## CAMBRIDGE INTERNATIONAL EXAMINATIONS International General Certificate of Secondary Education

# MATHEMATICS 

## Paper 4

Additional Materials: Answer Booklet/Paper Electronic calculator Geometric instruments Graph paper (2 sheets) Mathematical tables (optional) Tracing paper (optional)

May/June 2003

## 2 hours 30 minutes

## READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer.
Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142.

1 Tickets for the theatre cost either $\$ 10$ or $\$ 16$.
(a) Calculate the total cost of 197 tickets at $\$ 10$ each and 95 tickets at $\$ 16$ each.
(b) On Monday, 157 tickets at $\$ 10$ and $n$ tickets at $\$ 16$ were sold. The total cost was $\$ 4018$. Calculate the value of $n$.
(c) On Tuesday, 319 tickets were sold altogether. The total cost was $\$ 3784$.

Using $x$ for the number of $\$ 10$ tickets sold and $y$ for the number of $\$ 16$ tickets sold, write down two equations in $x$ and $y$.

Solve your equations to find the number of $\$ 10$ tickets and the number of $\$ 16$ tickets sold.
(d) On Wednesday, the cost of a $\$ 16$ ticket was reduced by $15 \%$. Calculate this new reduced cost.
(e) The $\$ 10$ ticket costs $25 \%$ more than it did last year. Calculate the cost last year.

2


In quadrilateral $A B C D, A B=77 \mathrm{~m}, B C=120 \mathrm{~m}, C D=60 \mathrm{~m}$ and diagonal $A C=55 \mathrm{~m}$.
Angle $C A D=45^{\circ}$, angle $B A C=x^{\circ}$ and angle $A D C=y^{\circ}$.
(a) Calculate the value of $x$.
(b) Calculate the value of $y$.
(c) The bearing of $D$ from $A$ is $090^{\circ}$.

Find the bearing of
(i) $A$ from $C$,
(ii) $B$ from $A$.

3 There are 2 sets of road signals on the direct 12 kilometre route from Acity to Beetown.
The signals say either "GO" or "STOP".
The probabilities that the signals are "GO" when a car arrives are shown in the tree diagram.
(a) Copy and complete the tree diagram for a car driver travelling along this route.

(b) Find the probability that a car driver
(i) finds both signals are "GO",
(ii) finds exactly one of the two signals is "GO",
(iii) does not find two "STOP" signals.
(c) With no stops, Damon completes the 12 kilometre journey at an average speed of 40 kilometres per hour.
(i) Find the time taken in minutes for this journey.
(ii) When Damon has to stop at a signal it adds 3 minutes to this journey time.

Calculate his average speed, in kilometres per hour, if he stops at both road signals.
(d) Elsa takes a different route from Acity to Beetown.

This route is 15 kilometres and there are no road signals.
Elsa's average speed for this journey is 40 kilometres per hour.
Find
(i) the time taken in minutes for this journey,
(ii) the probability that Damon takes more time than this on his 12 kilometre journey.

4 Answer the whole of this question on a sheet of graph paper.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | -8 | 4.5 | 8 | 5.5 | 0 | -5.5 | -8 | -4.5 | 8 |

(a) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 4 units on the $y$-axis, draw axes for $-4 \leqslant x \leqslant 4$ and $-8 \leqslant y \leqslant 8$.
Draw the curve $y=\mathrm{f}(x)$ using the table of values given above.
(b) Use your graph to solve the equation $\mathrm{f}(x)=0$.
(c) On the same grid, draw $y=\mathrm{g}(x)$ for $-4 \leqslant x \leqslant 4$, where $\mathrm{g}(x)=x+1$.
(d) Write down the value of
(i) $\mathrm{g}(1)$,
(ii) $\mathrm{fg}(1)$,
(iii) $\mathrm{g}^{-1}(4)$,
(iv) the positive solution of $\mathrm{f}(x)=\mathrm{g}(x)$.
(e) Draw the tangent to $y=\mathrm{f}(x)$ at $x=3$. Use it to calculate an estimate of the gradient of the curve at this point.

