## MATHEMATICS

Paper 4 (Extended)

Additional Materials:

Answer Booklet/Paper Geometrical instruments Mathematical tables (optional)

0580/04, 0581/04
May/June 2008
2 hours 30 minutes
Electronic calculator Graph paper (1 sheet) Tracing paper (optional)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all questions.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer.
Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For $\pi$ use either your calculator value or 3.142 .
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

## DO NOT WRITE YOUR ANSWERS ON THIS QUESTION PAPER.

## WRITE ALL YOUR WORKING AND ANSWERS ON THE SEPARATE ANSWER BOOK OR PAPER PROVIDED.

1 Vreni took part in a charity walk.
She walked a distance of 20 kilometres.
(a) She raised money at a rate of $\$ 12.50$ for each kilometre.
(i) How much money did she raise by walking the 20 kilometres?
(ii) The money she raised in part (a)(i) was $\frac{5}{52}$ of the total money raised.

Work out the total money raised.
(iii) In the previous year the total money raised was $\$ 2450$.

Calculate the percentage increase on the previous year's total.
(b) Part of the 20 kilometres was on a road and the rest was on a footpath.

The ratio road distance : footpath distance was 3:2.
(i) Work out the road distance.
(ii) Vreni walked along the road at $3 \mathrm{~km} / \mathrm{h}$ and along the footpath at $2.5 \mathrm{~km} / \mathrm{h}$.

How long, in hours and minutes, did Vreni take to walk the 20 kilometres?
(iii) Work out Vreni's average speed.
(iv) Vreni started at 0855 . At what time did she finish?
(c) On a map, the distance of 20 kilometres was represented by a length of 80 centimetres.

The scale of the map was $1: n$.
Calculate the value of $n$.

2 (a) (i) Factorise $x^{2}-x-20$.
(ii) Solve the equation $x^{2}-x-20=0$.
(b) Solve the equation $3 x^{2}-2 x-2=0$.

Show all your working and give your answers correct to 2 decimal places.
(c) $\quad y=m^{2}-4 n^{2}$.
(i) Factorise $m^{2}-4 n^{2}$.
(ii) Find the value of $y$ when $m=4.4$ and $n=2.8$.
(iii) $m=2 x+3$ and $n=x-1$.

Find $y$ in terms of $x$, in its simplest form.
(iv) Make $n$ the subject of the formula $y=m^{2}-4 n^{2}$.
(d) (i) $m^{4}-16 n^{4}$ can be written as $\left(m^{2}-k n^{2}\right)\left(m^{2}+k n^{2}\right)$.

Write down the value of $k$.
(ii) Factorise completely $m^{4} n-16 n^{5}$.

3 (a)


Bag A


Bag B

Nadia must choose a ball from Bag A or from Bag B.
The probability that she chooses Bag A is $\frac{2}{3}$.
Bag A contains 5 white and 3 black balls.
Bag B contains 6 white and 2 black balls.
The tree diagram below shows some of this information.

(i) Find the values of $p, q, r$ and $s$.
(ii) Find the probability that Nadia chooses Bag A and then a white ball.
(iii) Find the probability that Nadia chooses a white ball.
(b) Another bag contains 7 green balls and 3 yellow balls.

Sani takes three balls out of the bag, without replacement.
(i) Find the probability that all three balls he chooses are yellow.
(ii) Find the probability that at least one of the three balls he chooses is green.

4


200 people record the number of hours they work in a week.
The cumulative frequency graph shows this information.
(a) Use the graph to find
(i) the median,
(ii) the upper quartile,
(iii) the inter-quartile range,
(iv) the number of people who work more than 60 hours in a week.
(b) Omar uses the graph to make the following frequency table.

| Hours <br> worked $(h)$ | $0<h \leqslant 10$ | $10<h \leqslant 20$ | $20<h \leqslant 30$ | $30<h \leqslant 40$ | $40<h \leqslant 50$ | $50<h \leqslant 60$ | $60<h \leqslant 70$ | $70<h \leqslant 80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 12 | 34 | 36 | 30 | 38 | 30 | $p$ | $q$ |

(i) Use the graph to find the values of $p$ and $q$.
(ii) Calculate an estimate of the mean number of hours worked in a week.
(c) Shalini uses the graph to make a different frequency table.

| Hours worked (h) | $0<h \leqslant 30$ | $30<h \leqslant 40$ | $40<h \leqslant 50$ | $50<h \leqslant 80$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 82 | 30 | 38 | 50 |

When she draws a histogram, the height of the column for the interval $30<h \leqslant 40$ is 9 cm .

Calculate the height of each of the other three columns.


A circle, centre $O$, touches all the sides of the regular octagon $A B C D E F G H$ shaded in the diagram.
The sides of the octagon are of length 12 cm .
$B A$ and $G H$ are extended to meet at $P . H G$ and $E F$ are extended to meet at $Q$.
(a) (i) Show that angle $B A H$ is $135^{\circ}$.
(ii) Show that angle $A P H$ is $90^{\circ}$.
(b) Calculate
(i) the length of PH ,
(ii) the length of $P Q$,
(iii) the area of triangle $A P H$,
(iv) the area of the octagon.
(c) Calculate
(i) the radius of the circle,
(ii) the area of the circle as a percentage of the area of the octagon.


The pentagon $O A B C D$ is shown on the grid above.
(a) Write as column vectors
(i) $\overrightarrow{O D}$,
(ii) $\overrightarrow{B C}$.
(b) Describe fully the single transformation which maps the side $B C$ onto the side $O D$.
(c) The shaded area inside the pentagon is defined by 5 inequalities.

