## CAMBRIDGE INTERNATIONAL EXAMINATIONS International General Certificate of Secondary Education

# MATHEMATICS 

## Paper 4

October/November 2003
2 hours 30 minutes

Additional Materials: Answer Booklet/Paper<br>Electronic calculator<br>Geometrical instruments<br>Graph paper (2 sheets)<br>Mathematical tables (optional)<br>Tracing paper (optional)

## READ THESE INSTRUCTIONS FIRST

Write your answers and working on the separate Answer Booklet/Paper provided.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer.
Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
The total of the marks for this paper is 130.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Answers in degrees should be given to one decimal place.
For $\pi$, use either your calculator value or 3.142.

1 A train starts its journey with 240 passengers.
144 of the passengers are adults and the rest are children.
(a) Write the ratio Adults: Children in its lowest terms.
(b) At the first stop, $37 \frac{1}{2} \%$ of the adults and $\frac{1}{3}$ of the children get off the train. 20 adults and $x$ children get onto the train.
The total number of passengers on the train is now 200.
(i) How many children got off the train?
(ii) How many adults got off the train?
(iii) How many adult passengers are on the train as it sets off again?
(iv) What is the value of $x$ ?
(c) After a second stop, there are 300 passengers on the train and the ratio

Men:Women: Children is 6:5:4.
Calculate the number of children now on the train.
(d) On Tuesday the train journey took 7 hours and 20 minutes and began at 1353 .
(i) At what time did the train journey end?
(ii) Tuesday's time of 7 hours 20 minutes was $10 \%$ more than Monday's journey time. How many minutes longer was Tuesday's journey?

2 (a) The surface area of a person's body, $A$ square metres, is given by the formula

$$
A=\sqrt{\frac{h m}{3600}}
$$

where $h$ is the height in centimetres and $m$ is the mass in kilograms.
(i) Dolores is 167 cm high and has a mass of 70 kg . Calculate the surface area of her body.
(ii) Erik has a mass of 80 kg . Find his height if $A=1.99$.
(iii) Make $h$ the subject of the formula.
(b) Factorise
(i) $x^{2}-16$,
(ii) $x^{2}-16 x$,
(iii) $x^{2}-9 x+8$.
(c) Erik runs a race at an average speed of $x \mathrm{~m} / \mathrm{s}$.

His time is $(3 x-9)$ seconds and the race distance is $\left(2 x^{2}-8\right)$ metres.
(i) Write down an equation in $x$ and show that it simplifies to

$$
\begin{equation*}
x^{2}-9 x+8=0 \tag{2}
\end{equation*}
$$

(ii) Solve $x^{2}-9 x+8=0$.
(iii) Write down Erik's time and the race distance.


Felipe $(F)$ stands 17 metres from a bridge $(B)$ and 32 metres from a tree $(T)$.
The points $F, B$ and $T$ are on level ground and angle $B F T=40^{\circ}$.
(a) Calculate
(i) the distance $B T$,
(ii) the angle $B T F$.
(b) The bearing of $B$ from $F$ is $085^{\circ}$. Find the bearing of
(i) $T$ from $F$,
(ii) $F$ from $T$,
(iii) $B$ from $T$.
(c) The top of the tree is 30 metres vertically above $T$.

Calculate the angle of elevation of the top of the tree from $F$.

4 Answer the whole of this question on a sheet of graph paper.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(t)$ | 0 | 25 | 37.5 | 43.8 | 46.9 | 48.4 | 49.2 | 49.6 |

(a) Using a scale of 2 cm to represent 1 unit on the horizontal $t$-axis and 2 cm to represent 10 units on the $y$-axis, draw axes for $0 \leqslant t \leqslant 7$ and $0 \leqslant y \leqslant 60$.
Draw the graph of the curve $y=\mathrm{f}(t)$ using the table of values above.
(b) $\mathrm{f}(t)=50\left(1-2^{-t}\right)$.
(i) Calculate the value of $f(8)$ and the value of $f(9)$.
(ii) Estimate the value of $\mathrm{f}(t)$ when $t$ is large.
(c) (i) Draw the tangent to $y=\mathrm{f}(t)$ at $t=2$ and use it to calculate an estimate of the gradient of the curve at this point.
(ii) The function $\mathrm{f}(t)$ represents the speed of a particle at time $t$. Write down what quantity the gradient gives.
(d) (i) On the same grid, draw $y=\mathrm{g}(t)$ where $\mathrm{g}(t)=6 t+10$, for $0 \leqslant t \leqslant 7$.
(ii) Write down the range of values for $t$ where $\mathrm{f}(t)>\mathrm{g}(t)$.
(iii) The function $\mathrm{g}(t)$ represents the speed of a second particle at time $t$.

