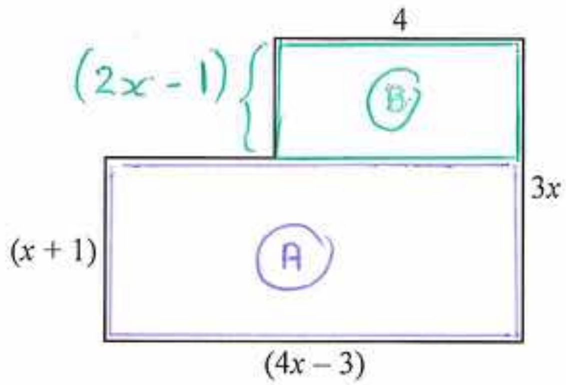


Diagram **NOT** accurately drawn

In the diagram, all the measurements are in centimetres.
All the corners are right angles.

The area of the hexagon is 40 cm² $\rightarrow$ A + B = 40

(a) Show that $4x^2 + 9x - 47 = 0$

$(4x-3)(x+1) + 4(2x-1) = 40$ B1 [EQUATION]

$4x^2 + 4x - 3x - 3 + 8x - 4 = 40$ M1 [EXPANDING BRACKETS]

$\Rightarrow 4x^2 + 9x - 7 = 40$ M1 [SIMPLIFYING]

$\Rightarrow 4x^2 + 9x - 47 = 0$

(b) Solve $4x^2 + 9x - 47 = 0$

Show your working clearly.
Give your solutions correct to 3 significant figures. $\rightarrow$ USE $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 4,\ b = 9,\ c = -47$

$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(4)(-47)}}{2(4)}$ M1 [CORRECT SUBSTITUTIONS]

$= \frac{-9 \pm \sqrt{81 + 752}}{8}$ M1 [SIMPLIFYING]

-4.73 $\quad$ 2.48 A1 [BOTH ANSWERS]

(c) Find the length of the longest side of the hexagon.
Give your answer correct to 3 significant figures.

LONGEST SIDE IS EITHER $(4x-3)$ OR $3x$

$3 \times (2.4827\ldots) = 7.4481\ldots$ m1

$4 \times (2.4827\ldots) - 3 = 6.93086\ldots$

LONGEST IS 7.45 cm A1

(a) Simplify $(16x^4y^2)^{\frac{1}{2}}$

$16^{\frac{1}{2}} = \sqrt{16} = 4$ B1

$x^{4 \times \frac{1}{2}} = x^2$

$y^{2 \times \frac{1}{2}} = y$

B1

$4x^2y$

(2)

(b) Simplify fully $\frac{2x^2 - 8}{4x^2 - 8x}$

$$= \frac{2(x^2 - 4)}{4x(x-2)}$$

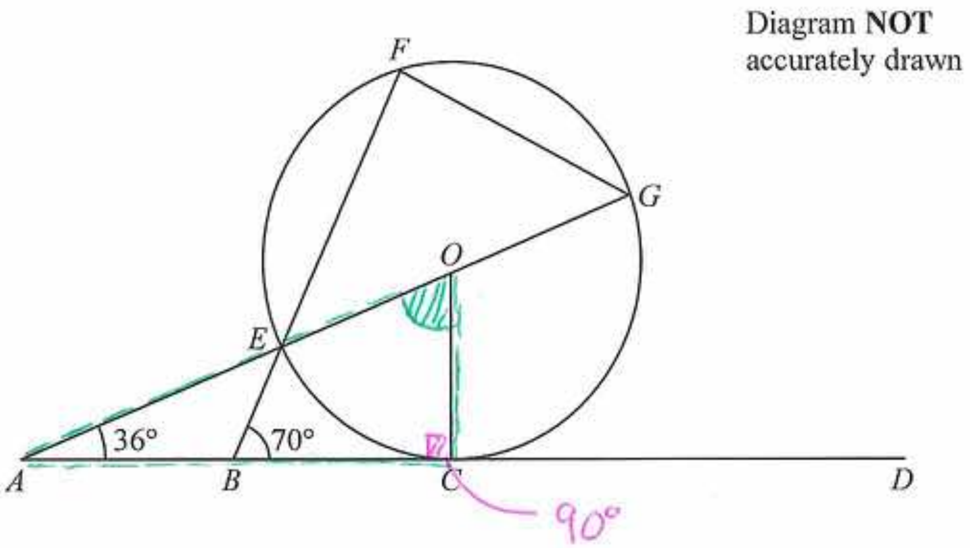
$$= \frac{\cancel{2}(x+2)\cancel{(x-2)}}{2\cancel{4}x\cancel{(x-2)}}$$