5 (a) Calculate the area of an equilateral triangle with sides 10 cm .
(b) Calculate the radius of a circle with circumference 10 cm .
(c)



Diagram 2


Diagram 3

The diagrams represent the nets of 3 solids. Each straight line is 10 cm long. Each circle has circumference 10 cm . The arc length in Diagram 3 is 10 cm .
(i) Name the solid whose net is Diagram 1. Calculate its surface area.
(ii) Name the solid whose net is Diagram 2. Calculate its volume.
(iii) Name the solid whose net is Diagram 3. Calculate its perpendicular height.


A rectangular-based open box has external dimensions of $2 x \mathrm{~cm},(x+4) \mathrm{cm}$ and $(x+1) \mathrm{cm}$.
(a) (i) Write down the volume of a cuboid with these dimensions.
(ii) Expand and simplify your answer.
(b) The box is made from wood 1 cm thick.
(i) Write down the internal dimensions of the box in terms of $x$.
(ii) Find the volume of the inside of the box and show that the volume of the wood is $8 x^{2}+12 x$ cubic centimetres.
(c) The volume of the wood is $1980 \mathrm{~cm}^{3}$.
(i) Show that $2 x^{2}+3 x-495=0$ and solve this equation.
(ii) Write down the external dimensions of the box.


A star is made up of a regular hexagon, centre $X$, surrounded by 6 equilateral triangles.
$\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O B}=\mathbf{b}$.
(a) Write the following vectors in terms of $\mathbf{a}$ and/or $\mathbf{b}$, giving your answers in their simplest form.
(i) $\overrightarrow{O S}$,
(ii) $\overrightarrow{A B}$,
(iii) $\overrightarrow{C D}$,
(iv) $\overrightarrow{O R}$,
(v) $\overrightarrow{C F}$.
(b) When $|\mathbf{a}|=5$, write down the value of
(i) $|\mathrm{b}|$,
(ii) $|\mathbf{a}-\mathbf{b}|$.
(c) Describe fully a single transformation which maps
(i) triangle $O B A$ onto triangle $O Q S$,
(ii) triangle $O B A$ onto triangle $R D E$, with $O$ mapped onto $R$ and $B$ mapped onto $D$.
(d) (i) How many lines of symmetry does the star have?
(ii) When triangle $O Q S$ is rotated clockwise about $X$, it lies on triangle $P R T$, with $O$ on $P$. Write down the angle of rotation.

## 8 Answer the whole of this question on a sheet of graph paper.

In a survey, 200 shoppers were asked how much they had just spent in a supermarket.
The results are shown in the table.

| Amount $(\$ x)$ | $0<x \leqslant 20$ | $20<x \leqslant 40$ | $40<x \leqslant 60$ | $60<x \leqslant 80$ | $80<x \leqslant 100$ | $100<x \leqslant 140$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of shoppers | 10 | 32 | 48 | 54 | 36 | 20 |

(a) (i) Write down the modal class.
(ii) Calculate an estimate of the mean amount, giving your answer correct to 2 decimal places.
(b) (i) Make a cumulative frequency table for these 200 shoppers.
(ii) Using a scale of 2 cm to represent $\$ 20$ on the horizontal axis and 2 cm to represent 20 shoppers on the vertical axis, draw a cumulative frequency diagram for this data. [4]
(c) Use your cumulative frequency diagram to find
(i) the median amount,
(ii) the upper quartile,
(iii) the interquartile range,
(iv) how many shoppers spent at least $\$ 75$.


Diagram 1


Diagram 2


Diagram 3


Diagram 4

Diagram 1 shows a triangle with its base divided in the ratio $1: 3$.
Diagram 2 shows a parallelogram with its base divided in the ratio $1: 3$.
Diagram 3 shows a kite with a diagonal divided in the ratio $1: 3$.
Diagram 4 shows two congruent triangles and a trapezium each of height 1 unit.
For each of the four diagrams, write down the percentage of the total area which is shaded. [7]
(b)


Diagram 6


Diagram 7

Diagram 5 shows a semicircle, centre $O$.
Diagram 6 shows two circles with radii 1 unit and 5 units.
Diagram 7 shows two sectors, centre $O$, with radii 2 units and 3 units.
For each of diagrams 5, 6 and 7, write down the fraction of the total area which is shaded. [6]