State whether the first or second particle travels the greater distance for $0 \leqslant t \leqslant 7$.
You must give a reason for your answer.


Adam writes his name on four red cards and Daniel writes his name on six white cards.
(a) One of the ten cards is chosen at random. Find the probability that
(i) the letter on the card is $\mathbf{D}$,
(ii) the card is red,
(iii) the card is red or the letter on the card is $\mathbf{D}$,
(iv) the card is red and the letter on the card is $\mathbf{D}$,
(v) the card is red and the letter on the card is $\mathbf{N}$.
(b) Adam chooses a card at random and then Daniel chooses one of the remaining 9 cards at random.
Giving your answers as fractions, find the probability that the letters on the two cards are
(i) both D ,
(ii) both A ,
(iii) the same,
(iv) different.

6 (a) Calculate the volume of a cylinder with radius 30 cm and height 50 cm .
(b)


NOT TO SCALE

A cylindrical tank, radius 30 cm and length 50 cm , lies on its side.
It is partially filled with water.
The shaded segment $A X B Y$ in the diagram shows the cross-section of the water.
The greatest depth, $X Y$, is 12 cm .
$O A=O B=30 \mathrm{~cm}$.
(i) Write down the length of $O X$.
(ii) Calculate the angle $A O B$ correct to two decimal places, showing all your working.
(c) Using angle $A O B=106.3^{\circ}$, find
(i) the area of the sector $A O B Y$,
(ii) the area of triangle $A O B$,
(iii) the area of the shaded segment $A X B Y$.
(d) Calculate the volume of water in the cylinder, giving your answer
(i) in cubic centimetres,
(ii) in litres.
(e) How many more litres must be added to make the tank half full?


Use one of the letters $A, B, C, D, E$ or $F$ to answer the following questions.
(i) Which triangle is $T$ mapped onto by a translation? Write down the translation vector.
(ii) Which triangle is $T$ mapped onto by a reflection? Write down the equation of the mirror line.
(iii) Which triangle is $T$ mapped onto by a rotation? Write down the coordinates of the centre of rotation.
(iv) Which triangle is $T$ mapped onto by a stretch with the $x$-axis invariant?

Write down the scale factor of the stretch.
(v) $\mathbf{M}=\left(\begin{array}{ll}1 & 4 \\ 0 & 1\end{array}\right)$. Which triangle is $T$ mapped onto by $\mathbf{M}$ ?

Write down the name of this transformation.
(b) $\mathbf{P}=\left(\begin{array}{ll}1 & 3 \\ 5 & 7\end{array}\right), \quad \mathbf{Q}=\left(\begin{array}{ll}-1 & -2\end{array}\right), \quad \mathbf{R}=\left(\begin{array}{lll}1 & 2 & 3\end{array}\right), \quad \mathbf{S}=\left(\begin{array}{r}-1 \\ 2 \\ 3\end{array}\right)$.

Only some of the following matrix operations are possible with matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ above.
PQ,
QP,
$\mathbf{P}+\mathbf{Q}$,
PR,
RS

Write down and calculate each matrix operation that is possible.

8 Answer the whole of this question on a sheet of graph paper.
120 passengers on an aircraft had their baggage weighed. The results are shown in the table.

| Mass of baggage $(M \mathrm{~kg})$ | $0<M \leqslant 10$ | $10<M \leqslant 15$ | $15<M \leqslant 20$ | $20<M \leqslant 25$ | $25<M \leqslant 40$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of passengers | 12 | 32 | 28 | 24 | 24 |

(a) (i) Write down the modal class.
(ii) Calculate an estimate of the mean mass of baggage for the 120 passengers. Show all your working.
(iii) Sophia draws a pie chart to show the data.

What angle should she have in the $0<M \leqslant 10$ sector?
(b) Using a scale of 2 cm to represent 5 kg , draw a horizontal axis for $0<M \leqslant 40$.

Using an area scale of $1 \mathrm{~cm}^{2}$ to represent 1 passenger, draw a histogram for this data.

9 In each of the diagrams below, triangle $A B C$ is an isosceles right-angled triangle.
$A B=A C=6 \mathrm{~cm}$.
A straight line or a circular arc divides the triangle into two parts, one of which is shaded.


Diagram 1


Diagram 2


Diagram 3


Diagram 4


Diagram 5
(a) Which diagram has a shaded region showing all the points in the triangle which are
(i) closer to $B C$ than to $B A$,
(ii) more than 3 cm from $A$,
(iii) closer to $C$ than to $A$ ?
(b) For each of the five diagrams, calculate the shaded area.

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