M1 FACTORISE

M1 FACTORISE

$\frac{x+2}{2x}$ A1

(3)



ABCD is the tangent at *C* to a circle, centre *O*.
E, *F* and *G* are points on the circle.
AEOG and *BEF* are straight lines.

Angle $BAE = 36°$
Angle $EBC = 70°$

(a) (i) Find the size of angle *AOC*.

$180 - (90 + 36)$

A1

54 °

(ii) Give reasons for your answer.

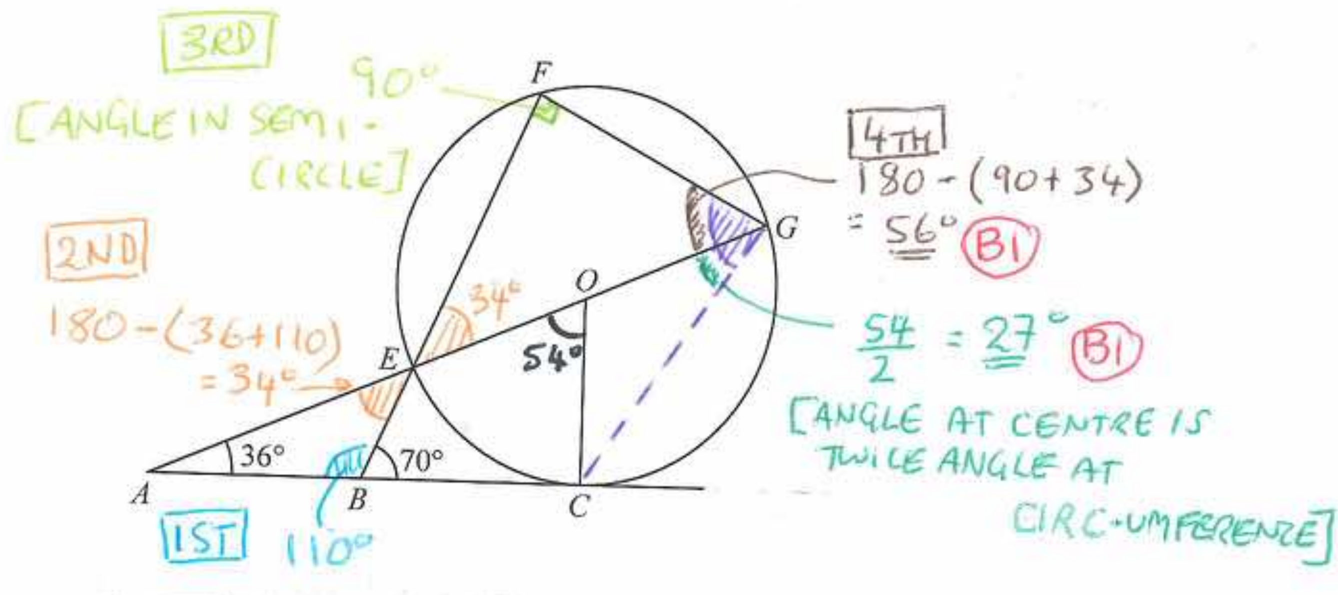
① ∠ ACO = 90° BECAUSE A TANGENT AND A RADIUS MAKE 90° B1

(2)