One of these inequalities is $y \leqslant \frac{1}{2} x+4$.

Find the other 4 inequalities.

7 (a)

$A, B, C$ and $D$ lie on a circle, centre $O$.
SCT is the tangent at $C$ and is parallel to $O B$.
Angle $A O B=130^{\circ}$, and angle $B C T=40^{\circ}$.
Angle $O B C=x^{\circ}$, angle $O B A=y^{\circ}$ and angle $A D C=z^{\circ}$.
(i) Write down the geometrical word which completes the following statement.

$$
\begin{equation*}
\text { " } A B C D \text { is a } \longrightarrow \text { quadrilateral." } \tag{1}
\end{equation*}
$$

(ii) Find the values of $x, y$ and $z$.
(iii) Write down the value of angle $O C T$.
(iv) Find the value of the reflex angle $A O C$.
(b)

$P, Q, R$ and $S$ lie on a circle.
$P Q=7 \mathrm{~cm}$ and $S R=10 \mathrm{~cm}$.
$P R$ and $Q S$ intersect at $X$.
The area of triangle $S R X=20 \mathrm{~cm}^{2}$.
(i) Write down the geometrical word which completes the following statement.
"Triangle $P Q X$ is
to triangle SRX."
(ii) Calculate the area of triangle $P Q X$.
(iii) Calculate the length of the perpendicular height from $X$ to $R S$.

8 Answer the whole of this question on a sheet of graph paper. Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4\% per year compound interest.
(a) How many dollars will Alaric have after 2 years?
(b) After $x$ years, Alaric will have $y$ dollars.

He knows a formula to calculate $y$.
The formula is $y=100 \times 1.04^{x}$

| $x$ (Years) | 0 | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ (Dollars) | 100 | $p$ | 219 | $q$ | 480 |

Use this formula to calculate the values of $p$ and $q$ in the table.
(c) Using a scale of 2 cm to represent 5 years on the $x$-axis and 2 cm to represent $\$ 50$ on the $y$-axis, draw an $x$-axis for $0 \leqslant x \leqslant 40$ and a $y$-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through them.
(d) Use your graph to estimate
(i) how many dollars Alaric will have after 25 years,
(ii) how many years, to the nearest year, it takes for Alaric to have $\$ 200$.
(e) Beatrice invests $\$ 100$ at $7 \%$ per year simple interest.
(i) Show that after 20 years Beatrice has $\$ 240$.
(ii) How many dollars will Beatrice have after 40 years?
(iii) On the same grid, draw a graph to show how the $\$ 100$ which Beatrice invests will increase during the 40 years.
(f) Alaric first has more than Beatrice after $n$ years.

Use your graphs to find the value of $n$.

$O P Q R$ is a parallelogram.
$O$ is the origin.
$\overrightarrow{O P}=\mathbf{p}$ and $\overrightarrow{O R}=\mathbf{r}$.
$M$ is the mid-point of $P Q$ and $L$ is on $O R$ such that $O L: L R=2: 1$.
The line $P L$ is extended to the point $S$.
(a) Find, in terms of $\mathbf{p}$ and $\mathbf{r}$, in their simplest forms,
(i) $\overrightarrow{O Q}$,
(ii) $\overrightarrow{P R}$,
(iii) $\overrightarrow{P L}$,
(iv) the position vector of $M$.
(b) $P L S$ is a straight line and $P S=\frac{3}{2} P L$.

Find, in terms of $\mathbf{p}$ and/or $\mathbf{r}$, in their simplest forms,
(i) $\overrightarrow{P S}$,
(ii) $\overrightarrow{Q S}$.
(c) What can you say about the points $Q, R$ and $S$ ?

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |

A 3 by 3 square

| $x$ | $b$ | $c$ |
| :--- | :--- | :--- |
| $d$ | $e$ | $f$ |
| $g$ | $h$ | $i$ | can be chosen from the 6 by 6 grid above.

(a) One of these squares is

| 8 | 9 | 10 |
| :---: | :---: | :---: |
| 14 | 15 | 16 |
| 20 | 21 | 22 |

In this square, $x=8, c=10, g=20$ and $i=22$.
For this square, calculate the value of
(i) $(i-x)-(g-c)$,
(ii) $c g-x i$.
(b)

| $x$ | $b$ | $c$ |
| :---: | :---: | :---: |
| $d$ | $e$ | $f$ |
| $g$ | $h$ | $i$ |

(i) $c=x+2$. Write down $g$ and $i$ in terms of $x$.
(ii) Use your answers to part(b)(i) to show that $(i-x)-(g-c)$ is constant.
(iii) Use your answers to $\operatorname{part}(\mathbf{b})(\mathbf{i})$ to show that $c g-x i$ is constant.
(c) The 6 by 6 grid is replaced by a 5 by 5 grid as shown.

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 |

A 3 by 3 square

| $x$ | $b$ | $c$ |
| :--- | :--- | :--- |
| $d$ | $e$ | $f$ |
| $g$ | $h$ | $i$ |

can be chosen from the 5 by 5 grid.

For any 3 by 3 square chosen from this 5 by 5 grid, calculate the value of
(i) $(i-x)-(g-c)$,
(ii) $c g-x i$.
(d) A 3 by 3 square is chosen from an $n$ by $n$ grid.
(i) Write down the value of $(i-x)-(g-c)$.
(ii) Find $g$ and $i$ in terms of $x$ and $n$.
(iii) Find $c g-x i$ in its simplest form.

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