② ACO IS A TRIANGLE

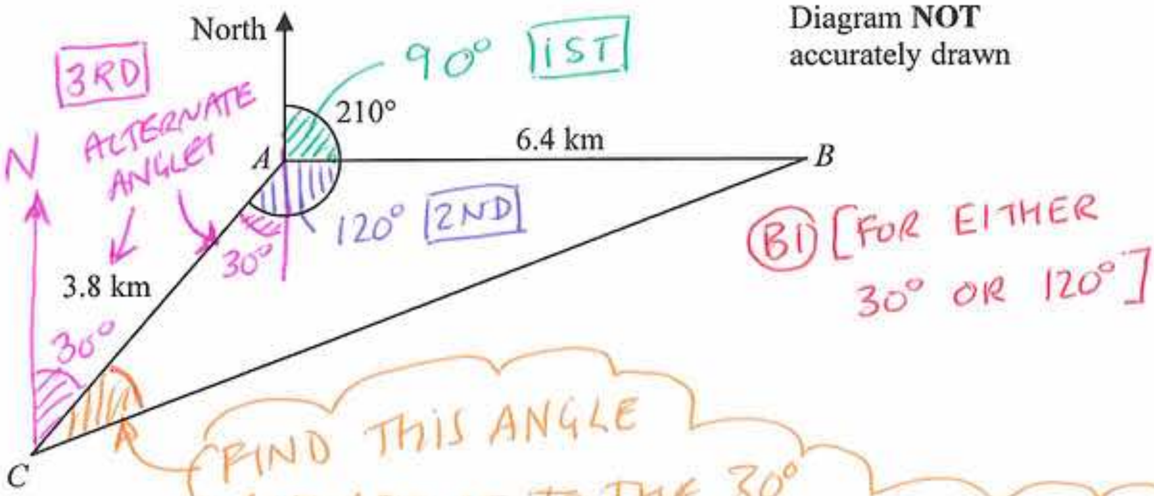
[NO MARK]

(b) Find the size of angle CGF.



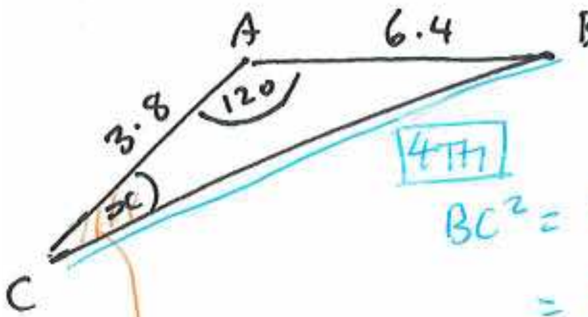
$$CGF = 56 + 27$$

$$= \underline{\underline{83°}} \quad \text{A1}$$



A, *B* and *C* are 3 villages.
B is 6.4 km due east of *A*.
C is 3.8 km from *A* on a bearing of 210°

Calculate the bearing of *B* from *C*.
Give your answer correct to the nearest degree.
Show your working clearly.



4TH

M1

$BC^2 = 3.8^2 + 6.4^2 - 2 \times 3.8 \times 6.4 \times \cos 120$

$= 79.72$

$\Rightarrow BC = 8.9286$ A1

5TH

$\dfrac{\sin x}{6.4} = \dfrac{\sin 120}{8.9286}$ M1 $\Rightarrow \sin x = 6.4 \times \dfrac{\sin 120}{8.9286}$

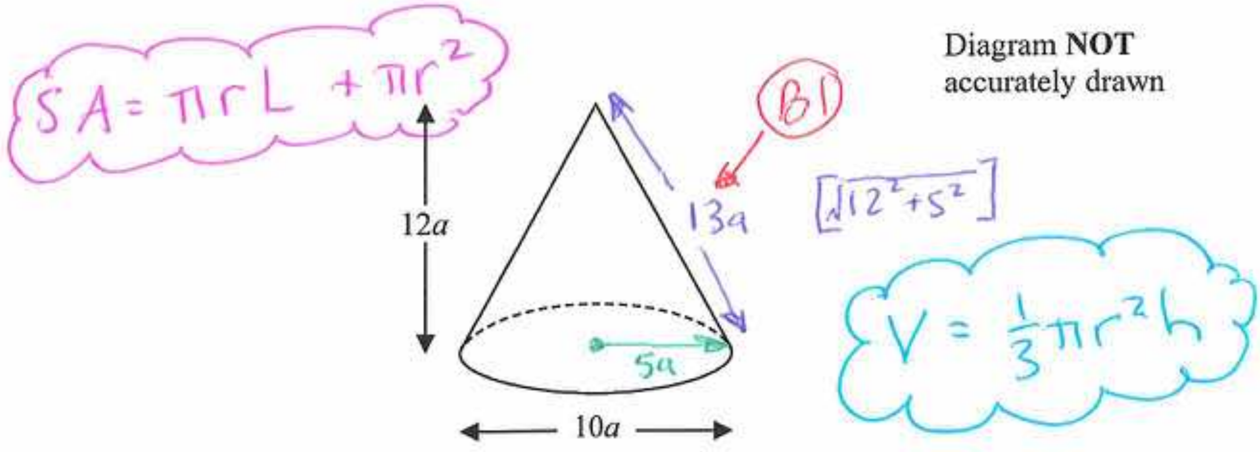
$\Rightarrow x = 38.4°$ A1

6TH!

BEARING $= 30 + 38.4$

$= 068°$ A1

The diagram shows a solid cone.



The diameter of the base of the cone is $10a$ cm.
The height of the cone is $12a$ cm.

The total surface area of the cone is 360π cm^2 1ST
The volume of the cone is $k\pi$ cm^3, where k is an integer. 2ND

Find the value of k.

1ST

$$\pi \times 5a \times 13a + \pi \times (5a)^2 = 360\pi \quad \text{M1 [EQUATION]}$$

$$\Rightarrow 65a^2\pi + 25a^2\pi = 360\pi$$

$$90a^2\cancel{\pi} = 360\cancel{\pi}$$

$$a^2 = \frac{360}{90}$$

$$a = 2 \quad \text{A1}$$

$$\frac{1}{3}\pi \times (5a)^2 \times 12a = k\pi \quad \text{M1 [EQUATION]}$$

$$\Rightarrow \frac{1}{\cancel{3}}\pi \times 25a^2 \times \overset{4}{\cancel{12}}a = k\pi$$

$$\Rightarrow 100a^3\pi = k\pi$$

$$100a^3 = k$$

$$k = 100 \times 2^3$$

$$= 800 \quad \text{A1}$$

M1 [SUBSTITUTION]

(a) Show that

$$(a^2 + 1)(c^2 + 1) = (ac - 1)^2 + (a + c)^2$$

LHS

F O I L

$(a^2+1)(c^2+1) = a^2c^2 + a^2 + c^2 + 1$ M1 [EXPANSION OF LHS]

RHS

$\underbrace{(ac-1)^2} + \underbrace{(a+c)^2} = \underbrace{a^2c^2 - 2ac + 1} + \underbrace{a^2 + 2ac + c^2}$ M1

$= a^2c^2 + 1 + a^2 + c^2$

$= a^2c^2 + a^2 + c^2 + 1 = \underline{\underline{LHS}}$! M1

(b) By finding suitable values of a and c, use part (a) to write 650065 as the sum of two square numbers.

HINT: 65 MUST BE A FACTOR!

$650065 = 65 \times 10001$ B1

$= (64+1) \times (10\,000+1)$

$= (8^2+1) \times (100^2+1)$

$\therefore \; a = 8, \; c = 100$ B1 [BOTH]

TAKING RHS

$(ac-1)^2 + (a+c)^2 = (8 \times 100 - 1)^2 + (8 + 100)^2$

$= \underline{\underline{799^2 + 108^2}}$ A1

[ACCEPT 638401 + 11664]